THE CBO MILITARY RETENTION MODEL

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PREFACE

Over the next several years, the costs and effects of changes in military compensation--including such items as increased educational benefits, bonuses, and special pay raises--are likely to be major issues confronting the Congress. Models relating military compensation to personnel retention are key in determining the effects of such proposals. This report offers a technical description of one such military retention model, developed by the Congressional Budget Office. Several previous CBO analyses of military pay for House and Senate committees have used this model. In keeping with CBO's mandate to provide objective and nonpartisan analysis, the report makes no recommendations.

The report was prepared by Joel N. Slackman of CBO's National Security and International Affairs Division, under the general supervision of Robert F. Hale. Valuable comments and assistance on earlier drafts were provided by Glenn Gotz of the Rand Corporation; Richard V.L. Cooper, formerly of the Rand Corporation; and Michael Sullivan and Nancy Swope, both of the CBO staff. Statistical analyses were based on work prepared by the Defense Studies Department of Mathtech, Inc.; key data were provided by the Rand Corporation and the Defense Manpower Data Center. (Responsibility for the final report rests solely with CBO, however, and not with any external reviewer.) Francis Pierce edited the manuscript; Janet Stafford prepared the report for release.

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CHAPTER I. INTRODUCTION

Concern about the retention of active-duty military personnel has prompted numerous proposals to improve military pay and benefits. These proposals raise two key questions:

- Would these pay and benefit changes, if enacted, permit the services to achieve their desired career retention levels?
- What effect would they have on the experience mix within the force?

To measure the effect of changes in military compensation on reenlistment decisions, CBO has developed a military retention model. The model provides a computationally simple basis for analyzing the effect of pay changes on retention, both for officers and for enlisted personnel.

Chapter II describes the model in detail, discussing its general nature, empirical specification, and empirical analysis. Chapter III examines the predicted retention effects of illustrative compensation changes.

CHAPTER II. DESCRIPTION OF THE MODEL

The CBO military retention model predicts the effects on retention of future compensation changes by assuming that reenlistment decisions are motivated by military and civilian compensation over an individual's entire remaining career. The model was formulated using a weighted average of future pays, called "perceived pay," where the weights are both discount rates and the person's probability of remaining in the military (assumed here to equal the probability of retention for all individuals because this probability is known: it represents the average chance of one individual staying). 1/ These weights are necessary because the pull of future compensation is lower than that created by next year's pay, both because of a time preference for money and because of the possibility that an individual will leave military service before completing a full career. The detailed formulation of the model is discussed in the next section.

Although military compensation comprises myriad special pays and fringe benefits, the model captures the effects only of the largest compensation components: regular military compensation (RMC), selective reenlistment bonuses, and retirement pay. Smaller benefits were excluded because their value varies widely among individuals. Civilian opportunities were represented in the perceived pay formulation by including civilian pay from the time an individual leaves the military until the end of his or her normal working career, assumed to be at age 60.

Taken by itself, the perceived pay formulation is deficient in two important respects. First, by including only monetary values, it ignores such intangible, but critical, factors as an individual's preference, or "taste," for military service. Second, it ignores the effect of past compensation practices on

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1/ This formulation was suggested in a 1974 Rand Corporation study of military retention. See David L. Jaquette and Gary R. Nelson, The Implications of Manpower Supply and Productivity for the Pay and Composition of the Military Force: An Optimization Model, R-1451-ARPA (The Rand Corporation, July 1974), pp. 11-14.

the current population eligible for reenlistment. 2/ For example, personnel induced by a bonus to remain in the service at the first-term reenlistment point may not be as likely to stay beyond the next reenlistment point as those who would have remained even without the bonus. 3/ Omitting taste factors from the model, then, would bias the estimated effect of perceived pay on personnel retention.

To reduce this bias, the CBO model incorporates a taste factor, using as a crude proxy the cumulative probability of leaving the military. Past studies have used year of service, which is positively correlated with the mean of the taste distribution. Year of service increases at a constant rate, however, while the CBO model's proxy increases more rapidly in the earliest years of service, similarly to the mean of the taste distribution. 4/

Length of military service also depends on how long the services allow a person to continue on active duty. At the services' discretion, personnel can be separated involuntarily. The CBO model allows for such involuntary separations by assigning lower values to some retention rates. Because they reduce perceived pay, involuntary separations affect an individual's willingness to continue in the service in years preceding the year when the involuntary separations occur.

EMPIRICAL SPECIFICATION

Perceived pay (PP) can be thought of as the present value of the returns to staying in the military one more year (VM) plus the weighted average of the returns to staying or leaving in future years (VMC), divided by the present value of the returns to leaving immediately (VC). The notation "$PP_{k,k+n}$" denotes the

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2/ Based on unpublished research by John T. Warner of the Center for Naval Analyses.

3/ For a rigorous derivation of taste's effect, see Glenn A. Gotz and John J. McCall, Estimating Military Personnel Retention Rates: Theory and Statistical Method, R-2541-AF (The Rand Corporation, June 1980).

4/ Ibid., pp. 31-32.

perceived pay of an individual with "k" years of service (YOS) over a time horizon of "n" years. "$PP_{k,\infty}$" corresponds to an individual's perceived pay over his or her remaining years in the work force (until age 60). The full specification for perceived pay is:

$$(1) \qquad PP_{k,k+n} = \frac{VM_{k,n} + VMC_{k,n}}{VC_{k,n}}$$

where

$$(2) \qquad VM_{k,n} = \sum_{i=k}^{k+n} \left[\left(\prod_{m=k}^{i} C_m \right) \left(MIL_i + EW_i \right) \left(\frac{1+RW}{1+DR} \right)^{i-k} S_{i,k} \right]$$

$$(3) \qquad VMC_{k,n} = \sum_{i=k}^{k+n} \left\{ \left[1 - \left(\prod_{m=k}^{i} C_m \right) \right] \left[\left(CIV_i \left(\frac{1+RW}{1+DR} \right)^{i-k} + (R_{i,j}) \frac{(1+RW)^{t-k}}{(1+DR)^{i-k}} \right) S_{i,k} \right] \right\}$$

$$t = \begin{cases} i \text{ if } i = j \\ T \text{ if } i > j \quad (T = \text{YOS } 30) \end{cases}$$

$$(4) \qquad VC_{k,n} = \sum_{i=k}^{k+n} \left\{ \left[(CIV_i) \left(\frac{1+RW}{1+DR} \right)^{i-k} + R_{i,k}(1+DR)^{k-i} \right] S_{i,k} \right\}$$

where

C_m = the probability of continuing in the military from the YOS interval m-1 to m to the YOS interval m to m+1.

MIL_i = regular military compensation (RMC) earned in YOS i to i+1.

EW_i = early withdrawal or bonus money earned by completing YOS i in YOS i to i+1.

CIV_i = civilian salary earned from YOS-age i to i+1 (for enlisted personnel, YOS-age equals i+19; for officers, i+23).

$S_{i,k}$ = probability of survival until YOS-age i given survival at YOS k.

DR = real personal discount rate.

RW = real wage growth.

$R_{i,j}$ = annuity earned in YOS-age i given that one leaves the service after completing year of service j.

Some military personnel, particularly junior personnel, may place greater emphasis on near-term pay than is captured by the discount rate. Evidence of this emphasis on near-term pay comes both from studies of first-term reenlistment, which often ignore all pay beyond four years or so after the reenlistment year, and from intuition. 5/ The choice of a specific time horizon, though,

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5/ See, for example, John H. Enns, Reenlistment Bonuses and First-Term Retention, R-1935-ARPA (The Rand Corporation, September 1977), pp. 35-39.

is essentially arbitrary. (CBO found that no specific horizon yielded better statistical results than any other.) When appropriate, the model uses a near-term horizon of seven years, allowing those at the first reenlistment point to place emphasis on the increase in pay that could occur after 10 years of service under some alternative pay system. Persons with 20 or more years' service were assumed to have but one time horizon because, with 10 or fewer years typically remaining in their military career, they were assumed less likely to distinguish between near- and far-term pay.

The CBO model computes perceived pay under alternative compensation policies by using estimated probabilities of continuing in the military from one YOS to the next. Beginning at the last YOS, the model works recursively to predict new reenlistment rates (R_k) using a logistic supply equation: 6/

$$R_k = \frac{1}{[1 + e^{-(L_k)}]} \tag{5}$$

where

$$L_k = a_k + b_1 \ln(PP_{k,k+n}) + b_2 \ln(PP_{k+n+1,\infty}) + b_3 T_k, \text{ for } K<20 \tag{6}$$

$$L_k = a_k + b_4 \ln(PP_{k,\infty}) + b_5 T_k, \text{ for } K \geq 20 \tag{7}$$

$$T_k = 1 - \prod_{i=1}^{k} C_i \tag{8}$$

where

T_k = taste proxy

6/ The logistic form ensures that zero and one bound the reenlistment rate. Also, the elasticities of response to pay decline with increases in the reenlistment rate, which is likely to happen in real life.

The predicted reenlistment rates are then used to compute the continuation rates (C_m) in the perceived pay equations (2) and (3) in each preceding YOS:

$$(9) \quad C_k = \left[F_k \cdot R_k\right] + \left[(1-F_k) \cdot NR_k\right] \quad \text{(for enlisted personnel)}$$

where

F_k = the fraction of personnel eligible to reenlist in YOS k;

NR_k = the fraction of those not eligible to reenlist who remain in the military at YOS k (the non-reenlistment retention rate).

Most officers are eligible to leave the service each year. Since data about fractions of officers eligible to reenlist in YOS "k" were not available, CBO assumed that all officers could reenlist in each year, so that $C_k = R_k$.

The perceived pay computation requires knowledge of probabilities beyond YOS "k," hence the backward working approach. But expression (8), T_k, requires continuation probabilities before the kth YOS, necessitating an iterative approach to yield predicted effects of compensation changes. In the first iteration, present continuation rates are entered in equation (8). In the second iteration, the continuation rates previously computed are entered. The model continues to work recursively until the continuation rates computed under two successive iterations converge. 7/

7/ Convergence was assumed when, for each YOS,

$$\frac{C^{I+1} - C^I}{C^I} < \begin{cases} .0001 \text{ for enlisted} \\ .001 \text{ for officers} \end{cases}$$

where

C^I = the continuation rate under the Ith iteration.

The parameters of the model, specifically the constant terms in (6) and (7), are fitted so that the model perfectly predicts the historical base started with, here fiscal year 1977 retention rates. Thus, reenlistment supply curves actually vary from one year of service to the next. Although statistically unorthodox, adjusting the constant terms in such a manner norms the model to the present system and recent behavior. For purposes of policy analysis, identifying changes in the pattern of retention may be more important than having precise point estimates of continuation rates. Table A-1 in Appendix A shows the estimated constant terms by YOS, along with the reenlistment rates estimated using the unadjusted intercepts. The unadjusted regression equations generally overestimate actual enlisted retention, while underestimating actual officer retention.

EMPIRICAL ANALYSIS

Sources of Data

A 1976 survey of officer and enlisted personnel conducted by the Department of Defense (DoD) provided the key reenlistment rate data. 8/ The survey asked three questions relevant to this study:

- How long personnel intended to remain in the military under the current compensation system;
- How long they would remain if retirement pay for those leaving with between 20 and 30 years of service were reduced by one-third until the retiree reached age 50; and
- How long they would remain if those leaving with 10 to 19 years of service were to receive a deferred annuity beginning at age 60 equal to 2.5 percent of the retiree's basic pay times years of service.

The Rand Corporation pooled responses to these questions for all four services, eliminated inconsistent responses (16 percent for enlisted personnel and 3 percent for officers), and calculated reenlistment rates from the consistent results.

8/ U.S. Department of Defense, 1976 Personnel Survey, Form A (Questions 55, 71, and 72).

The survey data had an advantage over historical cross-section data in that the effects of a variety of pay incentives were measured. The personnel surveyed were confronted with the sorts of pay changes that are now being considered, such as increases in pay at 10 years of service; cross-section data, of course, do not offer such flexibility. Reliance on survey data does introduce some potential for measurement error, however. The survey responses are assumed to predict actual reenlistment rates. In fact, they may be biased by the knowledge by some personnel that they cannot reenlist in a given year. Thus, the survey data may be capturing "voting" behavior: a respondent can indicate a preference or dislike for a particular pay system change by saying he would stay or leave even though he might not do so if he had to "vote with his feet." These uncertainties are outweighed, however, by the variation in the explanatory variables that the survey allows, making possible more precise estimates of the model's parameters than can be derived from historical data.

Data on fractions of enlisted personnel eligible to reenlist in fiscal year 1977 were supplied by the Defense Manpower Data Center. (As was noted above, fractions of officers eligible to reenlist were assumed to equal 1.0 in all years of service.) Data on civilian earnings came from staff papers of the President's Commission on Military Compensation (PCMC), which provided median earnings of white, full-time workers by age and education. (High school graduate earnings were used for enlisted personnel; college graduate earnings, for officers.) 9/ The estimates assume that officers and enlisted personnel remain at the median pay grade for their year of service. Tables 1 and 2 summarize the main items of data.

In constructing the model, CBO also assumed that real wages would grow at an average rate of 1 percent a year. Real discount rates were assumed to equal 5 percent for officers and 7.5 percent for enlisted personnel. 10/

9/ John T. Warner, "Analysis of the Retention Impact of the Proposed Retirement System," Supplementary Papers of the President's Commission on Military Compensation (April 1978), p. B-4.

10/ The discount rates were based on unpublished research by Dr. Harry Gilman of the Center for Naval Analyses.

TABLE 1. DEPARTMENT OF DEFENSE HISTORICAL DATA FOR ENLISTED PERSONNEL, 1977

Year of Service	Continuation Rates (Those Not Eligible to Reenlist)	Fraction Eligible to Reenlist
0-1 to 1-2	0.862	0.001
2	0.890	0.081
3	0.899	0.405
4	0.986	0.578
5	0.948	0.148
6	0.953	0.203
7	0.962	0.243
8	0.973	0.337
9	0.974	0.241
10	0.975	0.217
11	0.977	0.255
12	0.982	0.349
13	0.982	0.173
14	0.984	0.134
15	0.987	0.160
16	0.989	0.344
17	0.990	0.124
18	0.980	0.104
19	0.980	0.104
20	0.603	0.562
21	0.765	0.372
22	0.767	0.229
23	0.804	0.483
24	0.866	0.357
25	0.894	0.212
26	0.791	0.588
27	0.800	0.498
28	0.827	0.378
29	0.800	0.264
30	0.614	0.788

SOURCE: Defense Manpower Data Center.

TABLE 2. DATA FROM 1976 DEPARTMENT OF DEFENSE PERSONNEL SURVEY

Year of Service	Officers			Enlisted Personnel		
	Current Retirement System	Reduced Annuity at 20	Vesting at 10	Current Retirement System	Reduced Annuity at 20	Vesting at 10
0-1 to 1-2	0.991	0.991	0.991	0.955	0.955	0.955
2	0.806	0.795	0.808	0.629	0.619	0.656
3	0.861	0.847	0.869	0.572	0.553	0.609
4	0.836	0.785	0.860	0.455	0.416	0.493
5	0.870	0.809	0.896	0.955	0.947	0.963
6	0.871	0.826	0.911	0.814	0.782	0.855
7	0.946	0.909	0.972	0.934	0.915	0.951
8	0.958	0.922	0.981	0.886	0.848	0.924
9	0.983	0.962	0.992	0.958	0.933	0.975
10	0.953	0.917	0.851	0.954	0.916	0.840
11	0.987	0.971	0.980	0.987	0.972	0.974
12	0.989	0.977	0.976	0.987	0.962	0.966
13	0.996	0.985	0.993	0.997	0.987	0.994
14	0.996	0.986	0.992	0.997	0.986	0.994
15	0.997	0.987	0.987	0.998	0.991	0.988
16	1.000	0.994	0.997	0.999	0.991	0.996
17	1.000	0.998	0.999	1.000	0.998	0.998
18	1.000	0.999	0.999	0.999	0.998	0.999
19	1.000	1.000	1.000	0.996	0.995	0.997
20	0.564	0.573	0.503	0.323	0.507	0.285
21	0.955	0.958	0.964	0.841	0.921	0.877
22	0.849	0.935	0.875	0.752	0.910	0.792
23	0.933	0.970	0.944	0.867	0.964	0.900
24	0.864	0.947	0.877	0.830	0.951	0.863
25	0.725	0.849	0.746	0.827	0.876	0.830
26	0.689	0.892	0.725	0.651	0.922	0.753
27	0.940	0.956	0.950	0.936	0.990	0.968
28	0.845	0.926	0.885	0.930	0.984	0.956
29	0.960	0.984	0.978	0.983	0.996	0.993

SOURCE: The Rand Corporation.

Certain years were omitted from the empirical analysis. The first few years (two years for enlisted personnel, three for officers) were excluded because reenlistment rates in these years are determined as much by service policies as by willingness to remain in the military. Years near 20 were omitted because reenlistment rates were often one or so near to one (reflecting the "pull-to-20" effect of today's retirement system) that the left-hand side of the model was undefined or extremely large.

Estimating Model Parameters

After substituting expressions (6) and (7) into (5), a simple logarithmic transformation yields:

$$(10) \qquad \ln\left(\frac{R_k}{1-R_k}\right) = a_k + b_1 \ln(PP_{k,k+n}) + b_2 \ln(PP_{k+n+1,\infty}) + b_3 T_k, \quad k<20$$

$$(11) \qquad \ln\left(\frac{R_k}{1-R_k}\right) = a_k + b_4 \ln(PP_{k,\infty}) + b_5 T_k, \quad k \geq 20$$

Linear techniques are appropriate for estimating the model's parameters. By construction of the dependent variable, however, the error terms display heteroscedasticity. 11/ To improve the

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11/ The individual "logits" $\left[\ln\left(\frac{R_k}{1-R_k}\right)\right]$ have a sample variance estimated by:

$$Var_k = \frac{1}{n_k \cdot R_k \cdot (1-R_k)}$$

Thus, the estimator of the true model parameter is given by:

efficiency of the parameter estimates, the observations were weighted by a corrective factor, so that equations (10) and (11) appear as:

$$(12) \qquad WT \cdot \ln\left(\frac{R_k}{1-R_k}\right) = a_k \cdot WT + b_1 \ln(PP_{k,k+n}) \cdot WT$$

$$+ b_2 \ln(PP_{k+n+1,\infty}) \cdot WT + b_3 T_k \cdot WT, \quad k<20$$

$$(13) \qquad WT \cdot \ln\left(\frac{R_k}{1-R_k}\right) = a_k \cdot WT + b_4 \ln(PP_{k,\infty}) \cdot WT$$

$$+ b_5 T_k \cdot WT, \quad k \geq 20$$

where

$WT = R_k(1-R_k)n_k$

n_k = sample size

$$b = (X' \sum_e^{-1} X)^{-1} X' \sum_e^{-1} \text{logit}(R)$$

where

$\sum_e$ = a diagonal matrix with elements $\frac{1}{n_k \cdot R_k \cdot (1-R_k)}$

See Arnold Zellner and Tong Hun Lee, "Joint Estimation of Relationships Involving Discrete Random Variables," Econometrica (April 1965), p. 386.

The final regression results are summarized in Table 3; their derivations are discussed below.

TABLE 3. SUMMARY OF ESTIMATED COEFFICIENTS

	PPNT a/	PPFT	PP	T
Enlisted Personnel				
Junior	8.06	4.00	--	1.94
Senior	---	---	8.74	17.26
Officers				
Junior	---	---	8.95	3.61
Senior	---	---	6.74	7.28

Where

$$PPNT = WT \cdot \ln(PP_{k,k+n})$$
$$PPFT = WT \cdot \ln(PP_{k+n+1,\infty})$$
$$PP = WT \cdot \ln(PP_{k,\infty})$$
$$T = WT \cdot T_k$$

a/ Assumes near-term horizon of seven years.

Results for Junior Enlisted Personnel. The weighted least squares (WLS) method described above was applied to the junior enlisted subset. Forty-two observations were retained, corresponding to YOS 3 through 16 for each of the three survey options.

Severe multicollinearity rendered the WLS estimates nonsensical. Regardless of the time horizon used, the estimates for "b_1" (the near-term effect of perceived pay) were negative, as well as statistically insignificant. Table 4, showing the correlation matrix for key variables, strongly suggests that ill-conditioned data confounded estimation.

One response to debilitating multicollinearity is to relax the requirement of unbiasedness by using ridge regression. Ridge regression is based upon use of the matrix (X'X + kI) instead of

TABLE 4. CORRELATION MATRIX FOR KEY VARIABLES (AFTER WEIGHTING)

	Near-Term Perceived Pay	Far-Term Perceived Pay	Taste
Near-Term Perceived Pay	1.00	0.89	0.81
Far-Term Perceived Pay	0.89	1.00	0.71
Taste	0.81	0.71	1.00

X'X, where "k" represents a positive "bias" introduced into the data to obtain a smaller mean square error. Although no straightforward solution for determining the optimal "k" exists, the value at which the coefficients stabilize given increasing values of "k" is often taken to give the desired set of coefficients. 12/

Regression coefficients for various "k" values are shown in Table 5. The "k = 0" results match the initial WLS estimates. As expected, the near-term perceived pay coefficient changed sign as bias was introduced, increasing in size for smaller values of "k," then declining after "k" reached a value of 0.60. In this type of situation, earlier research suggests that good results are obtained by setting the value of "k" about where the coefficient passes through the maximum absolute value. 13/

Results for Senior Enlisted Personnel. Observations corresponding to YOS 20 displayed patterns different from observations corresponding to YOS 21 to 29, which was not surprising given that YOS 20 is a retirement system milestone. The introduction of a dummy constant term taking the value of 1.0 if the observation matched the 20th YOS sufficed to model this behavioral difference. The resulting equation, with t-values in parentheses, is:

12/ Donald W. Marquardt and Ronald D. Snee, "Ridge Regression in Practice," The American Statistician (February 1975), p. 11.

13/ Ibid., p. 12.

TABLE 5. RIDGE REGRESSION COEFFICIENTS FOR JUNIOR ENLISTED PERSONNEL (t-ratios in parentheses) a/

k	b_1	b_2	b_3
0.00	-2.54 (-0.24)	7.65 (2.22)	3.44 (2.94)
0.05	2.43 (0.29)	6.28 (2.20)	3.02 (2.91)
0.10	4.70 (0.63)	5.63 (2.18)	2.78 (2.84)
0.20	6.73 (1.06)	4.98 (2.20)	2.48 (2.78)
0.30	7.55 (1.34)	4.62 (2.23)	2.29 (2.74)
0.40	7.90 (1.52)	4.37 (2.26)	2.15 (2.71)
0.50	8.04 (1.65)	4.17 (2.28)	2.03 (2.68)
0.60 b/	8.06 (1.74)	4.00 (2.30)	1.94 (2.65)
0.65	8.05 (1.78)	3.93 (2.30)	1.89 (2.64)
0.70	8.02 (1.81)	3.86 (2.30)	1.85 (2.62)

a/ Based on 42 observations, and a near-term time horizon of seven years. The t-ratios apply to standardized ridge regression coefficients.

b/ "Optimal" k value.

$$WT \cdot \ln\left(\frac{R_k}{1-R_k}\right) = \underset{(4.04)}{8.74 \cdot \ln(PP_{k,\infty}) \cdot WT} + \underset{(3.76)}{17.26 \cdot T_k \cdot WT}$$

$$\underset{(-3.8)}{- 16.92 \cdot WT} \underset{(-5.5)}{- 1.58 \cdot D \cdot WT}$$

where

$$D = \begin{cases} 1 \text{ if } YOS = 20 \\ 0 \text{ if } YOS \neq 20 \end{cases}$$

Results for Junior Officers. Analogously to the enlisted regressions, observations for YOS 1 to 4 and YOS 17 to 19 were omitted, leaving 12 observations for each of the three survey options (YOS 5 to 16).

Table 6 summarizes the WLS results for various time horizons. Coefficients show the desired signs, but far-term perceived pay is statistically insignificant, its effect swamped by the near-term variable. This should be interpreted cautiously, since low t-statistics can result either because the true coefficient is close to zero or because the estimated coefficient has a high variance due to multicollinearity. Yet ridge regressions also yielded similar results, suggesting that for junior officers--unlike for junior enlisted personnel--the discount rate alone suffices to emphasize near-term pay. Insofar as time preference for money is inversely related to socioeconomic status and measured ability, one would expect officers to be less sensitive than enlistees to distinctions between near-term and far-term pay.

Thus near- and far-term perceived pay was replaced by perceived pay over all future years, giving:

$$WT \cdot \ln\left(\frac{R_k}{1-R_k}\right) = \underset{(3.37)}{8.95 \cdot \ln(PP_{k,\infty}) \cdot WT} + \underset{(3.21)}{3.61 \cdot T_k \cdot WT} - 6.24 \cdot WT$$

(The t-values are listed under their respective coefficients.)

TABLE 6. WLS COEFFICIENTS FOR JUNIOR OFFICERS (t-ratios in parentheses)

Near-Term Horizon	b_1	b_2	b_3	F-value
Five Years	15.29 (3.86)	3.48 (1.30)	3.78 (3.77)	187.12
Six Years	12.82 (3.06)	2.20 (1.01)	3.13 (2.92)	163.81
Seven Years	11.14 (2.93)	1.53 (0.65)	2.80 (2.51)	158.02

Results for Senior Officers. Regressions using all 30 observations gave meaningless results, largely because observations corresponding to YOS 20 again displayed aberrant patterns. Nor did introducing a dummy constant term equaling 1.0 for YOS 20 improve results. Consequently, the final WLS regression omitted the 20th YOS observation for each of the three survey options, giving:

$$WT \cdot \ln\left(\frac{R_k}{1-R_k}\right) = \underset{(2.52)}{6.74} \cdot \ln(PP_{k,\infty}) \cdot WT + \underset{(1.25)}{7.28} \cdot T_k \cdot WT - 6.12 \cdot WT$$

(The t-values are indicated in parentheses.) The estimates match prior expectations and display statistical significance.

CHAPTER III. RETENTION MODEL SIMULATIONS

To demonstrate the retention model's predictive effects, CBO simulated the model for a variety of compensation changes, ranging from targeted pay raises to a structural overhaul of the retirement system. CBO considered the following pay changes:

- A 10 percent increase in first-term pay (approximated by raising pay in YOS 3 to 6);
- A 10 percent increase in second-term pay (YOS 7 to 10);
- A 10 percent increase in third-term pay (YOS 11 to 14);
- A 10 percent increase in senior careerists' pay (YOS 9 to 30);
- A 10 percent across-the-board pay raise; and
- A two-tiered retirement reform plan, as proposed by the Uniformed Services Retirement Benefits Act (USRBA).

The following sections show the effects of these compensation changes both on reenlistment rates (R_k) and on survival rates (S_k), which are cumulative continuation rates indicating how many individuals would remain in the military from YOS 1 to YOS interval "k-1" to "k" were the system in steady state. For the sake of brevity, only the results for enlisted personnel are reported. Officer results, which are presented in Appendix B, were similar to response patterns for enlisted personnel in all but one case--reform of the retirement system.

EFFECTS OF CHANGES IN THE MILITARY PAY TABLE

Table 7 summarizes the effects of various types of RMC increases on retention of enlisted personnel. (Results for each YOS are presented in Appendix B.)

In general, the model predicts reenlistment rate increases only before and during the years in which pay is raised. Thereafter, reenlistment rates decline with YOS (relative to the

TABLE 7. EFFECTS ON ENLISTED RETENTION OF 10 PERCENT INCREASES IN PAY

	Year of Service								Average Man-Years per Accession
	3	4	7	11	15	20	21	26	
				Reenlistment Rates					
Base a/	.273	.308	.607	.823	.938	.365	.505	.483	4.73
First-Term Pay Raise	.352	.373	.594	.819	.937	.324	.494	.481	4.98
Second-Term Pay Raise	.314	.372	.696	.816	.936	.295	.487	.480	5.07
Third-Term Pay Raise	.283	.323	.675	.879	.936	.301	.488	.480	4.95
Career Pay Raise	.278	.473	.797	.918	.970	.189	.497	.498	5.67
Across-the-Board Pay Raise	.476	.508	.783	.914	.969	.140	.485	.496	6.12
				Survival Rates					
Base a/	.716	.462	.204	.126	.105	.095	.044	.011	4.73
First-Term Pay Raise	.716	.485	.230	.140	.116	.105	.047	.011	4.98
Second-Term Pay Raise	.716	.474	.230	.151	.125	.113	.049	.012	5.07
Third-Term Pay Raise	.716	.465	.212	.144	.123	.111	.048	.012	4.95
Career Pay Raise	.716	.464	.259	.195	.174	.160	.059	.015	5.67
Across-the-Board Pay Raise	.716	.521	.298	.234	.208	.191	.065	.016	6.12

NOTE: The simulations were subject to the constraints:

$$C_1 = .8614$$
$$C_2 = .8312$$
$$C_{30} \leq .2199$$

a/ Fiscal year 1977 continuation rates.

fiscal year 1977 base rates) at a diminishing rate, until the 20th year of service. Then reenlistment rates fall sharply, reflecting the weaker taste for military service held by personnel who were induced to remain in the service by earlier pay changes.

The general pattern for the effects of targeted raises is illustrated by the first-term pay increase case. Changes at the third and fourth YOS imply pay elasticities ranging from 2.1 to 2.9. By way of comparison, past studies found that first-term pay elasticities ranged from 2.0 to 6.0, with results concentrated around 2.0 to 3.0. 1/ In YOS 7 to 19, reenlistment rates decrease by diminishing percentages, from 2.2 percent to 0.04 percent. Unlike the reenlistment rates, the survival probabilities remain roughly 10 percent above the base rates through the fourth enlistment term. At YOS 20, however, most of the additional continuation induced by the earlier pay raise disappears: R_{20} decreases by 11 percent (translating into a continuation rate decline of about 5 percent). After YOS 20, the pattern of survival rates is very similar to that for the base year.

Of the three targeted raises considered, the second-term pay increase elicited the largest improvement in average man-years per accession, increasing by 7 percent over the base, to 5.07 years. In fact, more individuals would stay in the service to the third and fourth terms under the second-term pay raise case than under the third-term pay raise case.

A 10 percent increase in the entire career pay stream generated first-term retention increases as well as increases in later terms. Assuming that the services would not limit career continuation rates, the survival rate to YOS 20 would equal 0.160, versus the current rate of 0.095.

Not surprisingly, a 10 percent increase in the entire pay stream resulted in the largest retention improvements. The YOS 4 reenlistment rate rose from 0.308 to 0.508, implying a pay elasticity with respect to across-the-board raises of about 6.5. This is larger than the effects predicted with two other models; but, in the absence of apriority, these results should not be

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1/ John H. Enns, Reenlistment Bonuses and First-Term Retention, R-1935-ARPA (The Rand Corporation, September 1977).

dismissed as unreasonable. 2/ If, as the perceived pay framework posits, retention decisions are motivated by compensation over an individual's entire remaining career, then a pay change over the whole future pay stream should exert a strong effect on junior personnel.

EFFECTS OF THE UNIFORMED SERVICES RETIREMENT BENEFITS ACT

Significant Features

This retirement reform proposal, submitted to the Congress in 1979 as part of the Department of Defense's legislative program, would have established a two-tiered deferred compensation plan to replace the current single-level lifetime annuity system. Under USRBA, individuals would have been vested in an old-age annuity after completing 10 years of service, and would have been eligible to receive an immediate pre-old-age annuity from the date of retirement after 20 or more years of service until age 60. Members would have been entitled to withdraw portions of the accrued deferred compensation in a lump sum between their 10th and 15th years of service. If they reached 20 years of service without having paid back the cash, their pre-old-age annuity would have been reduced accordingly. For a member who separated before completing 20 years, the cash withdrawals would have replaced the entitlement to the old-age annuity. A Social Security offset would have been imposed on old-age retirement pay at age 65. (Appendix C discusses two key assumptions underlying the simulation: one concerns cash withdrawal rates; the other, fractions eligible to reenlist.)

2/ An Annualized Cost-of-Leaving Model (ACOL) developed by the Center for Naval Analyses predicts a first-term elasticity of 2.7 with respect to pay increases over the entire pay table. (This result is not much greater than presently predicted responses to one-term bonus increases.) A later version of the cost-of-leaving model, called SCOL (Stochastic COL), implies a short-run pay elasticity of 4.2 and a smaller steady-state elasticity of 3.6. (Based on unpublished research by John T. Warner of the Center for Naval Analyses.)

Retention Effects

The model predicts sizable reenlistment rate increases for enlisted members with between four and ten years of service, due largely to the plan's early cash withdrawal feature that would have encouraged people to "take their money and run." Reflecting the decreased attraction of staying for 20 years of service, mid-career reenlistment rates declined by amounts ranging from 2 to 17 percent. Rates then improved for those staying past YOS 21. Results for several YOS are presented in Table 8.

The net effect of these changes would have been an overall increase of 5.9 percent in enlisted retention from the current system. Despite declining reenlistment rates after YOS 10, CBO's model predicts that the USRBA would have more than offset those declines by increases previous to the tenth year of service.

TABLE 8. EFFECTS OF THE UNIFORMED SERVICES RETIREMENT BENEFITS ACT ON ENLISTED RETENTION

	Year of Service								Average Man-Years per Accession
	3	4	7	11	15	20	21	26	
Reenlistment Rates									
Base	.273	.308	.607	.823	.938	.365	.505	.483	4.73
USRBA	.226	.343	.728	.824	.873	.321	.491	.624	5.00
Survival Rates									
Base	.716	.462	.204	.126	.105	.095	.044	.011	4.73
USRBA	.716	.448	.212	.159	.129	.103	.046	.012	5.00

APPENDIXES

APPENDIX A. ACTUAL AND ESTIMATED REENLISTMENT RATES

Chapter II noted that, before fitting the retention model to predict actual fiscal year 1977 behavior, its empirically derived regression equations overpredicted actual enlisted retention and underpredicted officer retention. Table A-1 shows these over- and underpredictions, together with actual fiscal year 1977 reenlistment rates and the adjusted constant terms.

Estimates of enlisted retention most severely overpredicted results in the early and later years of service, while approaching quite closely the mid-career rates. Interestingly, the 1976 DoD survey responses display a similar pattern (see Table 2), suggesting that measurement error could be contributing to the overpredictions (because survey responses may reflect knowledge by some personnel that they cannot reenlist in a given year).

As in the enlisted results, errors in predicting officer retention rates were most severe in the early and later years. Actual retention rates in the first few YOS may be determined largely by service policies, which could account for some of the underprediction. The officer survey responses, however, do not show a like pattern.

An additional complication arose in the construction of the taste proxy. The survey responses to questions asking about alternative compensation systems reflect the taste distribution in effect at the time the survey was conducted, not the distribution that would exist were the alternative system in steady state. Thus, survey-derived reenlistment rates may be biased either below or above the "true" reenlistment rates. This could, of course, bias regression estimates of the taste coefficient. Nevertheless, survey responses appear reasonably consistent with prior expectations of behavior under the alternative systems, so that the magnitude of the bias may not be too severe.

TABLE A-1. ACTUAL AND ESTIMATED FISCAL YEAR 1977 REENLISTMENT RATES

YOS	Enlisted Personnel			Officers		
	Actual Reenlistment Rates	Estimated Reenlistment Rates	Fitted Intercepts	Actual Reenlistment Rates	Estimated Reenlistment Rates	Fitted Intercepts
0-1 to 1-2	.212	.263	-3.43	.999	.305	1.49
2	.164	.325	-4.04	.868	.351	-3.74
3	.273	.429	-3.83	.858	.531	-4.57
4	.308	.614	-4.42	.889	.687	-4.94
5	.458	.721	-4.27	.903	.782	-5.28
6	.441	.767	-4.57	.903	.843	-5.69
7	.607	.804	-4.12	.930	.887	-5.72
8	.642	.836	-4.19	.955	.913	-5.54
9	.635	.870	-4.49	.976	.930	-5.13
10	.708	.895	-4.41	.971	.943	-5.54
11	.823	.919	-4.04	.955	.956	-6.26
12	.886	.934	-3.75	.982	.966	-5.58
13	.863	.949	-4.21	.997	.972	-3.99
14	.890	.962	-4.29	.992	.977	-5.18
15	.938	.973	-4.03	.984	.982	-6.11
16	.973	.982	-3.57	.990	.986	-5.89
17	.948	.989	-4.70	.991	.989	-6.05
18	.946	.993	-5.25	.995	.991	-5.69
19	.946	.996	-5.68	.995	.993	-5.93
20	.365	.706	-18.34	.720	.722	-6.09
21	.505	.846	-18.60	.789	.722	-6.09
22	.581	.849	-18.32	.825	.760	-6.10
23	.611	.865	-18.32	.843	.720	-5.72
24	.710	.820	-17.54	.854	.733	-5.70
25	.790	.832	-17.19	.857	.740	-5.71
26	.483	.748	-18.07	.818	.664	-5.63
27	.549	.753	-17.83	.820	.665	-5.62
28	.461	.628	-17.59	.796	.651	-5.71
29	.507	.597	-17.27	.779	.633	-5.74
30	.114	.549	-19.16	.592	.609	-6.52

APPENDIX B. PREDICTED ENLISTMENT AND CONTINUATION RATES

This appendix contains tables showing predicted reenlistment and continuation rates under various pay changes for enlisted and officer personnel in each YOS. For each of the 10 percent pay raises, officer results parallel enlisted results. But because current officer retention is relatively high, officer pay elasticities are lower.

In contrast, the predicted effects of the USRBA are less believable for officers than for enlisted personnel. Despite the promise of early cash withdrawals after 10 years of service, officer continuation rates worsen before YOS 10. Differences in management polices for enlisted and officer personnel might account for the model's anomalous predictions. The known promotion points and "up-or-out" rules distinguishing officer management mean that service policies affect the pattern of officer retention more than they affect enlisted retention patterns. Yet the model cannot take account of likely personnel management changes resulting from structural overhauls of the compensation system--an imperfection especially weakening for officer predictions.

TABLE B-1. PREDICTED EFFECTS OF PAY CHANGES ON REENLISTMENT RATES FOR ENLISTED PERSONNEL

YOS	Fiscal Year 1977 Rate	10 Percent Pay Raise					
		First Term	Second Term	Third Term	Career	Across-the-Board	USRBA
1	.212	.251	.224	.216	.192	.364	.164
2	.164	.204	.181	.169	.159	.305	.125
3	.273	.352	.314	.283	.278	.476	.226
4	.308	.373	.372	.324	.473	.508	.343
5	.458	.501	.534	.492	.656	.653	.525
6	.441	.458	.529	.494	.667	.645	.547
7	.607	.594	.696	.675	.797	.783	.728
8	.642	.631	.706	.724	.818	.806	.767
9	.635	.626	.677	.723	.811	.801	.768
10	.708	.701	.722	.788	.855	.848	.835
11	.823	.819	.816	.879	.918	.914	.824
12	.886	.883	.882	.916	.948	.946	.867
13	.863	.860	.858	.887	.936	.934	.814
14	.890	.888	.887	.899	.948	.946	.822
15	.938	.937	.936	.937	.970	.969	.873
16	.973	.973	.972	.972	.987	.986	.924
17	.948	.947	.947	.947	.973	.972	.841
18	.946	.946	.945	.945	.971	.970	.814
19	.946	.946	.945	.945	.969	.968	.787
20	.365	.324	.295	.301	.189	.140	.321
21	.505	.494	.487	.488	.497	.485	.491
22	.581	.575	.571	.572	.591	.584	.625
23	.611	.607	.604	.605	.631	.627	.642
24	.710	.708	.706	.706	.726	.724	.799
25	.790	.788	.787	.788	.802	.801	.830
26	.483	.481	.480	.480	.498	.496	.624
27	.549	.548	.547	.547	.567	.566	.639
28	.461	.460	.460	.460	.473	.472	.635
29	.507	.507	.506	.506	.514	.514	.603
30	.114	.114	.114	.114	.116	.116	.128

TABLE B-2. PREDICTED EFFECTS OF PAY CHANGES ON CONTINUATION RATES FOR ENLISTED PERSONNEL

YOS	Fiscal Year 1977 Rate	10 Percent Pay Raise					
		First Term	Second Term	Third Term	Career	Across-the-Board	USRBA
1	.861	.861	.861	.861	.861	.861	.861
2	.831	.831	.831	.831	.831	.831	.831
3	.645	.677	.662	.650	.648	.728	.626
4	.594	.631	.631	.603	.690	.710	.615
5	.875	.882	.887	.881	.905	.904	.885
6	.849	.853	.867	.860	.895	.890	.871
7	.876	.873	.897	.892	.922	.919	.913
8	.861	.858	.883	.889	.921	.917	.938
9	.892	.890	.902	.913	.935	.932	.951
10	.917	.916	.920	.934	.949	.947	.919
11	.937	.937	.936	.952	.962	.961	.940
12	.949	.948	.947	.959	.970	.969	.964
13	.962	.961	.961	.966	.974	.974	.957
14	.971	.971	.971	.973	.979	.979	.936
15	.979	.979	.979	.979	.984	.984	.948
16	.983	.983	.983	.983	.988	.988	.980
17	.985	.985	.985	.985	.988	.988	.970
18	.976	.976	.976	.976	.979	.979	.935
19	.976 a/	.976	.976	.976	.979	.979	.948
20	.469 a/	.446	.430	.433	.370	.343	.444
21	.669	.664	.662	.662	.665	.661	.663
22	.724	.723	.722	.722	.727	.725	.734
23	.711	.709	.707	.708	.721	.718	.726
24	.810	.809	.809	.809	.816	.815	.842
25	.872	.872	.871	.872	.875	.874	.880
26	.610	.609	.608	.608	.619	.617	.693
27	.675	.674	.674	.674	.684	.684	.720
28	.689	.688	.688	.688	.693	.693	.755
29	.723	.723	.722	.722	.725	.724	.748
30	.220	.220	.220	.220	.220	.220	.220
Average Man-Years per Accession	4.73	4.98	5.07	4.95	5.67	6.12	5.00

NOTE: Simulations were subject to the following constraints:

$C_1 = .8614$

$C_2 = .8312$

$C_{30} \leq .2199$

a/ The actual retention rates for YOS 19 (.869) and YOS 20 (.527) were changed to remove 19-year retirements, while maintaining identical cumulative continuation: (.869)(.527) = (.976)(.469).

TABLE B-3. PREDICTED EFFECTS OF PAY CHANGES ON OFFICER CONTINUATION RATES

YOS	Fiscal Year 1977 Rates	10 Percent Pay Raise					
		First Term	Second Term	Third Term	Career	Across-the-Board	USRBA
1	.999	.999	.999	.999	.999	.999	.999
2	.868	.868	.868	.868	.868	.868	.868
3	.858	.858	.858	.858	.858	.858	.858
4	.889	.897	.898	.898	.944	.948	.792
5	.903	.906	.909	.909	.943	.946	.862
6	.903	.903	.909	.909	.939	.939	.878
7	.930	.928	.934	.934	.954	.953	.915
8	.955	.954	.956	.958	.969	.969	.946
9	.976	.976	.976	.977	.983	.983	.971
10	.971	.971	.970	.973	.980	.979	.966
11	.955	.954	.953	.958	.968	.967	.935
12	.982	.982	.981	.983	.987	.987	.973
13	.997	.997	.997	.997	.998	.998	.995
14	.992	.992	.992	.992	.994	.994	.986
15	.984	.984	.983	.983	.988	.988	.969
16	.990	.990	.990	.989	.992	.992	.979
17	.991	.991	.991	.991	.993	.993	.981
18	.995	.995	.995	.995	.996	.996	.989
19	.995	.995	.995	.995	.996	.996	.988
20	.720	.716	.706	.699	.588	.584	.720
21	.789	.789	.788	.787	.806	.807	.789
22	.825	.825	.825	.824	.840	.840	.923
23	.843	.843	.843	.843	.851	.851	.944
24	.854	.854	.854	.854	.860	.860	.921
25	.857	.857	.857	.857	.862	.862	.895
26	.818	.818	.818	.818	.818	.818	.818
27	.820	.820	.820	.820	.823	.823	.879
28	.796	.796	.796	.796	.796	.796	.828
29	.779	.779	.779	.779	.780	.780	.804
30	.592	.592	.592	.592	.591	.591	.592
Average Man-Years per Accession	10.86	10.92	11.02	10.90	12.24	12.27	9.38

NOTE: Officer continuation rates equal reenlistment rates, subject to the following constraints:

$C_1 = .999$ $C_{20} \leq .720$ $C_{26} \leq .818$

$C_2 = .868$ $C_{21} \leq .789$ $C_{28} \leq .796$

$C_3 = .858$ $C_{30} \leq .592$

APPENDIX C. ASSUMPTIONS UNDERLYING THE USRBA SIMULATION

CASH WITHDRAWAL RATES

An important assumption underlying the USRBA analysis concerns the rate at which persons withdraw their cash payments. To begin, CBO assumed that they behave optimally. Since the early withdrawal benefits are computed on the basis of pay at the time of election, members might choose to "game" the system in such a way as to maximize the present value of their early cash benefits. The optimal withdrawal strategy was approximated by computing the present value of the cash withdrawal payments for three options: in the first, members take their early benefits as soon as possible; in the second option, they wait until the 15th year of service to withdraw cash; in the last option, the first lump-sum withdrawal is not made until the completion of 12 years of service.

Table C-1 shows the present values of early cash benefits for officer and enlisted personnel under each option. The present value computations indicate that officers and enlisted personnel behave optimally when they withdraw their early cash benefits as soon as possible. Thus, CBO assumed that the maximum allowable payment is withdrawn each year.

FRACTIONS ELIGIBLE TO REENLIST

Another key assumption relates to the fraction of enlisted personnel eligible to reenlist. Under the USRBA, more personnel would be likely to time their reenlistment to occur after 10 years of service. On the other hand, fewer would probably reenlist in years just before and after the 10th year. To reflect these likely changes, CBO modified the fractions reenlisting between 7 and 10 years. CBO also altered most of the fractions eligible to reenlist in the 11th to 19th year-of-service interval. These changes are shown in Table C-2.

The alterations were based on service predictions of fractions likely to reenlist. CBO modified the service predictions in two ways: first, a weighted average of the service estimates was computed; second, the estimates were adjusted for the likely

TABLE C-1. PRESENT VALUE OF EARLY CASH BENEFITS FOR DIFFERENT WITHDRAWAL RATES (In discounted dollars)

Option a/	Enlisted Personnel b/	Officers c/
Withdraw As Soon As Possible	14,800	31,450
Withdraw After YOS 15	12,530	31,230
Withdraw After YOS 12	14,220	30,120

a/ Fiscal year 1979 was the base year. The discount rates were 7.5 and 5.0 percent for enlisted personnel and officers, respectively. Real pay was assumed to grow 1.0 percent annually.

b/ Enlisted promotions were assumed to be to E-6 at 10 years of service, and to E-7 after 15 years of service.

c/ The officer promotion schedule assumed that officers are promoted to O-4 at 10 years of service, and to O-5 after 15 years of service.

frequency of reenlistment. Modifications differ for the 7th to 19th, and for the 20th or more, year-of-service intervals.

Fractions in the 7th to 19th year-of-service interval were derived by first weighting the different predictions made by each service (the weights being the proportion of enlisted men in each service), plus an adjustment. The adjustment was an amount sufficient to ensure that the weighted average of current service fractions for fiscal year 1977 equalled the forcewide fractions for fiscal year 1977.

Each service implicitly assumed that the DoD plan would lead to increases in the frequency of reenlistment. As no strong *a priori* reason exists to support this assertion, the base case assumed that reenlistment frequencies remained at their fiscal

TABLE C-2. FRACTIONS OF ENLISTED PERSONNEL ELIGIBLE TO REENLIST

Year of Service	Fiscal Year 1977 System	USRBA a/
1-6	b/	c/
7	0.243	0.210
8	0.337	0.168
9	0.241	0.112
10	0.217	0.404
11	0.255	0.240
12	0.349	0.157
13	0.173	0.148
14	0.134	0.296
15	0.160	0.338
16	0.344	0.138
17	0.124	0.136
18	0.104	0.271
19	0.104	0.168
20 on	b/	c/

a/ CBO estimate based on service predictions.

b/ See Table 1.

c/ Same as fiscal year 1977.

year 1977 level. The forcewide service predictions were deflated accordingly. 1/

Changes in fractions eligible to reenlist beyond 20 years of service are less obvious. Beyond the 19th year, the service assumptions exhibited sharp differences. The Army and Air Force treated contracts as not binding by setting fractions equal to 1.0. The Navy and Marine Corps posited otherwise, assuming that

1/ Fractions were deflated by the percentage difference between the assumed service frequency and the current frequency in the 7th to 19th year-of-service interval (-17.11 percent).

fractions remained at their fiscal year 1977 level. CBO adopted the Navy fractions. Even if the contracts are not technically binding, CBO assumed that--under both the present system and the DoD proposal--personnel would behave as if they had an obligation to serve for a specified number of years.