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The CBO Infinite-Horizon Model with Idiosyncratic Uncertainty and Borrowing Constraints

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Abstract

This paper describes the infinite-horizon general equilibrium model that, among other models, CBO uses for its analysis of the President's budgetary proposals. Agents in the model live forever and face uninsurable, individual-specific working-ability shocks and borrowing constraints, and so they differ ex post in both income and wealth. The combination of idiosyncratic uncertainty and borrowing constraints affects household decisions about saving and working: households engage in precautionary saving to self-insure against uncertainty, and labor supply decisions are less elastic than in a standard infinite-horizon growth model. To show how the model behaves in an application, the paper analyzes a 10 percent reduction in income tax rates and compares the predictions under alternative assumptions about the level of uncertainty.

1 Introduction

The Congressional Budget Office (CBO) uses three models—a "textbook" growth model, a life-cycle growth model, and an infinite-horizon growth model—to estimate the supply-side effects of the President's budgetary proposals. This paper provides a technical description of the infinite-horizon growth model and illustrates the behavior of the model by applying it to simulate a 10 percent reduction in the tax rates on income from labor and capital.

1.1 Background on CBO's Growth Models

CBO's textbook growth model is an enhanced version of a model developed by Robert Solow, a pioneer in the theory of growth accounting.[1] That model incorporates the assumption that economic output is determined by the number of hours of labor that workers supply, the size and composition of the capital stock (for example, factories and information systems), and total factor productivity, which represents the state of technological expertise. The model is not forward-looking. The people it represents base their decisions about working and saving entirely on current economic conditions. In particular, they do not respond to expected future changes in government policy. Moreover, instead of incorporating effects from demand-side variations in the economy, the model assumes that output is always at its potential (or sustainable) level.

Like the textbook growth model, the life-cycle and infinite-horizon growth models ignore demand-side effects. However, these models differ from the textbook growth model in fundamental ways.[2] Each assumes that people decide how much to work and save so as to make themselves as well off as possible over a lifetime. They make those decisions not only on the basis of information about the present, but in keeping with their expectations about the future as well. In analyzing the President's proposals for any given year, for example, the life-cycle and infinite-horizon models can account for the ways in which those proposals would affect government spending and revenue over the 10-year projection period. Additionally, any deficits or surpluses that accumulate over that period can affect budgetary decisions in later years. Within the life-cycle and infinite-horizon models, people's expectations about those developments can affect their behavior before the changes materialize.

Economists disagree, however, about the extent to which expectations influence people's economic decisions, the time horizon over which people plan, or the future policy shifts that people actually expect. CBO therefore analyzes the President's proposals using alternative models that represent a wide range of assumptions about the extent of people's foresight and the expectations they might have about future policies. That approach yields a range of plausible

[1] For a detailed description of the textbook growth model, see Congressional Budget Office (2001).

[2] For a detailed description of the life-cycle model, see Nishiyama (2003) and Nishiyama and Smetters (2005). For a description of a model very similar to the infinite-horizon model, see Aiyagari (1994, 1995).

estimates about how those proposals might affect economic growth.

Although the life-cycle and infinite-horizon models do not provide a role for unpredictable fluctuations in aggregate output, CBO's models do assume that individual households face unforeseeable (and idiosyncratic) fluctuations in their income against which they cannot buy insurance. Faced with that uncertainty, households self-insure by holding some additional "precautionary" savings as a buffer against potential drops in income. The consideration of uncertainty and the precautionary motive it entails make household saving decisions less sensitive to the after-tax interest rate than they would be otherwise. That, in turn, makes CBO's models somewhat more realistic than models in which households are assumed to have no uncertainty about their future income.

The life-cycle and infinite-horizon models differ in their assumptions about how far ahead people look in making their plans. The life-cycle model is calibrated so that the probability of death at a given age matches current U.S. mortality rates, and people are assumed to consider the effects of future economic or policy changes only for themselves and not for their children. In the infinite-horizon model, in contrast, people behave as though they expect to live forever—such behavior is effectively equivalent to acting as though the well-being of their descendants is as important to them as their own well-being.[3] Although many people care about their descendants, there is evidence against the assumption that people care as much about their descendants as they do about themselves.[4] CBO uses both life-cycle and infinite-horizon models to characterize two extreme alternative assumptions about how people might adjust their economic decisions to account for future generations.[5]

1.2 Overview of the Infinite-Horizon Model

The economy of the infinite-horizon model is made up of households who live forever and are subject to uncertainty against which they cannot insure. This uncertainty is modeled as an idiosyncratic shock to each individual household's working ability, in a manner similar to Bewley (1986) and Aiyagari (1994). Households are forward looking and choose optimal levels of assets, consumption, and labor supply. Although households in the model are assumed to be identical at birth, they are heterogeneous ex post because of differences in their past histories of working-ability shocks, and in the consumption, saving, and labor supply decisions they have made in response to those shocks.

Although individual households face uncertainty about their own working ability, the model has no aggregate uncertainty. Aggregate variables such as

[3] Aiyagari (1994) highlights that although agents live forever and in principle care about future generations as much as they do about themselves, sequences of bad shocks will periodically lead to binding borrowing constraints. An agent's infinite-horizon optimization problem is thus broken up into a sequence of finite-horizon problems, and the agent's effective horizon is shortened.

[4] See Evans (1993), Hayashi, Altonji, and Kotlikoff (1996), and Stanley (1998).

[5] For a discussion of the bequest motive and how it affects the relationship between a life-cycle and an infinite-horizon model (with perfect certainty), see Barro and Sala-I-Martin (2004), pp. 198–200.

investment, labor supply, and consumption are affected by the decisions that households make in response to individual shocks. However, these aggregate variables do not become uncertain, because individual decisions about consumption, saving, and labor supply are averaged over a continuous distribution of heterogeneous households.

In reality, individuals face both individual-specific uncertainty about their future working ability and aggregate uncertainty that mainly stems from business-cycle episodes. But the model is constructed without aggregate uncertainty for two reasons. First, CBO uses the model mainly to analyze the potential impact of proposed changes in tax and transfer policies that often depend on household income. A model with individual-specific uncertainty allows for analysis of such proposals because the households in the model are heterogeneous, having a wide range of incomes from labor and capital at any point in time. In contrast, models constructed to recognize the effects of aggregate uncertainty on household behavior typically contain a single representative household, making them less appropriate for analysis of such proposals. Second, adding aggregate uncertainty on top of idiosyncratic shocks would substantially complicate the model and its solution.

The infinite-horizon model also places limits on household borrowing, to capture another market imperfection that is commonly observed in the real world. The possibility of being unable to borrow when faced with a period of bad shocks adds a further incentive for each household to accumulate a buffer of assets. Such precautionary saving implies that the aggregate accumulation of capital will be larger than it would be in a representative-agent framework without uncertainty. As a result, in contrast to a representative agent model with perfect certainty, the return to capital is less than the individual rate of time preference in a steady-state equilibrium.

In contrast to and as an extension of Bewley (1986) and Aiyagari (1994), household labor supply is determined endogenously in CBO's infinite-horizon model. The combination of idiosyncratic uncertainty and borrowing constraints affects household decisions about saving and working. Each household decides how to allocate its time between labor and leisure as it experiences variations to their working ability. During a period with a bad working-ability shock, a household might want to devote less time to work because it is less remunerative. But uncertainty about the duration of such a bad period also provides an offsetting incentive for such a household to work harder in order to earn more and build up savings without reducing consumption. Furthermore, when the borrowing constraints are binding, a household in the model can consume more only by working more. On net, as shown in the third section of this paper, that mix of incentives results in labor supply decisions that are substantially less elastic than they would be in a standard neoclassical infinite-horizon growth model that has no uncertainty and no borrowing constraints.

As discussed above, the possibility of representing heterogeneous earnings capacities and wealth accumulations provides a useful basis for analyzing tax policies that affect different income groups in the economy in different ways. For such policy experiments, overlapping-generation (OLG) models have been used to allow for some heterogeneity in income and wealth. However, OLG models become computationally difficult when uncertainty is introduced. Consequently, a rather flourishing strand of literature has applied infinite-horizon models with uncertainty and ex-post heterogeneity, similar to the one described here, as laboratories for policy analysis. See for example, Aiyagari (1995), Domeij and Heathcote (2004), and Heathcote (2005).

In order to build a laboratory for tax policy questions, CBO's infinite-horizon model also accounts for the presence of a government that redistributes income and consumes goods by collecting distorting taxes on capital and labor income, in a way similar to Domeij and Heathcote (2004).

To illustrate the effect of idiosyncratic uncertainty, this paper applies the infinite-horizon model to simulate a 10 percent reduction in capital and labor income tax rates, and compares the predictions under alternative assumptions about the level of uncertainty. The paper shows that in response to a tax cut, most of the variables react with a substantial increase over their values in the initial steady state. Furthermore, to show the role played by uncertainty coupled with borrowing constraints, we test the effect of increased uncertainty on asset accumulation and labor supply decisions. Greater uncertainty increases the capital stock at each level of the new tax rates and dampens the labor supply responses.

The rest of the paper is organized as follows. The next section formally presents the model and its solution method. The following section describes the calibration and examines its implications for labor supply behavior. The final section examines the tax policy experiment. Appendices provide additional technical details about model solution methods and about labor supply behavior in the case of an alternative possible habit-formation specification for the household's preferences that was considered but not adopted for CBO's current version of the infinite horizon model.

2 The model economy

The economy features an infinite number of households who consume a single type of good and decide how much to save and how much time to devote to work. In supplying labor, households face idiosyncratic shocks to their ability to transform the time devoted to work into productive labor (that is, working-ability shocks). The idiosyncratic working-ability state at the beginning of each calendar time t is known to the household when it decides how much to consume, work, and save, but working-ability states at future dates are uncertain. We assume that households cannot insure against that type of risk. Furthermore, if hit by a series of bad working-ability shocks, households cannot borrow against their future income.

Goods are produced in a representative firm using capital and labor supplied by the households. The firm operates in perfectly competitive markets with a constant-returns-to-scale technology. A government levies taxes, consumes goods, and distributes lump-sum transfers. We assume that one government rules forever and commits to a particular budget rule.

2.1 Household

Households are born identical and live forever. At each year t, each household receives a shock to its own working ability e_t, taking values in the set $E = \{e_1, ..., e_i\}$, and cannot insure against it. Incompleteness in insurance markets makes it possible for this idiosyncratic risk to transform ex ante identical households into heterogeneous ex post. Each household's earning ability or productivity follows a first-order Markov process, with transition probabilities between two states e_i, e_j in the space E given by $\pi_{i,j}(e_{t+1} = e_j \mid e_t = e_i)$, where π defines a probability distribution. The probability measure of households on E at each year t is represented by μ_t, with $\mu_t(E) = pr(e_t \in E) \geq 0$. If the initial measure of households with respect to their working ability is represented by a vector $\mu_{t=0}$, then the measure at some future date t will be $\mu_t = \mu_{t=0}\Pi^t$, where Π represents the transition probability matrix, whose elements are the $\pi_{i,j}$.

At each date t, after observing the realization of e_t the household decides how much to consume c_t, how much labor to supply h_t, and the next period's asset holding a_{t+1}, in the form of a single risk-free savings instrument. In the choice of the optimal assets to carry over to the next period, the household also faces borrowing constraints, which define the minimum assets it is allowed to hold. Households cannot carry negative assets; thus at each time $a_t > 0$. As a result, although agents live forever, sequences of bad shocks will lead to periods of binding borrowing constraints, breaking the horizon into a sequence of finite periods.

Let A be the asset space, assumed to be non-negative, $A \in \mathbb{R}_+$. The household's states are represented by the vector $s_t = (a_t, e_t)$ with values in the state space $(A \times E)$. Let $x_t(s)$ be the measure of households across both individual assets and working abilities at year t, and $X_t(s)$ be the corresponding cumulative measure such that $\int_{A \times E} dX_t(s) = 1$.

Because the economy does not experience any aggregate uncertainty, the households have perfect foresight of the aggregate return on capital r_t and of the aggregate wage rate w_t, although they do not know their own future wage (which is determined by w_t and e_t, the working-ability state). The aggregate states of the economy relevant for the individual vector of decision rules $d_t = (c_t, h_t, a_{t+1})$ are $S_t = (x_t(s), B_t)$, where B_t represents the government debt (or wealth) which is discussed later. The economy is then characterized by a set of government policy schedules, which affect the individual's decision rules. Let Θ_t denote the policy schedule set by the government. The optimization problem of the household can now be defined recursively as:

$$V(s_t, S_t; \Theta_t) = \max_{c_t, h_t, a_{t+1}} U(c_t, h_t) + \beta E_{e_{t+1}|e_t}[V(s_{t+1}, S_{t+1}; \Theta_{t+1})] \qquad (1)$$

subject to

$$a_{t+1} = (1+r_t)a_t + w_t e_t h_t - c_t - T(a_t, h_t; \Theta_t) + TR_t \quad (2)$$
$$A \in \mathbb{R}_+, \quad (3)$$

where β is the time preference parameter; $T(a_t, h_t; \Theta_t)$ represents the total individual tax function, with a tax base determined by the the asset holding a_t and by the labor supply h_t; and TR is a government lump-sum transfer. Each household faces a (normalized) time constraint 1. The utility function $u(c_t, h_t)$ expressing the individual preferences over consumption c_t and leisure $\ell_t = (1 - h_t)$ is specified as a time-separable isoelastic Cobb-Douglas function[6]:

$$U(c_t, h_t) = \frac{[c_t^{\alpha}(1-h_t)^{(1-\alpha)}]^{(1-\gamma)}}{1-\gamma}, \quad (4)$$

where γ is the coefficient of relative risk aversion and α is the share of consumption in the household's preferences. Given the household optimization problem, the law of motion of the measure $x_t(s)$ is determined by:

$$x_{t+1}(s) = \int_{A \times E} I_{[a_{t+1}=a_{t+1}(s_t, S_t; \Theta_t)]} \pi_{t,t+1}(e_{t+1} \mid e_t) dX(s_t), \quad (5)$$

where $I_{[a_{t+1}=a_{t+1}(s_t, S_t; \Theta_t)]}$ is an indicator function taking a value of 1 if the decision variable $a_{t+1} = a_{t+1}(s_t, S_t; \Theta_t)$.

2.2 Production

Production takes place in a representative firm operating with a Cobb-Douglas constant-returns-to-scale technology. At each year t, the firm uses aggregate capital K_t and aggregate labor L_t as inputs to produce a single output Y_t through the production function $F(K_t, L_t)$. The production technology is defined as follows:

$$F(K_t, L_t) = A_t K_t^{\theta} L_t^{(1-\theta)}. \quad (6)$$

The parameter $\theta \in [0, 1]$ represents the share of the capital input in the production process, and A_t is total factor productivity. Output can be transformed into future capital, private consumption, and government consumption according to:

$$C_t + G_t + K_{t+1} - (1-\delta)K_t = Y_t. \quad (7)$$

2.3 Government

The government consumes goods produced by the representative firm in the form of public consumption G and distributes lump-sum transfers to the households TR. To finance those outlays, the government issues one-period debt B_t and

[6] This utility specification makes preferences consistent with the analysis of Aiyagari (1994, 1995).

levies taxes T.[7] All those variables represent elements of the government's exogenous policy schedule Θ_t at each year t. From the households' perspective, government debt (or wealth) and capital are perfect substitutes, since both deliver a risk-free return in the absence of aggregate risk and transaction costs. An equilibrium condition equates aggregate household asset holdings in the economy to the sum of the government debt and the capital stock.

In this economic environment, by designing a certain tax system and distributing lump-sum transfers, the government also provides the only mechanism to share risk among households.

The government taxes capital and labor income in a linear fashion as follows:

$$T(a_t, h_t; \Theta_t) = \tau_{c,t} r_t a_t + \tau_{l,t} w_t e_t h_t, \tag{8}$$

where $\tau_{c,t}$ and $\tau_{l,t}$ are, respectively, the average effective marginal tax rates applied to capital income and labor earnings.[8] The tax system in reality is characterized by statutory marginal tax rates and by a mix of deductions and exemptions that de facto reduce the taxable base or taxable income. The effective marginal tax rates on capital and labor income account for the effective pressure of the fiscal system as a result of factors that shrink the tax base.

The government defines the policy schedule vector that the households use in the economy to solve their optimization problems:

$$\Theta_t = (\tau_{c,t}, \tau_{l,t}, G_t, TR_t, B_{t+1}).$$

Given the policy schedule, the law of motion for the government debt is the following:

$$B_{t+1} = (1 + r_t) B_t + G_t + TR_t - \int_{A \times E} T(a_t, h_t(s_t, S_t); \Theta_t) dX(s_t). \tag{9}$$

In words, government accumulates debt if expenses for the serving of debt $r_t B_t$, government consumption G_t, and transfers to the households TR_t exceed total receipts from collection of taxes at the individual level $\int_{A \times E} T(a_t, h_t(s_t, S_t); \Theta_t) dX(s_t)$.[9]

2.4 Equilibrium definition

A recursive equilibrium for this economy is a value function $\{V(s_s, S_s; \Theta_s)\}_{s=t}^{\infty}$, a vector of decision rules $\{c_s, h_s, a_{s+1}\}_{s=t}^{\infty}$ for the household optimization problem, a probability measure μ_0 and $\{\mu_s(E)\}_{s=t}^{\infty}$ for the initial level and time path of

[7] In the case in which the government runs a surplus, the negative of B_t can be interpreted as government wealth.

[8] The linear taxation assumed here is only an approximation to the US tax system, which is much more complex and taxes capital and labor income nonlinearly.

[9] The government debt cannot grow indefinitely faster than the interest rate in the long run. This boundary condition is imposed by the following: $B_0 + \sum_{t=0}^{\infty} \prod_{j=0}^{t} (1 + r_j)(G_t + TR_t) = \sum_{t=0}^{\infty} \prod_{j=0}^{t} (1 + r_j) \int_{A \times E} T(a_t, h_t(s_t, S_t); \Theta_t) dX(s_t)$.

the mass of the population in each working-ability state $e_s \in E$, the return on capital $\{r_s\}_{s=t}^{\infty}$ and the wage rate $\{w_s\}_{s=t}^{\infty}$, a measure of households across both the individual wealth and earning ability $\{x(s_s)\}_{s=t}^{\infty}$, a policy schedule vector $\{\Theta_s\}_{s=t}^{\infty}$, and a vector of aggregate variables $\{K_s, L_s\}_{s=t}^{\infty}$ such that:

1. $\forall t$ the decision rules $\{c_t, h_t, a_{t+1}\}$ solve the household's optimization problem, given r_t and w_t, the policy schedule in place Θ_t, and the sequence $\{\mu_s(E)\}_{s=0}^{t}$.

2. $\forall t$ for given Θ_t and $x(s_t)$, the solution of the firm's problem yields the return on capital and the wage rate:

$$r_t = \theta A_t K_t^{\theta-1} L_t^{1-\theta} \tag{10}$$

$$w_t = (1-\theta) A_t K_t^{\theta} L_t^{-\theta}. \tag{11}$$

3. $\forall t$ given the conditions (10)-(11), the factor markets clear:

$$K_t + B_t = \int_{A\times E} a_t dX(s_t) \tag{12}$$

$$L_t = \int_{A\times E} e_t h_t(s_t, S_t; \Theta_t) dX(s_t). \tag{13}$$

4. $\forall t$ given the policy schedule vector Θ_t, the prices r_t and w_t, and the household decision rules $\{c_t, h_t, a_{t+1}\}$, the government budget constraint (9) is satisfied, such that growth in government debt remains bounded.

5. $\forall t$ the goods market clears:

$$C_t + G + K_{t+1} - (1-\delta)K_t = F(K_t, L_t) \tag{14}$$

$$C_t = \int_{A\times E} c_t(s_t, S_t; \Theta_t) dX(s_t). \tag{15}$$

   The equilibrium definitions highlight the fact that households do not make any portfolio choice between shares of capital and government debt. Households are indifferent between capital and government debt because there is no aggregate uncertainty in the form of aggregate productivity shocks. The return on capital and the return applied for the government debt service are certain and in equilibrium are equal.

The economy is in a steady-state recursive equilibrium if the aggregate states of the economy are constant over, time which implies that $S_{t+1} = S_t$.

2.5 Solution method

We first solve for an initial steady-state equilibrium, where the economy is at $t = 0$, and then simulate a tax reform and solve for the transition path to a final steady state.

To compute an equilibrium, we start by guessing the return on capital and the wage rate and then using a series of iterations to solve for the households' stochastic optimization problem. We create a discrete state space $A \times E$, using $g \times 7$ grid points, to determine the finite space of the possibilities for assets and earnings, as a function of the scale and the standard deviation of the earning ability shock σ: $E = \{e_1, ..., e_7\}$ and $A = \{a_1, ..., a_g\}$. The discretization of the space makes it possible to find a solution of the household Euler equations for each possible grid point. One can also find the value function that satisfies the Euler equations over the entire asset space for the different households' working abilities in the economy.

Another series of iterations is meant to compute the general equilibrium. We aggregate the optimal decisions computed at the household level. With the levels of aggregate capital and labor and a new return on capital and a wage rate, we search for a new solution to the households' problem. This process is repeated until the return on capital and the wage rate converge. The solution method is described in further detail in Appendix A1.

3 Parameterization

Because the time period in the model is one year, all parameter values are expressed in yearly terms. Some of the parameters are taken from the literature; others are calibrated to relevant facts of the U.S. economy.

Table 1 reports the values assigned to some of the parameters characterizing the steady-state equilibrium at time $t = 0$ (i.e., before any policy experiment).

Table 1

Parameter values in the initial steady-state equilibrium

Capital share in the production function	θ	0.3
Depreciation rate	δ	0.05
Time preference parameter	β	0.94
Share of consumption in the utility function	α	0.63
Relative risk-aversion parameter	γ	2.0

3.1 Production

The parameters for the specification of the Cobb-Douglas technology and the depreciation of physical capital are standard. Capital's share θ in the production function is set to 0.3, and the depreciation rate δ is set to 0.05. In each year t, capital K and the government debt B correspond to the sum of private asset holdings. Given the specified technology, the capital-to-output ratio is targeted to 2.74, as in Nishiyama and Smetters (2005). To reproduce this fact, the time preference parameter β is set to 0.94. Total factor productivity A is derived consequently and is set to 0.95.

3.2 Households

The share of consumption α in the utility function is 0.63 and the coefficient of relative risk aversion γ is 2.0.[10] Given the latter, α is chosen as to make the model households work on average 50 percent of the maximum available time.[11]

Households' earnings abilities and the related stochastic properties are key features of the model because they will generate agents' ex post heterogeneity in wealth and income. The probability measure of households across the working-ability states, $\mu(E)$, and the matrix that defines the transition probabilities between two states, Π, are crucial to the introduction of heterogeneity in the distribution of capital and labor income. It is also necessary to define the persistence ρ and variance σ^2 of the working-ability shocks and to derive the condition that ensures that μ_t converges to a unique ergodic distribution μ^*, independent of the initial measure μ_0. The solution involves finding the eigenvector μ associated with the unit eigenvalue of the matrix Π, such that $\mu = \mu\Pi$.

The state space for earnings ability consists of seven grid points, $E = \{e_1, ..., e_7\}$. A Markov chain with seven states is used to approximate a first-order autoregressive process for the logarithm of the working-ability shock e_t, as in Aiyagari (1994). The autoregressive process that is then approximated can be represented as:

$$\log(e_t) = \rho \log(e_{t-1}) + \sigma(1-\rho^2)^{\frac{1}{2}} \epsilon_t, \tag{16}$$

where ρ is the persistence, σ the coefficient of variation, and ϵ_t the innovation of the working-ability shock e_t. The model converts this continuous representation into the corresponding discrete Markov process following Tauchen (1986). The algorithm is implemented with a serial correlation $\rho = 0.9$ and a coefficient of variation $\sigma = 0.6$ (i.e., a variance of $0.065/(1-0.9^2)$ for working ability), which implies a standard deviation of the residual of autoregressive representation of the earnings ability process (16), $\sigma(1-\rho^2)^{\frac{1}{2}} = 0.25$. Both values of the serial correlation and the standard deviation of the working-ability shock are consistent with the range of values found in many empirical studies of data from Panel Study of Income Dynamics, PSID, (Card, 1991; Domeij and Heathcote, 2004; Storesletten, Telmer, and Yaron, 2001).

[10] Existing estimates of γ are highly disperse, ranging from 1 to 5 and more. Most macroeconomists use numbers toward the bottom end of that range: for example, Chetty (2003) estimates that γ is in a range around 1, and Nishiyama and Smetters (2005) set it to 2. We follow Nishiyama and Smetters (2005) and assume γ is 2.

[11] See Nishiyama and Smetters (2005).

3.2.1 Elasticity of labor supply

Labor supply in the model is determined by the dynamic responses of a population of ex post heterogeneous, forward-looking agents. The compensated and uncompensated labor supply elasticities typically applied in a static context are not adequate for characterizing labor supply responses in models where agents optimize through extended periods of time.

The validity of dynamic models for analyzing policy experiments relies on the possibility that rational, optimizing agents can substitute both intratemporally and intertemporally in quantities of consumption and labor supply. The intertemporal labor supply responsiveness in dynamic models is often measured using a Frisch elasticity.[12] The Frisch elasticity of labor supply is defined as the percentage change in labor supply resulting from a 1 percent increase in the expected wage rate, holding the marginal utility of wealth constant. It has a dynamic representation.

The Frisch elasticity can be derived from the first-order conditions of the household's decision problem, sketched out in Section 2, with respect to the labor supply choice:

$$\lambda_t w_t = -\frac{\partial U(c_t, h_t)}{\partial h_t} \tag{17}$$

$$\lambda_t = \beta(1 + r_{t+1})E\left[\lambda_{t+1}\right] + \eta_t, \tag{18}$$

where λ_t represents the marginal utility of wealth and η_t the marginal utility of borrowing in period t (i.e., η_t is the Lagrange multiplier attached to the borrowing constraint in equation 2). Those conditions characterize how individuals can substitute labor hours intertemporally and highlight the role of (binding) borrowing constraints in the dynamic response of labor supply. When borrowing constraints are binding, i.e., $\eta_t > 0$, individuals are not able to borrow against future earnings from a planned increase in future labor supply. The labor supply response (elasticity) of borrowing-constrained individuals is smaller than that predicted by analytical expressions that ignore such constraints (Domeij and Floden, 2006).

In a world without uncertainty and borrowing constraints, using (17), (18), and the utility specification (4), we can derive an analytical expression for the Frisch elasticity:

$$\left.\frac{\partial \log h}{\partial \log w}\right|_{\lambda} = \frac{1}{h_t}\left[\frac{U_h}{U_{hh} - \frac{U_{hc}^2}{U_{cc}}}\right] \tag{19}$$

where the notation F_y for a function F represents the first-order derivative with respect to the argument y, and F_{yy} and F_{yz} represent second-order derivatives of F with respect to y, or with respect first to y and then to z. Taking into account the utility specification (4), we can rewrite (19) as:

$$\left.\frac{\partial \log h}{\partial \log w}\right|_{\lambda} = \frac{(1 - h_t)}{h_t}\left[\frac{1 - \alpha(1 - \gamma)}{\gamma}\right]. \tag{20}$$

[12] An early application of the concept of Frisch elasticity in dynamic models can be found in Browning, Deaton, and Irish (1985).

Given the parameter values for γ and α, the theoretical Frisch elasticity, ignoring uncertainty and borrowing constraints, should take the value of 0.82.

However, that expression does not account for borrowing constraints (18) or for uncertainty about working ability in future periods. In a model that includes those factors, the "empirical" Frisch elasticity (calculated from model simulations) will be lower than equation (20) indicates. That is, the presence of borrowing constraints and earnings uncertainty means that the model's simulations will exhibit a substantially lower Frisch elasticity than would be calculated from the model's utility function. Contreras and Sinclair (2008) estimated this effect in a stochastic OLG model, using a large number of model simulations to construct synthetic longitudinal data on labor supply decisions and on expected and unexpected values of labor earnings. Those data were used in an econometric investigation of the Frisch elasticity.

In such estimations, however, a simple regression of labor supply on wages would *underestimate* the Frisch elasticity. Pistaferri (2003) showed that if the empirical analysis of the labor supply response to wage changes does not control for the variance of unexpected changes in future wages and for the level of those changes, the Frisch elasticity estimates are significantly biased downward.[13] Contreras and Sinclair (2008) used the same approach as Pistaferri (2003) to control for borrowing constraints and earnings uncertainty. In the stochastic overlapping-generations model they used, they found that the correctly estimated Frisch elasticity in simulations was about one-fourth the value implied by (20).[14]

Following the same procedure as Contreras and Sinclair (2008), our model produces a Frisch elasticity of 0.57 in simulations, which is substantially below that implied in (20), though not as dramatically so as Contreras and Sinclair found. It is also lower than the elasticity estimated by Pistaferri (2003).

The estimated elasticity of labor supply with respect to unexpected changes in the wage or ability profile is 0.07.[15] That is much lower than the elasticity estimated by Pistaferri (2003), but in line with MaCurdy (1981), who estimated a value of 0.08 using PSID data. That coefficient represents an important parameter of interest for policy analysis. In fact, the behavioral response to policy reforms, such as tax reforms, is characterized usually by a permanent (unexpected) shock to the individual's disposable earnings and a shift of the entire

[13] The Pistaferri (2003) labor supply response specification, adapted to the present model economy, is the following:

$$\Delta \log h_{it} = \omega_i + \varphi \Delta \log w_{it} + \upsilon Var_{t-1}(\epsilon_{it}) + (\varphi + \Gamma)\epsilon_{it},$$

where $Var_{t-1}(\epsilon_{it})$ and ϵ_{it} are, respectively, the variance and the level of the unexpected wage change or innovation. φ represents the Frisch elasticity and $(\varphi + \Gamma)$ the coefficient that determines the response of labor supply to unexpected wage (or working ability) changes.

[14] See Contreras and Sinclair (2008) for further details.

[15] The stochastic specification assumed for the working-ability process implies the serial correlation between working abilities $\rho < 1$. Pistaferri, instead, assumed a martingale process and $\rho = 1$. This allows one to characterize $\Gamma = \sum_{\tau=0}^{T-t} \gamma_\tau$, a representation that perfectly identifies the wealth effect represented by $\sum_{\tau=0}^{T-t} \gamma_\tau$. In the case of $\rho < 1$, we cannot identify the wealth effect, since $\Gamma = \sum_{\tau=0}^{T-t} \gamma_\tau \rho^\tau$.

earnings profile.

3.3 Government policy schedule

Government expenditure, the value for the next-period debt, and the taxation system are designed by picking the relevant parameters to match those variables for the U.S. economy, thus determining the elements of the policy schedule Θ_t. Table 2 summarizes the values of the parameters characterizing the schedule at time $t = 0$.

We assume that government consumption G, government lump-sum transfers TR, and government debt B are set as percentages of aggregate output Y in the initial steady state. G is chosen to match the actual ratio of government consumption to Y of 0.06 (from 2006 National Income and Product Account data), and B is set to match the actual ratio of government debt held by the public to Y of 0.36 (2006 data from the Budget of the U.S. Government). In the initial steady state at $t = 0$, TR is used to close the budget constraint of the government and to match the actual ratio of tax revenue to output, which is 0.11 (excluding revenue from payroll taxes). Globally, the lump-sum transfer value TR is 7.8 percent of aggregate output.[16]

3.3.1 Taxation system

The fiscal parameters for capital and labor income taxation were chosen to match the effective marginal tax rates on capital and labor income for the US federal tax system. Effective marginal income tax rates are lower than the statutory rates because the household's taxable income is lower than its economic income as a result of various deductions and exemptions available to individuals and firms.

Calibration of the progressive tax system involves choosing of the parameters that specify the tax function in (8). The effective marginal tax rates on capital and labor income are calibrated to match the values estimated by CBO for 2007. The effective marginal tax rate on capital income $\tau_c$is set equal to 13.7 percent and accounts for taxation of persons at both the individual and the corporate level. The effective marginal tax rate on labor income τ_l is set equal to 18.2 percent and does not include payroll taxes.

Table 2

Policy schedule parameters at $t = 0$

G/Y	0.06
B/Y	0.36
TR/Y	0.078
t_l	0.182
τ_c	0.137

[16] The ratio of the lump-sum transfer to GDP, TR/Y, does not account for the part of those trasfers that is due to Social Security expenditure, because agents in the model do not receive any Social Security benefits.

The calibrated vector of parameters and variables characterizing the policy schedule is:

$$\overline{\Theta} = (\tau_l, \tau_c, G, TR, B). \quad (21)$$

4 Policy experiment

In order to show the properties of the model, this section reports results of a fairly simple policy experiment. At time $t = 0$ the economy is in the steady state, implied by the parameters and policy schedule described in the previous sections. We then assume that at the beginning of $t = 1$ the economy is shocked by a permanent 10 percent cut in labor and capital income tax rates. The government balances its budget through an adjustment in lump-sum transfers, that is, without using debt financing.

Table 3 reports the impact of such a policy change on the main macroeconomic variables. Capital and labor supply both increase as a result of a decrease in the income tax rates distorting the marginal incentive to save and invest. Capital income increases by 3.5 percent relative to the initial steady state in the long run, while labor supply, after increasing sharply in response to unexpected disposable wage change in the short run, eventually reaches a value that is 0.9 percent higher than the initial steady state.[17] These dynamics create a 0.19-percentage-point decrease in the real (pre-tax) interest rate r and a 0.74-percentage-point increase in the wage rate w.

The decrease in capital taxation also represents a decrease in the level of risk sharing among individuals in the economy. In the presence of uninsurable uncertainty, individuals would accumulate a buffer stock of assets during good earning states to insure themselves against a consumption drop during bad times. Capital income taxation makes agents who receive good earning shocks save more but also pay higher taxes than agents suffering an unfavorable shock. Furthermore, a tax cut financed with a reduction in lump-sum transfers reduces the level of redistribution from "lucky" to "unlucky" individuals. In that way, capital income taxation and the transfers system provide for risk sharing among agents in the absence of efficient insurance markets. A decrease in capital income taxation weakens this risk sharing mechanism, so individuals choose to accumulate more capital, also for insurance purposes, not only for the relaxation of tax distortions.

As a result of the increase in capital and labor, output increases by 1.7 percent and consumption by 1.6 percent in the long run above their initial steady-state levels.

[17] Appendix A2 presents results of the same policy experiment with a different preference specification that allows for a slower labor supply response.

Table 3

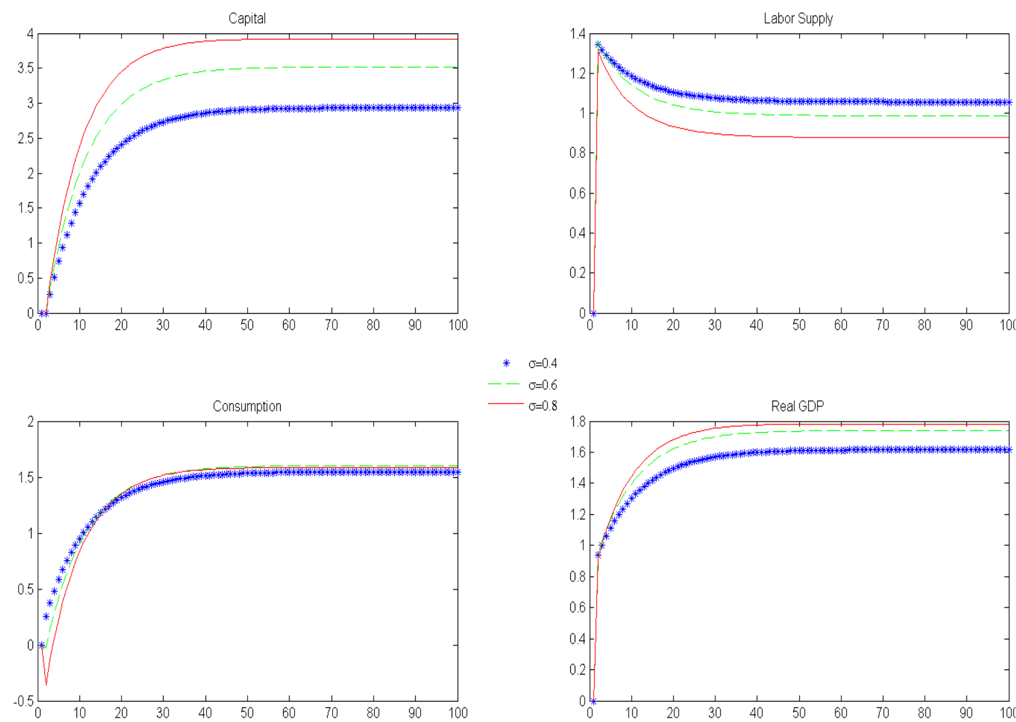
Aggregate variables: percentage deviation from the initial steady state ($t = 0$)

	$reform: 10\%\ \ income\ \ tax\ \ cut$					
	$t+1$	$t+5$	$t+10$	$t+20$	$t+50$	$long-run$
$capital\ (K)$	0.0	0.95	1.98	2.98	3.40	3.48
$labor\ (L)$	1.34	1.22	1.10	1.01	0.97	0.098
$output\ (Y)$	0.94	1.15	1.38	1.61	1.73	1.74
$consumption\ (C)$	0.02	0.42	0.90	1.35	1.59	1.60
$pre-tax\ return\ (r)$	0.10	0.02	-0.06	-0.14	-0.18	-0.19
$pre-tax\ wage\ (w)$	-0.40	-0.09	0.25	0.57	0.73	0.74

Capital and labor supply show very interesting behavior in the presence of increasing uncertainty (see Figure 1). With higher uncertainty, capital accumulation is higher in the new long-run equilibrium. The response to a reduction in capital income taxation in the long run is stronger because, with higher uncertainty (i.e., higher σ), individuals need more capital to self-insure against risk. Capital income taxes provide insurance in this environment. Therefore, for any reduction in taxes, individuals have to accumulate more capital to insure themselves in the presence of greater risk (i.e., higher σ).

Consumption increases in the long run in all cases, but in the short run individuals have to sacrifice more, the higher the uncertainty, in order to accumulate more capital. The labor supply reaction to a decrease in labor income taxes also shows interesting behavior in the presence of risk. When uncertainty is higher, labor is less responsive to changes in the after-tax wage. Uncertainty depresses the labor supply Frisch elasticity in the way described in Section 3.2.1.

Figure 1: Effect of higher uncertainty on main macroeconomic variables



Note: We use three possible values for the coefficient of variation: $\sigma = 0.4$ (standard deviation of the working-ability innovation=0.17), $\sigma = 0.6$ (standard deviation of the working-ability innovation=0.25), $\sigma = 0.8$ (standard deviation of the working-ability innovation=0.35)

Appendix

A1. Solution method

The household state space $s \in A \times E$ is discretized using g grid points for the asset space, $A = \{a_1, ..., a_g\}$, and 7 grid points for the working-ability space $E = \{e_1, ..., e_7\}$. Consistent with the individual state space, the aggregate state space of the economy will be $S = (x(s), \cdot)$. We first compute an initial steady-state equilibrium in $t = 0$ and then, after applying the policy experiments, we compute a new final steady state and the transition path between the two steady states.

Steady-state equilibrium: The initial steady state is characterized by a time-invariant government policy schedule $\overline{\Theta}$. Given this schedule, the algorithm uses an inner loop to compute the individual optimal behavior, as follows:

1. Set the initial values for the capital-to-labor ratio, $K/L^0_{t=0}$, and given the production function specification and equilibrium conditions, compute the return on capital $r^0_{t=0}$ and the wage rate $w^0_{t=0}$.

2. Given $r^0_{t=0}$ and $w^0_{t=0}$ (and an initial government policy variable G^0), find the optimal household decision rules $d_t(s, S_{t=0}; \overline{\Theta})$ for all points in the state space $s \in A \times E$ as follows. Guess an initial value for next-period asset holdings $a^0_{t+1}(s, S_{t=0}; \overline{\Theta})$, and compute the optimal consumption and working hours using first-order conditions, so that:

$$\begin{aligned} c^0_{t=0}(s, S_{t=0}; \overline{\Theta}) &\in (0, c^{\max}(a^0_{t+1})] \\ h^0_{t=0}(s, S_{t=0}; \overline{\Theta}) &\in [0, 1] \end{aligned}$$

3. Compute the numerical derivative of the value function with respect to the asset holding $V_a(s, S_{t=0}; \overline{\Theta})$, and the value function $V(s, S_{t=0}; \overline{\Theta})$.

4. Plug optimal decision rules $c^0_{t=0}$, $h^0_{t=0}$ (found for each possible individual state) in the Euler equation (for consumption) along with $V_a(s, S_{t=0}; \overline{\Theta})$. Stop if the error in the Euler equation is small.[18] If the error is not small enough, update the guess for a^0_{t+1} and repeat the process from step 2 through 4 again. The algorithm here uses a bisection search to update and find the optimal next-period asset holding, given the individual states.[19]

5. Compute the measure of households in the asset and working-ability state space $x(s, S_{t=0}; \overline{\Theta})$ through linear interpolation using the decision rules found with steps 2 through 4.

[18] The convergence criterion for the Euler equation is set to a tolerance of 10^{-5}.

[19] For a series of similar methods to construct iterative algorithms and to solve for first-order conditions and fixed points, see Judd (1998).

6. An outer loop at this point computes the aggregate variables, the new $r^1_{t=0}$ and $w^1_{t=0}$, and the new policy variables satisfying the government budget constraint (in this case government consumption G^1) consistent with the measure $x(s, S_{t=0}; \overline{\Theta})$.

7. Compare $r^1_{t=0}$, $w^1_{t=0}$ with $r^0_{t=0}$, $w^0_{t=0}$ and G^1 with G^0. If the difference is sufficiently small[20], then stop. Otherwise update the guesses and start from step 1 again.

Transition path equilibrium. The economy is in the initial steady-state equilibrium when, at the beginning of year $t = 0$, the government announces a new policy schedule $\Theta_{t=0}$characterized by new tax rates and tax parameters. The aggregate state of the economy at the beginning of $t = 0$ is the initial steady state. The algorithm computes the transition to a new steady state, assumed to be reached at some date T.

1. Guess a path for the return on capital $\{r^0_s\}^T_{s=t+1}$ and the wage rate $\{w^0_s\}^T_{s=t+1}$ and the government policy. Keeping G fixed at the initial steady-state level, consistently guess a path for the new labor taxation parameters (e.g., apply a multiplicative factor ϕ_{adj} to the individual tax base, depending on the type of reform implemented, to make the experiment revenue neutral).

2. Given the previous set of guesses, since there is no aggregate uncertainty, the path for the aggregate state S_t is deterministic and so is $\{r_s, w_s, \Theta_s\}^T_{s=t+1}$ from the household point of view. So find final steady-state decision rules $d(s, S_T; \Theta_T)$, the numerical derivative $V_a(s, S_T; \Theta_T)$, the value function $V(s, S_T; \Theta_T)$, and the measure of households $x(s, S_T; \Theta_T)$ for all the states $s \in A \times E$, using the steady-state algorithm described above, and then update the guesses made in step 1.

3. Given the guesses $\{r^0_s, w^0_s\}^T_{s=t+1}$ and the new policy parameter to balance the government budget, compute the decision rules $d(s, S_t; \Theta_t)$, the numerical derivative $V_a(s, S_t; \Theta_t)$, and the value function $V(s, S_t; \Theta_t)$, working backward from the final steady state, $t = \{T-1, ..., 0\}$, and using $V_a(s, S_{t-1}; \Theta_{t+1})$ and the value function $V(s, S_t; \Theta_{t+1})$ recursively in the Euler equation.

4. For $t = \{1, ..., T-1\}$ now compute forward the new path $\{r^1_s, w^1_s\}^T_{s=t+1}$, the new policy parameters to balance the government budget, and the measure of households $x(s, S_{t+1}; \Theta_{t+1})$ using the decision rules $d(s, S_t; \Theta_t)$ computed in step 3.

[20] The tolerance in this case is set as follows:

$$\max\left\{\left|K/L^1 - K/L^0\right|, \left|G^1 - G^0\right|\right\} < 10^{-5}.$$

5. Compare the new $\{r_s^1, w_s^1\}_{s=t+1}^T$, the new policy parameters to balance the government budget, with the initial guess made in step 1. If the difference is small enough (using criteria specified in the steady-state algorithm), then stop. Otherwise start from step 2, using these new guesses again.

A2. Frisch Elasticity with Habit Persistence in Leisure

The introduction of habit persistence in leisure slows down the labor supply response to wage changes and consequently reduces the Frisch elasticity. Introducing that friction in the model is not an easy task for two main reasons: agents face idiosyncratic uncertainty even in the steady state of the economy and our solution method implies the iteration of nonlinear functions. Those two features make the model and the solution method different from most models exploited in the real business cycle literature that deal with a representative agent, facing no uncertainty in the economy in steady state, and solved through linearization methods.

Preferences are assumed to be nonseparable in consumption and leisure as follows:

$$U(c_t, h_t, h_{t-1}) = u(c_t)v(h_t, h_{t-1}),$$

And the following utility function specification is adopted:

$$U(c_t, h_t, h_{t-1}) = \frac{c_t^{\alpha(1-\gamma)}[(1-h_t) - b(1-h_{t-1})]^{(1-\alpha)(1-\gamma)}}{1-\gamma} \tag{a1}$$

This specification takes into account that the household's utility depends not only on current levels of consumption and leisure, c_t and $\ell_{t-1} = 1 - h_t$, but also on the values of leisure in the previous period, h_{t-1}. The parameter $b \in (0,1)$ denotes the intensity of habit formation and introduces nonseparability of preferences over time. Under habit persistence, an increase in current leisure lowers the marginal utility of leisure in the current period and increases it in the next period. Intuitively, the more the household enjoys leisure today, the more it wants to enjoy it tomorrow. In the preferences above, past leisure represents the household's stock of habit in year t.

Using first-order conditions, we can derive the labor supply elasticity, holding the marginal utility of wealth constant, or Frisch elasticity:

$$\left.\frac{\partial \log h}{\partial \log w}\right|_{\lambda} = \frac{1}{h_t}\left[\frac{U_h - b\beta E[U'_h]}{U_{hh} - b\beta E[U'_{hh}] - \frac{U_{hc}^2}{U_{cc}}}\right], \tag{a2}$$

where $E[U'_h]$ and $E[U'_{hh}]$ represent, respectively, the first- and second-order derivatives of the next-period expected utility function with respect to hours worked in the current period. The introduction of leisure habit persistence introduces two additional terms ($b\beta E[U'_h]$ and $b\beta E[U'_{hh}]$, which are both zero if $b = 0$) into the Frisch elasticity. In accordance with the utility specification (a1), $U'_h > 0$ whereas $U'_{hh} < 0$ if $\gamma > 0$ and $0 < \alpha < 1$. As a result, these two additional factors reduce the Frisch elasticity.

When $b = 0.7$, the response of labor to unexpected shocks to disposable earnings, such as the 10 percent tax cut simulated in Section 4, is around 39 percent smaller in the first five years after the shock (for example, 0.7 versus 1.2 in $t + 5$).

CBO does not adopt this preference specification which, as shown, would slow-down the labor supply response to wage changes for two main reasons:

1. The addition of an extra state variable, the time devoted to leisure in the previous period ℓ_{t-1}, or to work h_{t-1}, increases the dimension of the state space and then significantly the increase the solution time.

2. More important, there is virtually no literature documenting on the measure of the parameter b.

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