

## EXHIBIT 4

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# Expert Report: Educational Value of Neodymium Magnet Spheres in the matter of Zen Magnets, LLC, CPSC Docket No. 12-2

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Neodymium magnet spheres provide unique opportunities for teaching mathematics, physics, chemistry, biology, and engineering both in and out of the classroom. These magnets have spawned a learning community engaged in designing, building, and understanding magnetic sculptures. Banning the sale of these magnets would be an educational, creative, and artistic opportunity lost.

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## I. BACKGROUND AND QUALIFICATIONS

I hold a 1985 PhD in applied physics from Stanford University. Since November 2010, I have served as professor of physics and dean and executive director of Utah State University Uintah Basin. Previously, I taught physics at West Virginia University for 24 years, the last five as Russell and Ruth Bolton WVU Professor, an endowed professorship awarded for excellence in teaching. I have received the June Harless Award for Exceptional Teaching and the John R. Williams Outstanding Teacher Award. I am the author of more than 50 peer-reviewed scientific publications, including five on magnetic phenomena, and have received millions of dollars of research funding from the National Aeronautics and Space Administration, the United States Department of Energy, the United States National Science Foundation, and other agencies. I have taught electromagnetism at the introductory undergraduate, advanced undergraduate, and graduate levels. Copies of my curriculum vitae and publications are available online [1].

In the fall of 2012, my son brought a set of 216 Buckyballs home from college, and I quickly became fascinated by their magnetic properties. Buckyballs are coated magnet spheres of a neodymium iron boron alloy,  $\text{Nd}_2\text{Fe}_{14}\text{B}$ , of approximate diameter 5 mm. Searching online, I found that magnet spheres can be used to create beautiful sculptures, such as the caged bubble star by Magnenaut [2] (Fig. 1). The magnets, being spherical, can be connected to each other at any angle, opening the door to shapes that would be virtually impossible to build with Legos or other fixed-angle construction sets (Fig. 2, Video "Connections\_Zen\*.mp4"). Magnet spheres are like a jigsaw puzzle with a solution that you devise for yourself.

In November 2012, I placed an online order for 18 "booster" sets of Zen Magnets, another manufacturer of magnet spheres, at a price of \$32.98 per set, each set with 216 magnets and six spare magnets. In March 2013, I placed a second order, for three booster sets, and in April 2013, I placed a third order, for five booster sets. For the magnets in this third order, I measured the lengths of

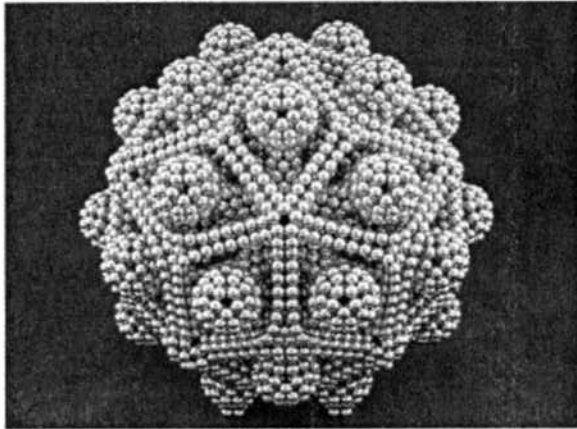


FIG. 1. Photograph of a caged bubble star designed, built, and photographed by Magnenaut using 6205 Zen Magnets [3]

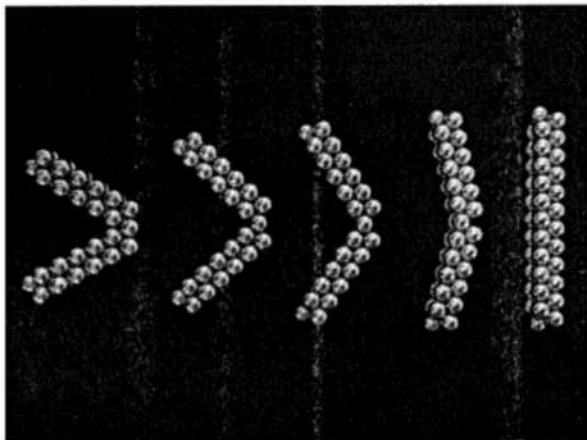


FIG. 2. Demonstration of the range of stable angles of an elbow in a  $2 \times 2$  Zen Magnets strut, illustrating the versatility of spherical magnets in building shapes involving a variety of angles.

chains of 200+ magnets with a tape measure, and determined that the magnet diameters were slightly larger than the advertised diameter range of  $5 \text{ mm} \pm 0.01 \text{ mm}$ . I notified the company of the matter, and returned the magnets. This cycle was repeated twice. Zen Magnets founder, Shihan Qu, then informed me that he had discovered that the tape measure he was using to determine magnet chain lengths was faulty. In August 2013, I received replacement magnets of the correct size, together with 10 additional booster sets, minus the spares. These additional sets were awarded for pointing out the oversize problem and for my help as Qu designed a reliable way to measure chain lengths.

I have participated in two photography contests hosted by Zen Magnets [4]. In November 2013, I placed second in Contest 34: Shadow Projection: Second Generation, and was awarded \$55 in Zen Magnets credit. In

May 2014, I placed first in Contest 39: The Flagship Video Contest (Video “Playing\_with\_Plato\*.mp4”), and was awarded \$2500 and a “Mandala” set including 1728 magnets and 8 spares. Excluding spares, the Mandala set contains the same number of magnets as eight booster sets. The Mandala set retails for \$263.84, which is eight times the cost of a booster set.

I have submitted 64 photographs of magnetic sculptures of my own creation to the online Zen Magnets gallery, which accepts four photographs per day and rewards successful photographers with one booster set for every three photographs accepted. Of these submissions, 46 photographs have been accepted, and 42 of these have been redeemed for 14 booster sets. I have authored 26 YouTube videos describing how to build various shapes with Zen Magnets [5].

I now own the equivalent of 84 booster sets of Zen Magnets, some that came with spare magnets and some that did not, for a total of over 18,000 magnets. Of these 84 sets, 34 were purchased with cash, 14 were received for photographs accepted into the Zen Magnets gallery, 8 were awarded for winning Zen Magnets Contest 39, 18 sets were traded for cash winnings from Zen Magnets Contests 34 and 39, and 10 sets were awarded for my help with the oversize problem.

In this report, I refute two paragraphs in the CPSC Complaint Counsel’s Second Amended Complaint in the matter of Zen Magnets, LLC, Docket No. 12-2:

105. Upon information and belief, the Subject Products have low utility to consumers.

106. Upon information and belief, the Subject Products are not necessary to consumers.

Zen Magnets have educational, creative, and artistic utility, and thousands of consumers have expressed the need for these magnets, as shown below.

## II. PHYSICS

### A. Electromagnetism

Despite having taught undergraduate and graduate electromagnetism for years, and despite knowing the mathematical equations describing how dipole magnets attract or repel each other depending on their relative orientations [6], I was surprised at how Zen Magnets always seem to find a way to attract each other. Parallel bar magnets repel each other when they are placed side-by-side, yet parallel chains of magnets attract each other when brought into close proximity (Fig. 3). Consequently, adjacent chains and adjacent rings connect in two distinct ways depending on their magnetic orientations (Fig. 4, Video “Connections.Zen\*.mp4”).

This hands-on experience taught me practical concepts of electromagnetism, a subject that I thought I knew well. The key was experimenting with the magnets, which

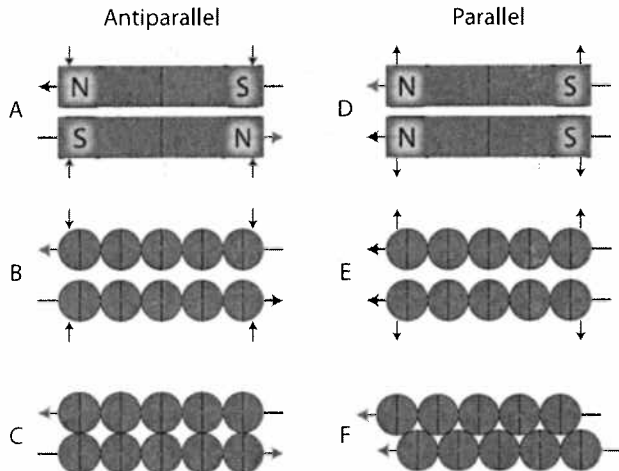


FIG. 3. Schematic diagram illustrating the stable configurations of parallel and antiparallel chains of magnet spheres. North magnetic poles are denoted by the color red and, in some cases, by the letter “N.” South magnetic poles are denoted by the color green and, in some cases, by the letter “S.” Axial magnetic field lines are denoted by horizontal (violet) arrows, and always point from south to north poles. Forces between magnets are denoted by vertical (black) arrows. A: Bar magnets with antiparallel magnetic fields attract each other when they are placed side-by-side because the north pole of one magnet attracts the south pole of the other. B: Antiparallel chains of magnet spheres attract each other for the same reason. C: Stable configuration for antiparallel chains. D: Parallel bar magnets repel each other when they are placed side-by-side because their north and south poles repel each other. E: Parallel chains of magnet spheres repel each other for the same reason. F: However, when forced into close proximity, parallel chains shift by half a magnet so that north and south poles line up, yielding a stable configuration with spheres in one chain fitting into the gaps in the other chain.

caused me to consider magnetic configurations that I hadn’t studied before. Once I had seen the behavior of the magnets, it was straightforward to explain it. This is education at its best, letting exploration drive questions that can be answered using the scientific method.

Magnet spheres present unique opportunities for the physics classroom and laboratory. In physics departments in universities in the United States, electromagnetism is often taught to students of physics, mathematics, chemistry, and engineering at four levels: (1) as the second semester of a general physics classroom/laboratory course, for freshman and sophomore students, (2) in a classroom-only electromagnetism course for juniors and seniors, (3) as part of an advanced physics laboratory course for juniors and seniors, and (4) as a specialized classroom-only electromagnetism course for graduate students. Students build their physical understanding and mathematical proficiency as they proceed from one level to the next.

Zen Magnets closely approximate uniformly magne-

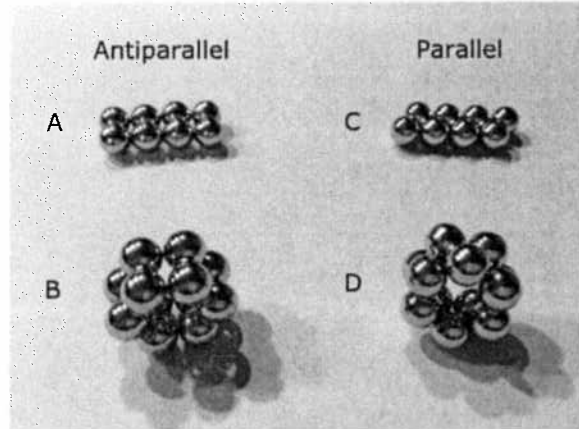


FIG. 4. Photograph of parallel and antiparallel configurations of Zen Magnets. A: Two antiparallel chains with four magnets each (Fig. 3C). B: Two antiparallel chains with six magnets each whose ends are joined to form a double ring, with axial magnetic field lines now forming two circles that circulate in opposite directions. C: Two parallel chains with four magnets each (Fig. 3F). D: Two parallel chains with six magnets each whose ends are joined to form a double ring, with axial magnetic field lines now forming two circles that circulate in the same directions. For parallel connections, spheres in one chain or ring fit into the gaps in the other chain or ring. For antiparallel connections, spheres in one chain or ring line up side-by-side with spheres in the other chain or ring. Since the north and south poles of Zen Magnets are not marked, it’s not possible to tell visually whether the magnetic field of a ring of magnets is clockwise or counterclockwise. The way that magnets connect together can therefore seem mysterious until builders understand these basic magnetic principles. Once these principles are understood, the behavior of the magnets is predictable.

tized spheres, one of the most important examples of magnetism in the undergraduate and graduate physics curriculum. Figure 5 shows the magnetic field lines of a uniformly magnetized sphere, which are straight and parallel within the sphere, and follow circular paths outside of the sphere, except for the axial magnetic field line, which is an infinite straight line directed from the south pole to the north pole, as seen in Fig. (3).

The magnetic field  $\mathbf{B}$  at a position  $\mathbf{r}$  outside of a sphere of radius  $a$  and uniform magnetization  $\mathbf{M}$  is among the simplest of all magnets, being the magnetic field of a point dipole  $\mathbf{m}$  located at the center of the sphere [7, 8],

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi r^3} [3(\mathbf{m} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}], \quad (1)$$

where  $\hat{\mathbf{r}} = \mathbf{r}/r$ . Here,

$$\mathbf{m} = \frac{4}{3}\pi a^3 \mathbf{M} \quad (2)$$

is the dipole moment. These equations mathematically describe the magnetic field lines outside of the sphere shown in Fig. 5. Inside the sphere, the magnetic field is

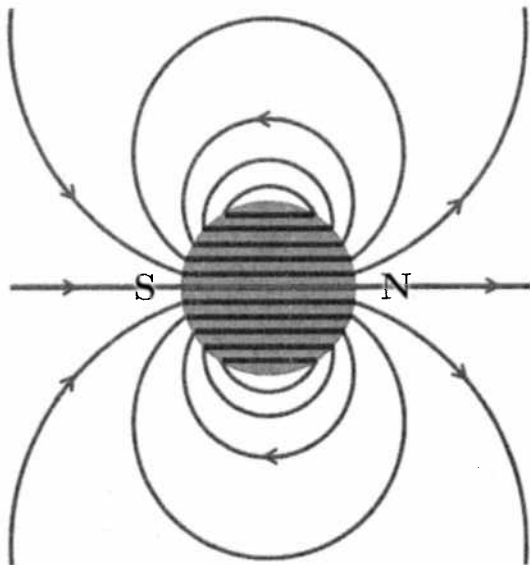


FIG. 5. Lines of magnetic field  $\mathbf{B}$  produced by a sphere of uniform magnetization  $\mathbf{M}$ . These lines approximate the magnetic fields produced by a single magnet sphere, such as those produced by Zen Magnets. This magnetic field is one of the most important solvable magnetic field configurations, and is a staple of undergraduate and graduate physics education. Inside the sphere, the magnetic field is uniform and parallel to the magnetization, according to Eq. (3). Outside the sphere, the magnetic field replicates the dipole magnetic field given by Eqs. (1) and (2). The similarity between the measured north pole and south pole magnetic fields (Table I) indicates that the magnetic fields produced by Zen Magnets are similar to the fields shown in this figure. Diagram courtesy <http://en.citizendium.org>.

uniform and obeys

$$\mathbf{B} = \frac{2\mu_0}{3}\mathbf{M}. \quad (3)$$

Evaluated along the axis of the magnetic poles ( $\hat{\mathbf{r}}$  parallel to  $\mathbf{m}$ ) at a distance  $r \geq a$  from the center of the magnet, Eq. (1) yields the axial magnetic field,

$$B = \frac{\mu_0 m}{2\pi r^3}. \quad (4)$$

An instructive undergraduate exercise is to determine the magnetic field produced by a chain of magnets. A chain of magnets produces a stronger magnetic field than a single magnet. But how much stronger? The answer comes through superposition, a key principle of physics, which states that the net magnetic field is the vector sum of fields from all sources. In contrast with iron and steel, neodymium magnets have high coercivity, meaning that they have a high resistance to demagnetization by external magnetic fields [9]. We can therefore assume that nearby magnets do not affect the magnetization of such magnets, and we can simply add up the magnetic fields produced by all of the magnets in the chain in order to

determine the net magnetic field. The following exercise is suitable for advanced physics laboratory students:

**Problem 1:** (a) Use a gauss meter to measure the north and south polar magnetic fields of four magnet spheres. (b) Deduce their magnetic moments from Eq. (4), assuming that they are uniformly magnetized. (c) Apply the principle of superposition to compare predictions and measurements of the axial magnetic field at the end of chains of two, three, and four magnets.

## B. Energy and Forces

The energy  $U = -\mathbf{m} \cdot \mathbf{B}$  of a magnetic dipole moment  $\mathbf{m}$  in the presence of a magnetic field  $\mathbf{B}$  reaches a minimum when  $\mathbf{m}$  is parallel to  $\mathbf{B}$  and when  $\mathbf{B}$  is at its maximum possible value. For two magnets, this energy minimum is achieved when the south pole of one magnet is in direct contact with the north pole of its neighbor, as seen in Fig. 3. If one of the magnets is held fixed, and the other is moved from this minimum energy position, the latter magnet will experience a force

$$\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B}) \quad (5)$$

and a torque

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B} \quad (6)$$

that tend to move and reorient the magnet back to its minimum energy position.

The following exercise is suitable for advanced undergraduate physics students, and is pertinent to angle strain in organic molecules (Sec. III C).

**Problem 2:** Calculate the ground-state energies of symmetric rings of 3, 4, 5, 6, 7, and 8 uniformly magnetized spheres. Treat the spheres as magnetic dipoles, each with magnetic moment  $\mathbf{m}$ . Include only the energies of nearest neighbor interactions. Show that the energy per magnet decreases with increasing ring size.

## III. CHEMISTRY

### A. Crystal Structure

Crystal structure is covered in general chemistry courses, solid state physics courses, and metallurgy courses. Using Zen Magnets to model various crystal structures can make the subject come alive for students, and can help students to visualize difficult three-dimensional lattices. This subject is particularly well suited for demonstration and exploration using Zen Magnets (Video “Lattices\_Zen\*.mp4”).

The simple cubic lattice is the simplest of them all. Figure 6 shows how to build a  $6 \times 6 \times 6$  simple cubic lattice out of Zen Magnets by stacking six  $6 \times 6$  square lattices on top of each other. This  $6 \times 6 \times 6$  cube has exactly 216 magnets, and is the form in which sets of 216

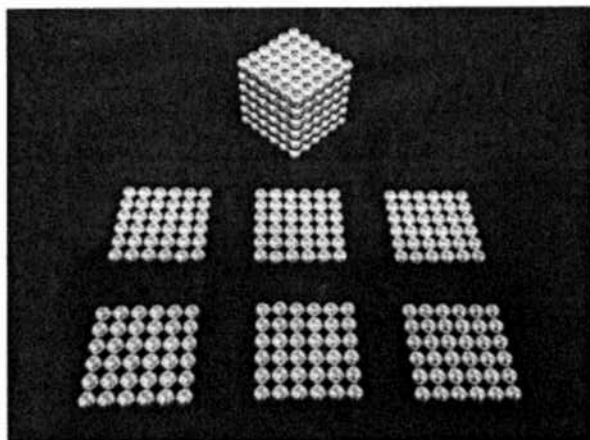


FIG. 6. Photograph of a simple cubic lattice built using Zen Magnets. Shown are six  $6 \times 6$  square layers and a  $6 \times 6 \times 6$  cube formed by stacking such layers directly on top of each other. The resulting lattice has antiparallel connections between all magnets.

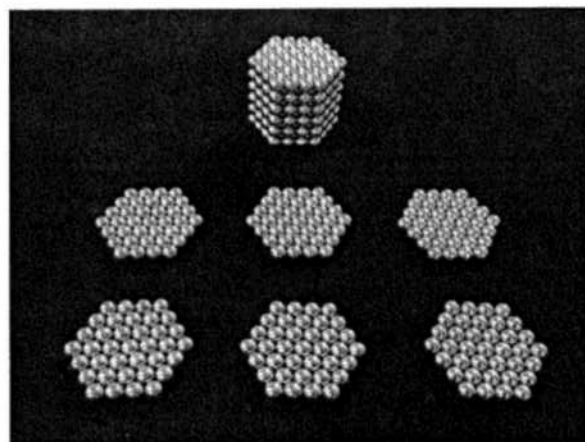


FIG. 8. Photograph of a hexagonal lattice built using Zen Magnets. Shown are six hexagonally-packed layers, each with parallel connections, and a hexagonal lattice formed by stacking six such layers directly on top of each other, with antiparallel connections between layers.

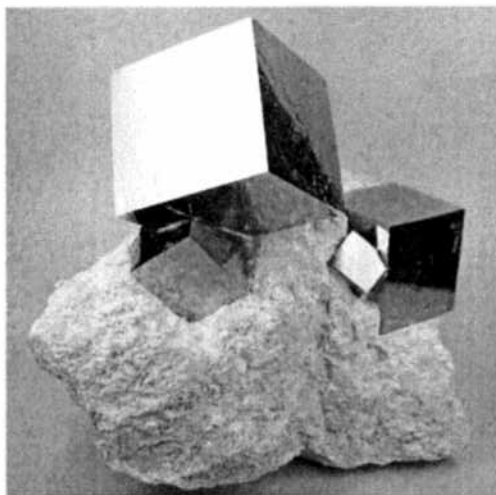


FIG. 7. Photograph of the simple-cubic crystal structure of pyrite, courtesy Wikipedia [10].

BuckyBalls were sold. Polonium and pyrite (a sulfide of iron,  $\text{FeS}_2$ ) have simple cubic lattice structures [10]. Figure 7 shows pyrite crystals with this lattice structure.

The hexagonal lattice has hexagonally packed layers that stack directly on top of each other. Figure 8 shows how to build a hexagonal lattice out of Zen Magnets by stacking six hexagonally packed layers on top of each other. It is the crystal structure of Beryl, a gemstone that comes in several varieties, including aquamarine, emerald, golden beryl, goshenite, and red beryl [11]. Shown in Fig. 9 is a crystal of red beryl. Zen Magnets ships its magnets packed into a hexagonal lattice similar to that shown in Fig. 8.

The hexagonal close-packed lattice has hexagonally packed layers that mesh with each other instead of stack-

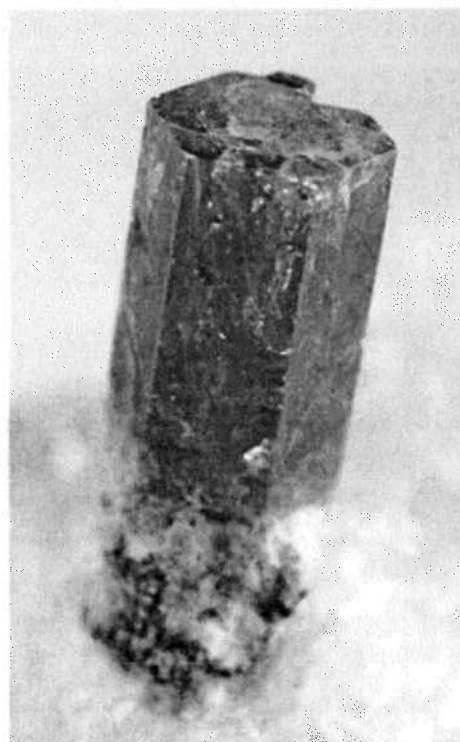


FIG. 9. Photograph of the hexagonal crystal structure of red beryl, courtesy Wikipedia [11].

ing directly on top of each other. That is, magnets in one layer fit into the spaces between magnets in the layer just below, and in the layer just above. The hexagonally close-packed lattice alternates between two types of layers, A and B. Shown in Fig. 10 is a diagram used to teach the structure of this lattice. Figure 11 shows

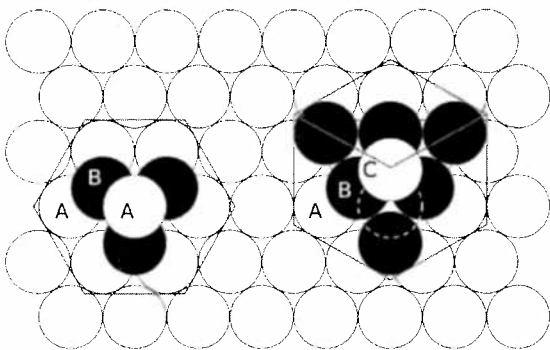


FIG. 10. Diagram of hexagonal close packing (left) and face-centered cubic packing (right), courtesy Wikipedia [13].

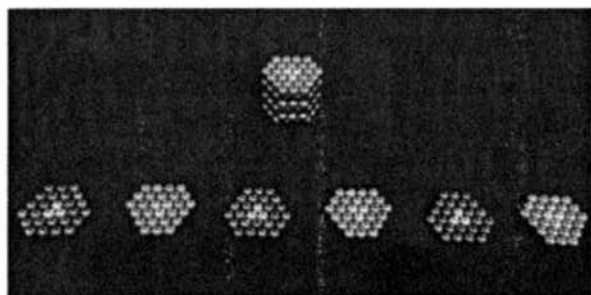


FIG. 11. Photograph of a hexagonally close-packed lattice made from Zen Magnets and colored Neoballs, with two kinds of alternating hexagonally packed layers (with parallel connections) that connect in parallel with the layers above and below.

how to build a hexagonal lattice out of Zen Magnets and colored Neoballs by alternating violet layers A, with upright 3-magnet triangles at their cores, and orange layers B, with inverted 3-magnet triangles at their cores. Hexagonal close packing is the crystal structure of zinc, titanium, and cobalt [12]. Shown in Fig. 12 is the crystalline form of titanium. Colored Neoballs are sold by Zen Magnets, LLC.

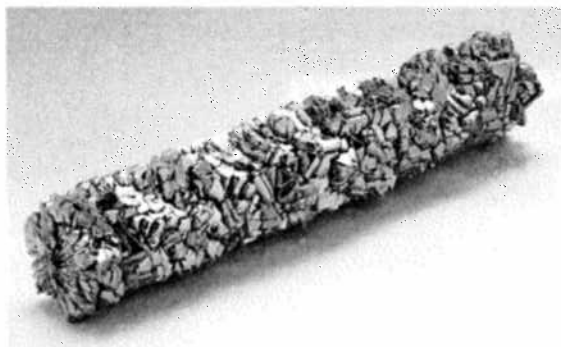


FIG. 12. Photograph of the hexagonal close-packed crystal structure of titanium, courtesy Wikipedia [14].

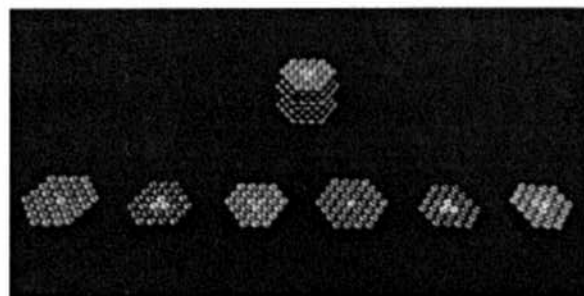


FIG. 13. Photograph of a face-centered cubic lattice made from Zen Magnets and colored Neoballs, with three kinds of alternating hexagonally packed layers (with parallel connections) that connect in parallel with the layers above and below.

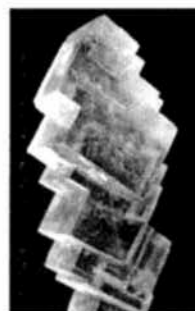


FIG. 14. Photograph of the face-centered cubic crystal structure of halite, or rock salt, the mineral form of table salt, courtesy Wikipedia [17].

Face-centered cubic packing has the same density as hexagonal close packing, which has the maximum possible packing density. The difference between the two is that face-centered cubic packing has three types of layers, alternating ABCABC... Figure 13 shows how to build a face-centered cubic lattice out of Zen Magnets and colored Neoballs by alternating green layers A with single-magnet cores, violet layers B with upright 3-magnet triangular cores, and orange layers C with inverted 3-magnet triangular cores. In this case, the single magnet at the core of an A layer fits into the hollow created by the 3-magnet triangles above and below it. Face-centered cubic metals include aluminum, lead, copper, silver, and gold [15]. The ionic solids sodium chloride (NaCl, table salt) and zinc blende (ZnS) also have face-centered cubic structure [16]. Figure 14 shows the crystal structure of halite, or rock salt, which is the mineral form of table salt. Figure 15 shows the structure of crystalline gold.

In Fall 2012, my wife, Nadine Edwards, took a college chemistry class (CHEM 1210) from Dr. Michael Christiansen at Utah State University Uintah Basin, which included a discussion of the face-centered-cubic lattice [15]. She mentioned to me that my Zen Magnet models of this lattice helped her to understand it better.

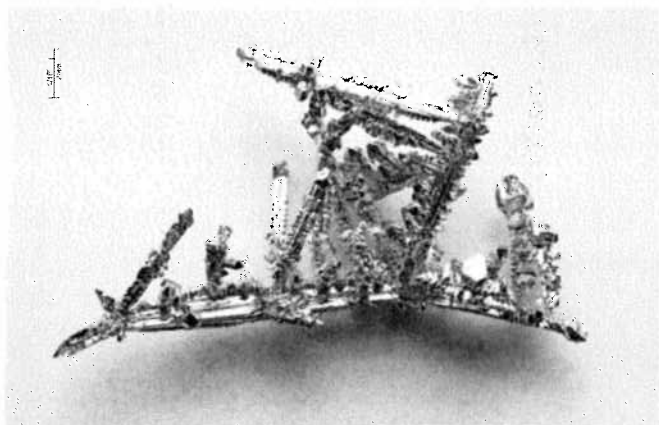


FIG. 15. Photograph of the face-centered cubic crystal structure of gold, courtesy Wikipedia [18].

Face-centered-cubic packing is used in building the diagonal cube, a popular cube shape built from diagonal layers of magnet spheres. Producing a tutorial video on this shape [19] taught me better than any textbook how the face-centered-cubic lattice fits together in three dimensions because I was forced to confront exactly how each layer connects to the next. Figure 16 shows a side view (a) and a top view (b) of a family of diagonal cubes that I built using this technique. The top view inspired Zen Magnets photography contest #34, Shadow Projection: Second Generation [20].

Magnenaut published an alternate construction method for the diagonal cube [21]. His cube looks identical to the standard diagonal cube – it has the same face-centered-cubic packing with magnets in the same locations. But it is magnetically quite different. The magnets are rotated in their places and connect to each other differently to yield circular magnetic polarities at all eight corners, instead of just at two, allowing greater flexibility in attaching the cube to other structures. The fact that two seemingly identical cubes could have such different magnetic properties reflects the amazing versatility of these magnets, the large numbers of ways of connecting them, and their value as a tool for creative exploration.

Figure 17 shows how face-centered cubic packing is used to build the diagonal cube with Zen Magnets and Neoballs, with the same three types of layers as Fig. 13. Video “Lattices.Zen\*.mp4” shows how to transform Fig. 13 into Fig. 17.

Building these lattices using Zen Magnets helped me to fully appreciate their structure for the first time. I studied these lattices in solid state courses that I took as a student, trying to make sense of them using diagrams like Fig. 10. But such diagrams simply do not effectively communicate how lattices fit together. It takes a tactile experience, an opportunity to examine each layer and place it atop the previous. Thus I can personally attest to the power of magnet spheres in bringing understand-

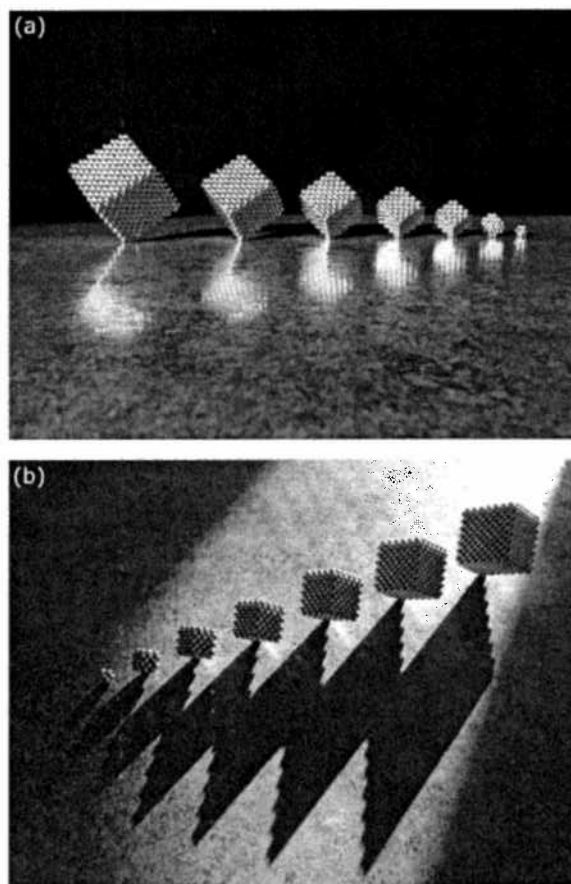


FIG. 16. Side-view (a) and top-view (b) photographs of a family of diagonal cubes built by the author and illuminated by the setting sun. The top view inspired a Zen Magnets shadow projection photography contest (see text).

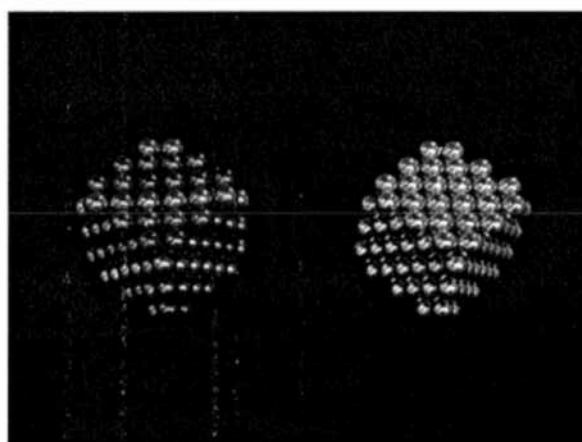


FIG. 17. Photograph of two diagonal cubes, one made of colored Neoballs with the same face-centered cubic structure as Fig. 13, and the second identical in every respect except that all of the magnets are the same color.



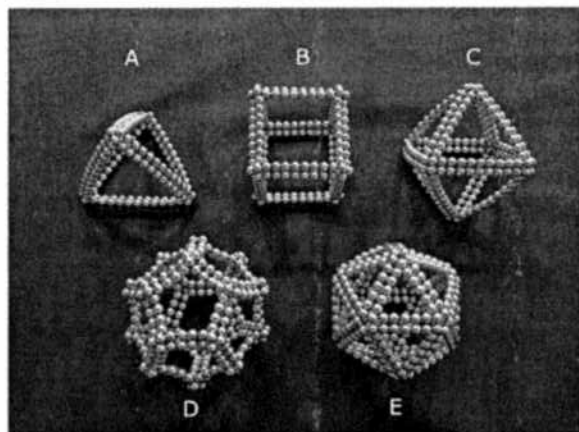


FIG. 18. Photograph of Platonic solid frames built using Zen Magnets, including a tetrahedron with 4 triangular faces (A), a cube with 6 square faces (B), an octahedron with 8 triangular faces (C), a dodecahedron with 12 pentagonal faces (D), and an icosahedron with 20 triangular faces (E). Molecules with more than one symmetry axis have tetrahedral, octahedral, or icosahedral symmetry.

ing of crystal structure. While exploring these lattices using Zen Magnets, I also tried models using ping-pong balls, which proved very difficult to hold together during construction - the slightest jostle and I had ping-pong balls all over the table and floor. With magnets, the magnetic force supplies all of the force necessary to hold the structures elegantly together.

### B. Molecular Structure

A staple of chemistry education is the study of molecular structure, that is, how molecules are formed out of atoms. And here, Zen Magnets have great utility. Magnet spheres can be used to teach molecular structure and symmetry through models of molecules and atomic clusters. Of particular interest are molecules with more than one symmetry axis, including methane ( $\text{CH}_4$ , with tetrahedral symmetry, Fig. 18A), sulfur hexafluoride ( $\text{SF}_6$ , with octahedral symmetry, Fig. 18C), and the buckminsterfullerene ( $\text{C}_{60}$ , with icosahedral symmetry, Fig. 18E) [22], as well as 13-atom icosahedral clusters formed by neon, argon, krypton, and xenon [23]. Figure 19 shows a photograph of a model of a 13-atom cluster, an icosahedron frame with atoms located at its 12 vertices and with one atom located at its center. Care must be taken to ensure correct magnetic polarities in building this model so that all of the components fit together correctly [24]. That is, success in constructing the shape is contingent upon understanding its underlying magnetic structure.

On June 28, 2014, my wife Nadine Edwards described to me how magnet spheres help students visualize electron and molecular configurations in her college chemistry class, saying:

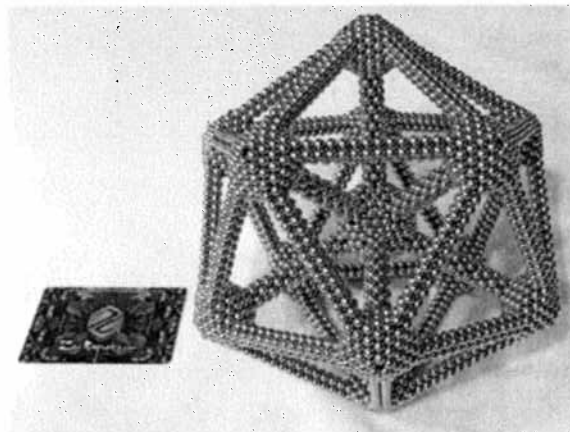


FIG. 19. Photograph of a model of a 13-atom icosahedral cluster made with 3860 Zen Magnets. Such clusters can be formed by neon, argon, krypton, and xenon, and have 12 atoms at the icosahedral vertices and one atom at the center.

In Fall 2012, I took a college chemistry class (CHEM 1210) from Dr. Michael Christiansen at Utah State University Uintah Basin. In class, we learned about electron domains of atoms and molecular geometry of molecules, including linear, trigonal planar, tetrahedral, trigonal bipyramidal, and octahedral [25]. I left the classes and went home where my husband showed me Zen Magnet shapes including a tetrahedron, octahedron, and dodecahedron, some of which I had learned about in chemistry. As he showed me the shapes he had built and as he used their names, the Zen Magnets reinforced what I was learning in chemistry.

### C. Angle Strain

Brian Kirk, author of statement 411 in Appendix E and holder of a 1984 B. A. in Chemistry from Hanover College in Hanover, Indiana, invented a way to use magnet spheres to illustrate strain-induced reactivity of organic molecules [26]. Atoms have optimal angles at which they like to bind with each other to form molecules. For example, the optimal angle for carbon is  $109.5^\circ$  [27]. Molecules with bond angles that are at or near these optimal angles tend to be more stable, and to have lower energies, than molecules that deviate significantly from these angles. Molecules with non-optimal bond angles can rearrange, via chemical reactions, into lower energy configurations with bond angles that are closer to optimal. “Angle strain” is the term used to describe the increase in energy, reactivity, and heat of combustion of molecules with non-optimal bond angles [27].

Magnet spheres prefer to connect in straight lines with

north and south poles of adjacent magnets in direct contact with each other (Fig. 3). Thus, magnet spheres can be used to illustrate angle strain for molecules with optimal bond angles of  $180^\circ$ . As discussed in Sec. II B, straight-line connections between magnets minimize the energy, and any deviation from a straight line leads to forces and torques that tend to reorient and realign the magnets into a straight line. Bending a chain of magnets increases its energy and introduces angle strain. The smaller the bend radius, the greater the angle strain.

Consequently, small rings of magnets have large angle strain and high reactivity, as seen by the following simple demonstration. Roll a single magnet toward a 3-magnet ring (Fig. 20). The ring quickly opens to accommodate the new magnet, relieving some angle strain by rearranging to form a symmetric ring of 4 magnets. Four-magnet rings behave similarly when a fifth magnet is rolled toward them, producing a symmetric ring of 5 magnets. As magnets are incorporated into the expanding ring, its angle strain decreases until it no longer accommodates new magnets, which stick instead to the outside of the ring (Fig. 20, Video "Angle\_Strain\_Zen\*.mp4"). The maximum number of magnets accommodated by a ring ranges between 6 and 9 magnets. Larger rings have less angle strain and find it energetically less favorable to open to accommodate a new sphere. Thus, small rings have large angle strain and high reactivity, while large rings have small angle strain and low reactivity.

Weak magnets (Sec. VIII B) are ill suited for this demonstration (Video "Angle\_Strain\_Weak.mp4"). Following the procedures in the previous paragraph 100 consecutive times using weak magnets, I succeeded in replicating the demonstration only 16 times because the magnet that was rolled toward the 3-magnet ring generally produced some shape other than a symmetric or nearly-symmetric ring of 4 magnets (Video "Angle\_Strain\_100\_Trials\_Weak.mp4"). Following the procedures in the previous paragraph 100 consecutive times using regular Zen Magnets, I succeeded in replicating the demonstration 87 times (Video "Angle\_Strain\_100\_Trials\_Zen.mp4"). For both Zen Magnets and weak magnets, a trial was considered successful if it produced symmetric or nearly symmetric 4-magnet and 5-magnet rings.

This reactivity can also be seen by sliding one small ring into another identical ring; these rings quickly combine to form a symmetric ring with twice as many magnets. This works for rings of 3 and 4 magnets, but not for rings of 5 or more, which simply attach together without opening into a larger ring, as shown in Fig. 21.

I explained this demonstration to Dr. Michael Christiansen, Assistant Professor of Chemistry at Utah State University - Uintah Basin [28] and asked him whether the demonstration would be useful in teaching strain-induced reactivity. He confirmed that it would, and expressed interest in obtaining magnet spheres in order to incorporate the demonstration into his organic chemistry classes. He mentioned that the experience of tactile learning in-

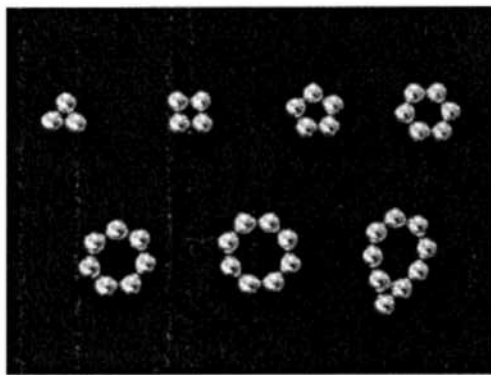


FIG. 20. Rings of 3, 4, 5, 6, 7, 8, and 9 sphere magnets used to illustrate strain-induced reactivity in organic molecules. The ring of 4 was produced by rolling a single magnet into the ring of 3, which quickly opened to accommodate the new magnet to produce a ring of 4. The ring of 5 was produced by rolling a single magnet into the ring of 4, and so on. Large rings with 8 or more magnets have less angle strain, and do not generally open to accommodate new magnets, which instead attach to the outside of the ring. Thus, small rings have high angle strain and high reactivity, while larger rings are stable.

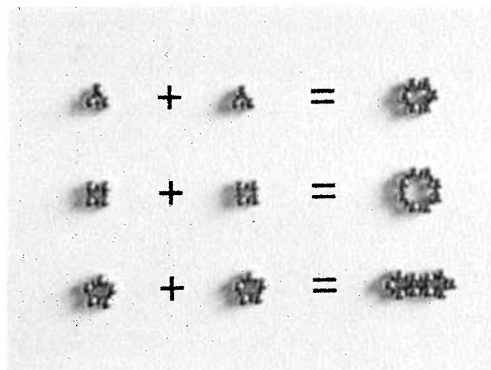


FIG. 21. Second illustration of chemical reactivity resulting from angle strain in organic molecules. Two highly-strained rings of 3 magnets combine to form a ring of 6 magnets. Two rings of 4 magnets combine to form a ring of 8 magnets. Two rings of 5 magnets attach to each other, but do not have enough angle strain to produce an open ring of 10 magnets.

creases comprehension over simple visual and auditory learning, because it adds one more sense to the learning process. He said that magnet spheres can convert the abstract microscopic concept of atoms into something macroscopic that students can see and touch. He pointed out that the dynamics of this angle strain demonstration could not be accomplished with standard ball-and-stick chemistry models, which have fixed angles and lack an attractive force to bring the "atoms" together.

I also explained the demonstration to Dr. Alvan Hengge, Professor of Organic Chemistry and Head of the Department of Chemistry and Biochemistry at Utah



FIG. 22. Motor built by Graham Nash out of magnet spheres. This motor is the subject of the second-most popular magnet-sphere YouTube video, with over 8 million views [30].

State University [29]. Dr. Hengge pointed out that, while the demonstration does not replicate the details of chemical reactions, it is useful in illustrating the concept of reactivity associated with angle strain, especially when coupled with understanding of the energetics of magnetic dipoles (Sec. II B).

The demonstration illustrates the unique utility of magnet spheres for chemistry education. Necessary for this angle strain demonstration are the strong attractive forces of magnet spheres and their ability to connect with each other at a continuous range of angles. As pointed out by Mr. Kirk, the demonstration emphasizes that molecular bonds are not like sticks in ball-and-stick molecular models, but are energy fields, modeled here by magnetic energy fields.

Mr. Kirk reviewed this section and consented to have me include it in this report.

## IV. ENGINEERING

### A. Motors

Electrical engineers must understand how motors work. The second-most popular magnet-sphere YouTube video, with over 8 million views, shows a motor built by Graham Nash out of magnet spheres [30] (Fig. 22).

### B. Crystal formation and defects

On July 4, 2014, I talked with Dr. Stephen Niezgoda, Assistant Professor of Materials Science Engineering at The Ohio State University about his educational use of magnet spheres [31]. I approached Niezgoda after seeing his comment on the proposed CPSC rule: Safety Standard for Magnet Sets [32], transcribed the essence of our conversation, and sent the transcript to him. He replied and granted me permission to include this transcript in this report.

In 2009-2010, while a graduate student at Drexel University, Niezgoda used magnet spheres to demonstrate the complex structure of crystalline materials, including

defects such as dislocations and vacancies, at a week-long materials camp sponsored by the American Society for Materials (ASM) and at internal recruitment efforts within the university. The purpose of these events was to recruit undergraduate students to the Drexel University Materials Science Department. Attendees at the camp included Drexel University undergraduate engineering students who had not yet chosen a major and high school students who were considering attending Drexel University.

Niezgoda owned some Zen Magnets at the time, and realized that they could be used at the materials camp to demonstrate crystal defects, a central concept in metallurgy. He worked with his department to purchase several sets of magnet spheres for this purpose. At the camp, students formed a single-thickness layer of magnets on a table by smashing an amorphous blob of magnets onto the surface, and investigated the structure of this layer (Fig. 23, Video “Lattice\_Defects\_Zen\*.mp4”). These layers include regions of cohesive hexagonal lattice structure separated by defects, dislocations, cleavages, crystallites, and vacancies.

Niezgoda used this activity to teach the students that such defects in crystalline structure help to define differences between materials, and that the temperature (the extent of thermal agitation) during the crystal formation process affects the equilibrium concentration of these defects. This simple exercise demonstrates key thermodynamic concepts of how crystalline materials self organize into complex structures on solidification, and how engineering materials such as metals that appear homogenous on the macroscopic scale can be quite heterogeneous on the microscopic level.

Niezgoda pointed out that this learning activity could not be accomplished with standard ball-and-stick molecular models because of the dynamic and imperfect nature of the simulated crystal growth process.

### C. Deformations in close-packed lattices

Niezgoda also commented on the utility of magnet spheres for demonstrating deformation in defect-free close-packed regular three-dimensional lattices. He pointed out that such demonstrations help metallurgy students to understand why different metals have different material properties. For example, magnet spheres can be used to build the face-centered cubic (FCC) lattice and to demonstrate its well-known mechanisms of accommodating permanent, or “plastic” deformation. Such plastic deformation is key to the malleability and ductility of aluminum, copper, gold, and silver.

To build the lattice, stack hexagonal layers in an ABCABC arrangement as in Fig. 13. To demonstrate the deformation mechanisms, apply a shear force and deformation will occur by sliding between the 111 (cube diagonal) planes, with atoms in one layer slipping past the atoms in the next. The magnets naturally slide along

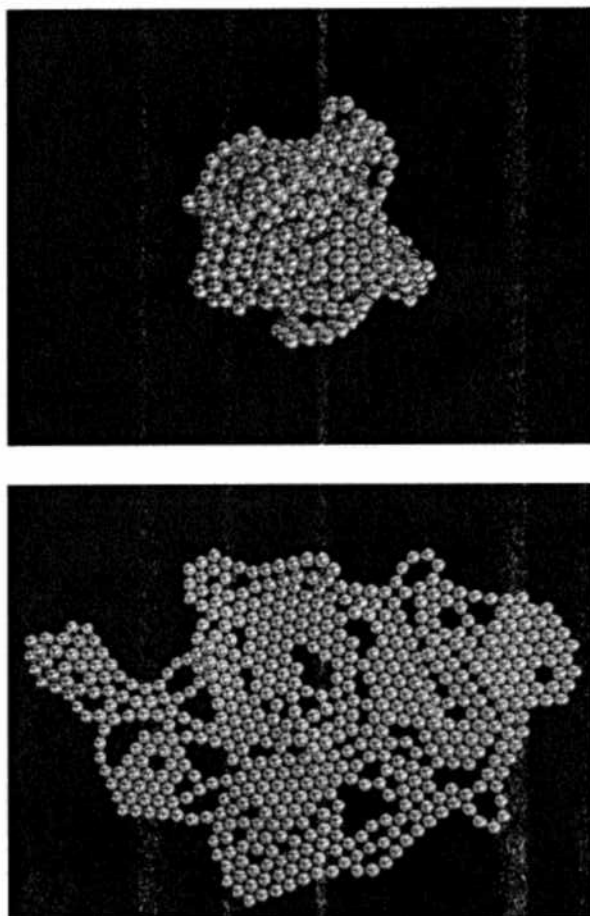


FIG. 23. Photograph of an amorphous blob of magnets (top), and this same blob after being smashed into a single-thickness layer of magnets (bottom). This process was used by Dr. Stephen Niezgoda at a materials camp to simulate the formation of defects during crystal growth.

this direction, just as atoms in FCC lattices do in nature. Figure 24 and Video “Lattices\_Zen\*.mp4” show how this mechanism works, by sliding a close-packed layer relative to another. Niezgoda mentioned that the capability for magnet spheres to demonstrate such slippage along crystallographic planes gives them a significant educational advantage over ball-and-stick models, which lack this capability.

#### D. Structural Engineering

Zen Magnets can also be used to teach and learn principles of structural engineering. A cuboctahedron frame that I built using Zen Magnets has generated many comments about the engineering principles that enable the structure to support itself (Fig. 25), principles that are taught in structural engineering courses designed to train civil engineers to design buildings and bridges [33].

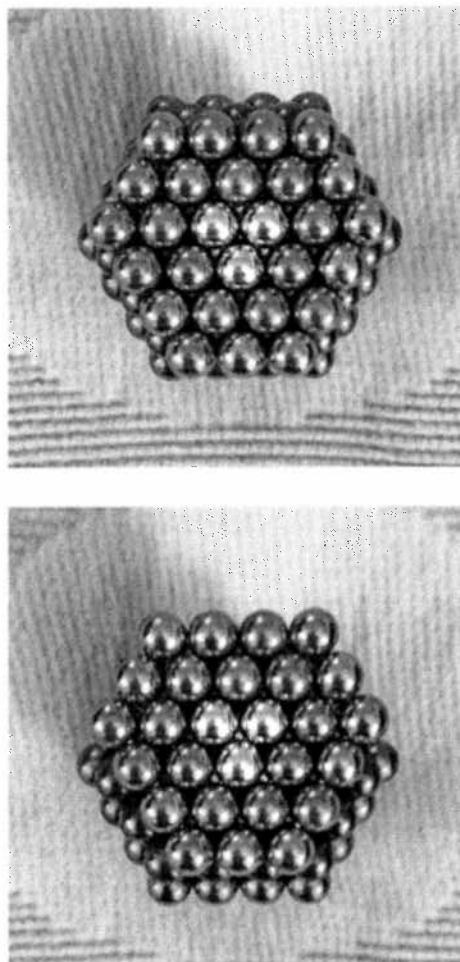


FIG. 24. Demonstration, using Neoballs and Zen Magnets, of the slip mechanism that is responsible for the softness of metals with face-centered cubic crystal lattices, such as aluminum, copper, gold, and silver, showing the lattice before (top) and after (bottom) displacing the top layer.

## V. BIOLOGY

### A. Protein Structure and Function

Anthony J. Pelletier uses magnet spheres to teach protein structure and function at the high school level. Using magnet spheres, he has built helical molecules, boxes to encase DNA, an icosahedron to model an ando virus, and a dodecahedron to model a rhinovirus [36]. His students use Zen Magnets and BuckyBalls to aid in understanding how simple repeating subunits can be used to build more complex structure (See Appendix E, statement 212).

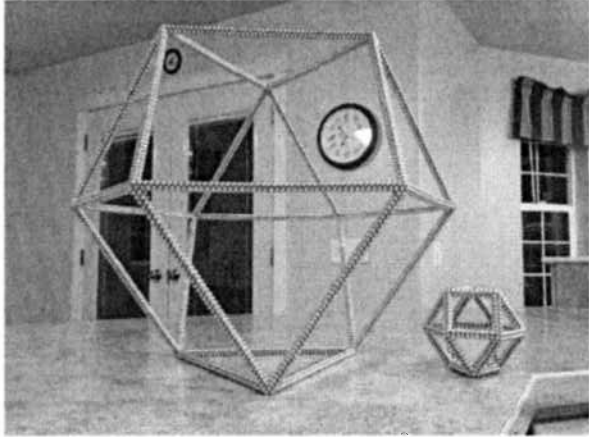


FIG. 25. Photograph of a large 4,416-magnet cuboctahedron frame designed by the author and a small 912-magnet cuboctahedron frame designed by Magnenaut [34, 35]. Comments marvel at the power of Zen Magnets to support such a large structure, and discuss engineering principles that enable the structure to support itself.

### B. Amoeba Movement

W. Beaty has used magnet spheres to model the movement of amoebas [37].

### C. Cell Division

A U.S. teacher uses magnet spheres in her seventh grade science class to demonstrate cell division and chromosome splitting [38]. Figure 26 shows how she mimics cell division by creating a “parent” ring of 16 magnets, pinching it together in the middle, and separating two 8-magnet “daughter” rings [39].

I published a YouTube video entitled “Playing with Plato” that shows how to build the five Platonic solids using Zen Magnets [40]. In the video, transitions between these shapes include ring divisions similar to those shown in Fig. 26 (Video “Playing\_with\_Plato\*.mp4”). These divisions prompted one viewer to comment on the similarity with cell division [41].

Magnet spheres also aid in understanding DNA replication and chromosome segregation, essential processes during cell division. Figure 27 shows a model of a DNA strand with 10 base pairs, with nitrogenous bases represented by magnet spheres of different colors:

- A = adenine (silver)
- C = cytosine (violet)
- G = guanine (green)
- T = thymine (orange)

The model obeys the DNA base pairing rules, with adenine pairing only with thymine and cytosine pairing only with guanine [42]. During DNA replication and chromosome segregation, the two strands in the molecule sepa-

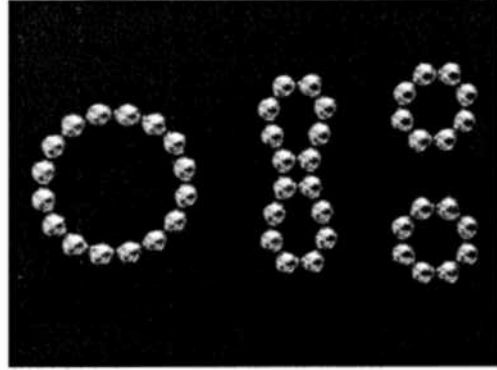


FIG. 26. Demonstration of cell division using Zen Magnets.

rate and each strand serves as a template for a new identical molecule, constructed by adding complementary nitrogenous bases according to the pairing rules.

A valuable classroom learning activity is to ask groups of students to follow the base pairing rules in designing and creating a double DNA strand like that shown in Fig. 27, to separate it into two single strands, to have different groups use these as templates to create two new double strands, and to compare the two new strands. If they are identical, then both groups succeeded in the activity. If not, then the activity serves to illustrate genetic mutation.

I described this activity to Dr. Edmund Brodie, Professor of Biology at Utah State University [43], who stated that the demonstration might be useful at the junior high level.

The double strand shown in Fig. 27 can be twisted by hand to demonstrate the helical structure of the DNA molecule. Alternatively, this helical structure can be demonstrated by building a more rigid model of the DNA molecule (Fig. 28).

The classroom activity mentioned above can be carried out using pennies, nickels, dimes, and quarters, for example, to represent the four nitrogenous bases. The advantage of magnets for this activity is that they attract each other, mimicking the chemical bonds and producing a structure that holds itself together, as the DNA molecule does. Coins cannot easily be used to replicate the helical structure shown in Fig. 28.

## VI. MATHEMATICS

Magnet spheres can be used as a math manipulative, a physical object that can help students learn mathematics. Using manipulatives in mathematics instruction produces a small- to medium-sized effect on student learning when compared with instruction that uses abstract symbols alone [45].

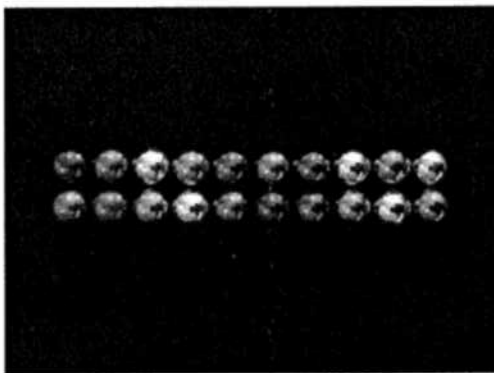


FIG. 27. Model of a DNA strand with 10 base pairs, made of two 10-magnet antiparallel strands of Zen Magnets and Neoballs, with four magnet colors representing the four nitrogenous bases.

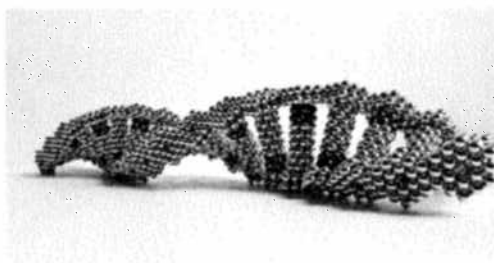


FIG. 28. Zen Magnets Double Helix, a Flickr photograph by Thomas [44].

### A. Solid Geometry

Magnet spheres excel at solid geometry. In fact, it was the amazing replicas of solid geometrical shapes that people had built that motivated me to buy Zen Magnets in the first place.

At a 2013 New Year's party with family members and friends, I used a PowerPoint presentation to teach partygoers about the Platonic solids (Appendix A), and taught them how to use Zen Magnets to build an icosahedron, a Platonic solid with 20 triangular faces (Fig. 29). The five Platonic solids, each with a different number of identical regular polygonal faces (Fig. 18), are the five most common shapes of gaming dice [46] and were once thought to be spacers for planetary orbits [47]. My video showing how to build the Platonic solids won first place in Zen Magnets Contest 39: The Flagship Video Contest [40] (Video "Playing\_with\_Plato\*.mp4")

Another of my videos shows how to build a giant spherical rhombicosidodecahedron frame [48] (Fig. 30), a highly symmetric shape with triangles, squares, and pentagons arranged in a fascinating pattern. The rhombicosidodecahedron is one of 13 Archimedean solids, each employing two or more regular polygonal faces meeting in identical vertices [49].

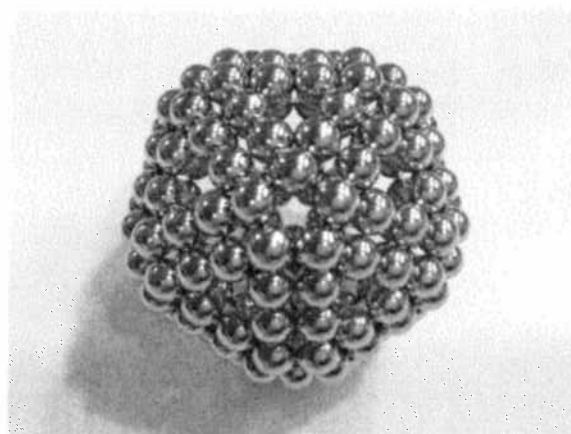


FIG. 29. Photograph of an icosahedron, a Platonic solid with 20 triangular faces, built using 180 Zen Magnets. The author taught nine teams of attendees at a 2013 New Year's party how to build this shape as part of a lesson on Platonic solids.



FIG. 30. Photograph of a hollow spherical rhombicosidodecahedron frame designed and built by the author using 3660 Zen Magnets. This structure barely supports itself under Earth's gravity, and cannot be built using weaker magnets.

### B. Tessellations

Magnet spheres are well suited to demonstrate tessellations. Some solids can be replicated and stacked together to fill all space, with no gaps. These are called tessellations, or honeycombs. Spheres cannot tessellate space, for example, but cubes can. Another example of a tessellation is shown in Fig. 31, which uses rhombic dodecahedra to fill an octahedral volume [50]. Garnet crystals take the shape of rhombic dodecahedra.

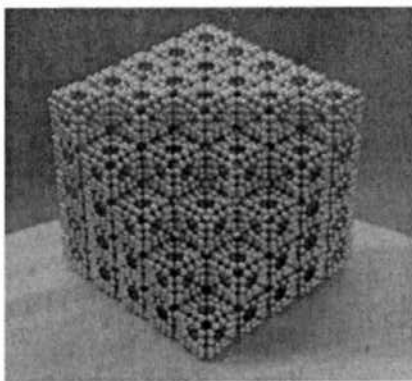


FIG. 31. Tessellating Rhombic Dodecahedra, by Edo Timmerman [50].

### C. Elementary Geometry

Michele LaForge, author of statement 357 in Appendix E, has used magnet spheres to demonstrate concepts in her high-school geometry classes. She holds a 1991 B. A. in Russian and Mathematics from the University of New Hampshire, Durham, New Hampshire, and a 1994 M. A. in Slavic Languages and Literatures from Northwestern University, Evanston, Illinois. She is Head of School at Baxter Academy, in Portland, Maine, a “rigorous, college-preparatory high school promoting student ownership of learning through inquiry and project based curriculum focused specifically on science, technology, engineering, and math (STEM)” [51].

I contacted Ms. LaForge via e-mail for more information about her teaching methods. She responded with a description of her teaching philosophy:

I am a 44 year old, married woman from Maine with degrees in Math and Russian Literature. I have two daughters, twelve and fourteen, and for the last eleven years have been an educator. The first ten were as a math teacher at Freeport High School, the local public high school in Freeport, Maine and I was the department chair for six years of that time. Last year I accepted a new challenge, to start a new school. Until a few years ago, Maine didn't have any charter schools. The legislature invited applications for up to ten charter schools, all *public*. And so I am now the Head of School for Baxter Academy of Technology and Science right in downtown Portland, Maine. Our first year was extraordinary, in my opinion, and our 133 students, and around 15 staff and faculty, did the wonderful work last year of building a curriculum based on hands-on work. The same kind of hands-on work that the best teachers in every school I know have been working into their classrooms for years.

What colleges and businesses are calling for are young people who can solve problems, who can persist in the face of obstacles, new laws, new software, new management, new challenges in the work environment. To do that kids need the same kind of growth experiences that produce resilient thinkers. That means designing, trying, failing, revising, trying, failing. Iteration. They need experience with the tools, the same tools, that adults use to design their projects, build their businesses and buildings and boats: paint and chalk, wood working and metal tools, computers, pencils and ink, paper and tablet.

Buckyballs are one of those tools and like any tool, can be misused without proper training, but they are invaluable aids to learning in physics class. I am not an expert in physics but three of my physics teachers have independently mentioned the fact that the magnets were so hard to procure these days. In my math class, in my home, I used buckyballs to teach my kids persistence—ever tried to get the magnets to easily form the shape in your mind?—and to look for and capitalize on patterns. It was great to provide to students in my Geometry and Honors Geometry classes when we were working on Tiling or the unit on Translations/Rotations/Reflections. For the last ten years, I have used buckyballs consistently in my classroom and at home.

I followed up with Ms. LaForge via telephone to better understand how she uses magnet spheres in her teaching. In her geometry classes at Freeport High School, she distributed sets of magnet spheres and other construction materials for in-class projects that students would carry out at their desks. She finds that students prefer magnet spheres over other construction materials, and she rewards good behavior by granting access to these magnets for classroom activities. I asked for specific examples of such activities. She gave me three:

1. She uses magnet spheres to teach two-dimensional geometrical packing. She asks students to make flat, filled hexagons from the magnet spheres, which can be done by winding concentric rings of magnets around a central magnet (Fig. 32). She then asks them to use the spheres to build flat, filled pentagons and heptagons, and, when they fail, asks the students to explain why these shapes are not possible to build with magnet spheres.
2. She uses magnet spheres to teach symmetry. She asks students to build hexagons and squares and to draw their lines of symmetry, like those shown in Fig. 32, to explore the differences between square and hexagonal symmetries. She teaches rotation symmetry to the students by helping them observe

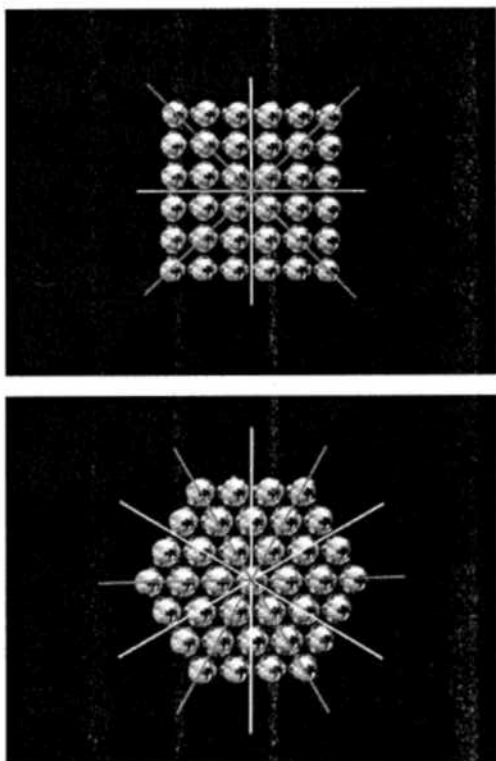


FIG. 32. A filled square and a filled hexagon built using Zen Magnets, with lines of reflection symmetry overlaid. Red lines represent lines of symmetry through corners and white lines represent lines of symmetry through edges. Such figures are used by Michele LaForge to teach symmetries in her high school geometry classes.

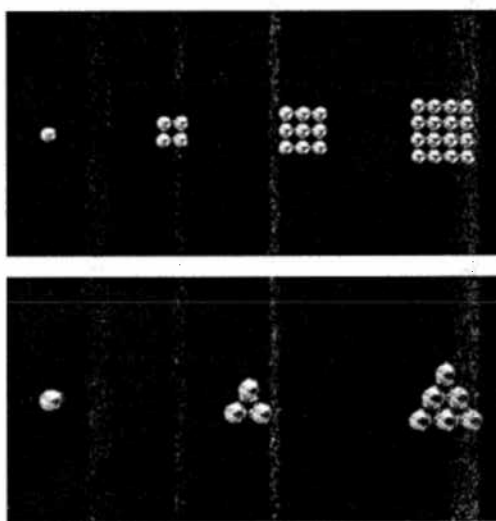


FIG. 33. Filled squares and triangles of edge lengths 1, 2, 3, ..., made of Zen Magnets. Sequences of filled squares and triangles are used by Michele LaForge to teach the sequence of square numbers and the relationships between the square numbers and the triangle numbers.

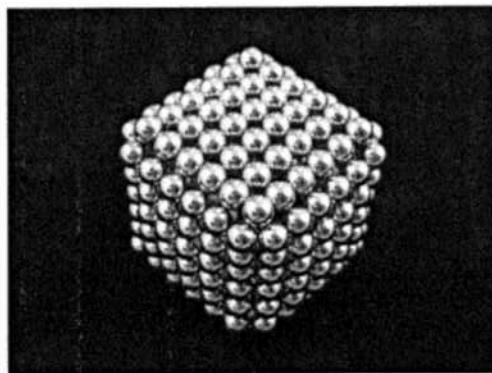


FIG. 34. Hollow cube, made with 215 Zen Magnets, illustrating how magnet spheres can be used to teach geometrical concepts that are linked to Common Core curriculum standards.

that a hexagon rotated by  $60^\circ$  looks the same as the original hexagon, and a square rotated by  $90^\circ$  looks the same as the original square. She also uses mirrors to help the students understand reflection symmetries.

3. She uses magnet spheres to teach the geometrical basis for number sequences. To help students understand the sequence of square numbers, 1, 4, 9, 16, ..., she asks them to build filled squares of increasing sizes ( $1 \times 1$ ,  $2 \times 2$ ,  $3 \times 3$ , etc., Fig. 33). She also asks her students to explore the sequence of “triangle numbers,” that is, the number of magnets in filled equilateral triangles. This sequence is 1, 3, 6, 10, ... (Fig. 33). She then points out a relationship between these sequences: The number of magnets in the  $2 \times 2$  square (4 magnets) is the sum of the numbers of magnets in the 1-magnet and 3-magnet triangles. The number of magnets in the  $3 \times 3$  square (9 magnets) is the sum of the numbers of magnets in the 3-magnet and 6-magnet triangles. In general, the number of magnets in a square is the sum of the number of magnets in the triangle of the same edge length and the number of magnets in the next-smaller triangle. To see why this works geometrically, shift rows to morph each of the two equilateral triangles into a right isosceles triangle, and observe that the smaller triangle fits into the corner of the larger one to form a square of the correct size.

Ms. LaForge reviewed this section and consented to have me include it in this report.

Another U.S. teacher uses magnet spheres in her seventh grade science class to teach concepts of geometry linked to the Common Core educational standards [38]. The seventh grade mathematics standards include the following requirement [52]:

CCSS.MATH.CONTENT.7.G.B.6



Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

One example is to use the six  $6 \times 6$  filled squares shown in Fig. 6 to form the six faces of the hollow cube shown in Fig. 34. A similar procedure can be used to build hollow tetrahedra, octahedra, dodecahedra, and icosahedra, thus helping students understand the difference between the surface area of three-dimensional objects (represented by the magnets, whose total number is a measure of the surface area) and the volume of these objects (represented by the space enclosed by these magnets).

In addition to such standards-based demonstrations, this U. S. teacher uses magnets for warm-up exercises in her seventh grade science classes. She shows the class a three-dimensional model such as the solid cube shown in Fig. 6, distributes magnet spheres to the class, and asks the students to figure out how to build it.

The solid cube is challenging to build. Engagement with such tough problems builds persistence and critical thinking skills that are crucial to education at all levels. Magnet spheres serve as a vehicle to teach these skills.

## VII. ZEN MAGNETS, LLC

Zen Magnets, LLC promotes and incentivizes the educational, artistic, and creative use of Zen Magnets by curating a photography gallery of Zen Magnets sculptures on Flickr, a photo-sharing website [53]. Membership in the gallery requires administrative approval. Four photographs are accepted into the gallery per day. Successful photographers receive a coupon for one booster set of 216 Zen Magnets, valued at \$32.98, for every three photographs accepted. The gallery began on September 25, 2009, and now contains over 5200 photographs that have generated over 1.2 million views [54]. A list of photographs that have generated over 1000 views each is shown in Appendix B.

I consider the following paragraph in the Second Amended Complaint to be incorrect:

80. Upon information and belief, despite making no significant design or other physical changes to Zen Magnets since their introduction in 2009, Respondent has attempted subsequently to rebrand Zen Magnets as, *inter alia*, an educational “science kit,” suitable for 8 year olds, although the Firm has provided no educational material with the Subject Product.

Zen Magnets includes an educational Zen Magnets Guide (Appendix C) with their gift sets and Mandala sets, and have been doing so since I first became aware of

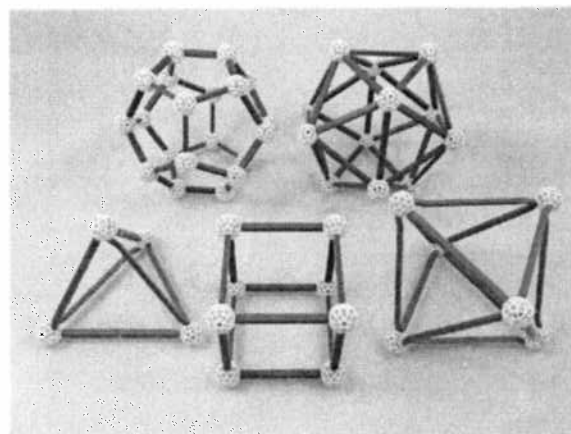


FIG. 35. Platonic solids constructed using Zometool, including (counterclockwise from bottom left) a tetrahedron, cube, octahedron, icosahedron, and dodecahedron.

the company, in Fall 2012. This guide describes how magnetic polarities work, shows how to build various shapes, and generally introduces the reader to the universe of building with magnets.

I consider the following paragraph in the Second Amended Complaint to be misleading:

101. The Subject Products also move in unexpected, incongruous ways as the poles on the magnets move to align properly, which can evoke a degree of awe and amusement among children, enticing them to play with the Subject Products.

As discussed in Fig. 4, once an individual understands the basic magnetic principles discussed in the Zen Magnets Guide (Appendix C), the movement of magnets is quite understandable.

## VIII. ALTERNATIVE PRODUCTS

### A. Ball-And-Stick Models

Magnet spheres have educational utility that cannot be found in other products. They offer several advantages over ball-and-stick models, which are used to study geometry, symmetry, and structure of molecules and lattices. In ball-and-stick models, balls represent atoms and are called “nodes.” Sticks connect nodes together and represent molecular bonds between atoms [55]. For concreteness, we compare magnet spheres with Zometool (<http://www.zometool.com/>), an innovative ball-and-stick modeling system that is capable of building a wide variety of shapes.

1. *Magnet spheres are inexpensive.* The least expensive Zometool system kit that is capable of building

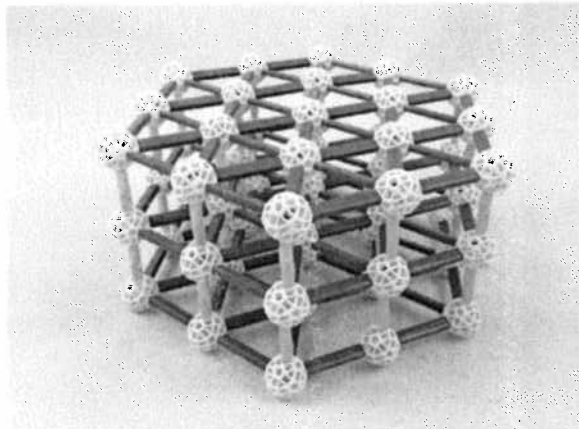


FIG. 36. A hexagonal lattice constructed using Zometool. The construction violates 6-fold rotational symmetry about axes occupied by yellow struts.

all five Platonic solids is Advanced Math Creator 4 (\$329.00), which includes the “advanced” green struts needed to build the regular tetrahedron and octahedron (Fig. 35). In contrast, all five Platonic solids can be built using one set of 216 Zen Magnets (\$32.98), as shown in my *Playing with Plato* video [40]. The Advanced Math Creator 4 kit has insufficient numbers of parts to build the simple-cubic lattice of Fig. 6, the hexagonal lattice of Fig. 8, the hexagonally close-packed lattice of Fig. 11, or the face-centered cubic lattice of Fig. 13, each of which can be built using a single set of Zen Magnets, and each of which clearly shows the lattice structure. Attempts to build Zometool versions of these lattices are disappointing. For example, Fig. 36 shows a hexagonal lattice built using Zometool, with three layers of  $3 \times 3$  hexagons, 57 white nodes, 120 blue struts, and 38 yellow struts. This shape uses all 120 of the short blue struts in the kit, which is 6 struts short of the 126 struts required (note missing blue struts on the bottom layer of Fig. 36). In contrast, the hexagonal lattice of Fig. 8 has six layers of  $4 \times 4$  hexagons and 222 nodes (magnets, in this case), and can be built with a single set of 216 Zen Magnets, including the six spares. In this and other explorations with the Zometool Advanced Math Creator 4 kit, I found myself quickly exhausting its capability. For the price of this kit, one can buy 10 sets of Zen Magnets (2,160 magnets) with which one can build a huge variety of shapes.

2. *Magnet spheres are versatile.* Ball and stick models have fixed bond angles and bond lengths that are specialized to particular structures, and cannot be used to build many shapes that are possible with magnet spheres. For example, Zometool cannot be used to build two of the most fasci-

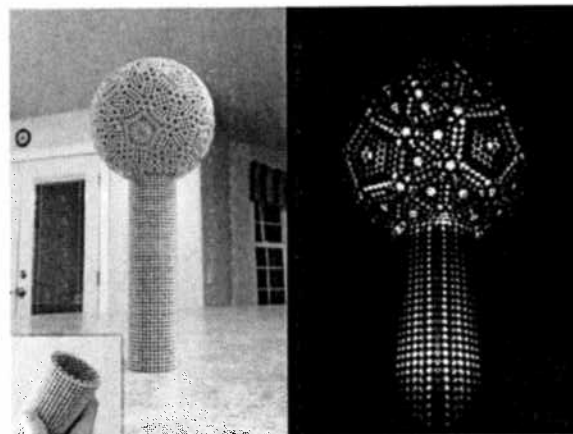


FIG. 37. Seven-pound 6590-magnet snub dodecahedron ball of my own invention, supported by a 4060-magnet double-walled tube invented by Damian OConnor, included to show the educational utility of Zen Magnets for teaching mathematics, lattice packing, and geometry. The left photograph shows the structure with normal lighting. The right photograph was taken at night with the room lights off and a flashlight inside of the tube. The snub dodecahedron has 12 pentagonal faces and 80 triangular faces. It has the largest number of faces and the highest sphericity of the eighteen highly-symmetric Platonic and Archimedean solids, and it cannot be built with Zometool. It has two chiral forms, a clockwise form and a counterclockwise form, each the mirror image of the other. Shown is the clockwise form, with each pentagon rotated slightly clockwise from its position in a regular dodecahedron. The design of the snub dodecahedron ball is the subject of a YouTube tutorial video of mine (<http://youtu.be/biEMiD2mClw>), and the procedures for mounting it atop a double-walled tube are the subject of a second tutorial video (<http://youtu.be/cJ6ZebVTMOA>). This snub dodecahedron ball might be the largest self-supporting double-thickness hollow sphere that can be made with Zen Magnets. As shown in the inset, the double-walled tube features a square-packed outer wall and a hexagonally-packed inner wall. In the written description accompanying my tutorial video for this tube (<http://youtu.be/TuDL1TvAEY4>), I show mathematically that a wall circumference of 40 magnets (for both the inner and outer wall) best matches the natural curvature set by the difference between the packing densities of the inner and outer walls.

nating Archimedean solids, the snub cube and the snub dodecahedron (Fig. 37). Objects like Fig. 38 might also be quite challenging for Zometool and other ball-and-stick models. The fixed bond angles and bond lengths of ball-and-stick models preclude a continuous range of stable connection angles (Fig. 2). Consequently, ball-and-stick models cannot replicate the dynamic nature of strain reactivity (Fig. 20), crystal defect formation (Fig. 23), plastic deformations (Fig. 24), and cell division (Fig. 26).

3. *Magnet spheres honor symmetry.* The Zometool node is a small, white rhombicosidodecahedron

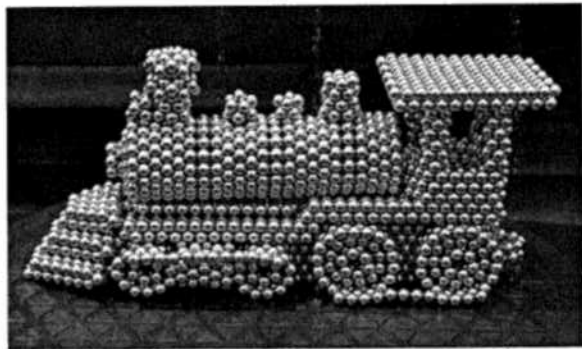


FIG. 38. Train constructed using Zen Magnets, by EraZorX13 [56].

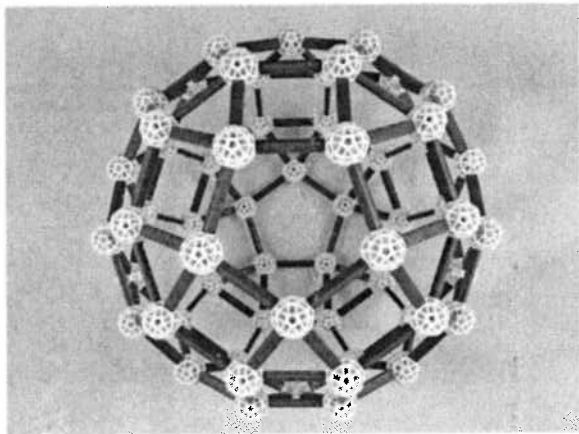


FIG. 39. Rhombicosidodecahedron constructed using Zometool, containing 60 white nodes and 120 blue struts. The rhombicosidodecahedron is a lovely and highly spherical Archimedean solid with 20 triangular faces, like an icosahedron, 12 pentagonal faces, like a dodecahedron, and 30 square faces.

with icosahedral symmetry, and can be used to build icosahedra, dodecahedra, and rhombicosidodecahedra that honor the full symmetries of these shapes (Fig. 39). Zometool cannot be used to build tetrahedra, cubes, octahedra, face-centered cubic lattices, hexagonal lattices, and other shapes without violating various symmetries of these structures (Figs. 40, 41, 42, 43, and 36), leading to less aesthetic structures and to possible student misperceptions about the full symmetries of these structures.

4. *Magnet spheres are efficient.* Magnetic sculptures are generally built by incorporating magnet chain segments into a structure, with the magnets in the segment remaining in contact with each other during the construction process. This procedure speeds construction because it does not require the builder to handle and orient each magnet. For example, the 1260-magnet icosahedron shown in Fig. 44 can be built in 7 min-

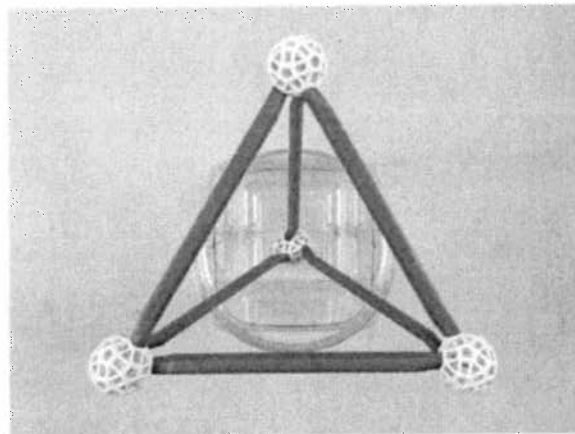


FIG. 40. Tetrahedron constructed using Zometool. The construction violates left-right reflection symmetry.

utes and 22 seconds, which amounts to 0.35 seconds spent on each magnet (Video “Icosahedron\_Zen\*.mp4”). Ball-and-stick models require the builder to handle and orient each node, and also require the builder to worry about struts and where they connect to nodes. The 60-node rhombicosidodecahedron shown in Fig. 39 took me over 16 minutes to build (Video “Rhombicosidodecahedron\_Zome.mp4”), while the 60-magnet rhombicosidodecahedron shown in Fig. 45 took less than a minute (Video “Rhombicosidodecahedron\_Zen.mp4”). Even though the 60 nodes in the Zometool construction are arranged just like the 60 magnets in the Zen Magnets structure, the Zometool construction requires much more time.

5. *Magnet spheres are elementally simple.* With magnets, shapes are built with only one building block, a smooth shiny sphere that provides its own connection with neighboring spheres through the magnetic force. No struts of various shapes and sizes whose supply can be exhausted during construction. No worries about where to insert struts into nodes in order to build a particular structure. The elemental simplicity of magnet spheres lends them a certain aesthetic appeal that captures the imagination and invites deep exploration.

Perhaps for the advantages outlined above, magnet spheres have spawned a large and vibrant grassroots learning community, where enthusiasts share their techniques and their creations (Sec. IX).

## B. Weak Magnets

The Second Amended Complaint presents evidence, in paragraphs 9-12 and 130, that Zen Magnets and Neoballs qualify as hazardous magnets according to section 3.1.37

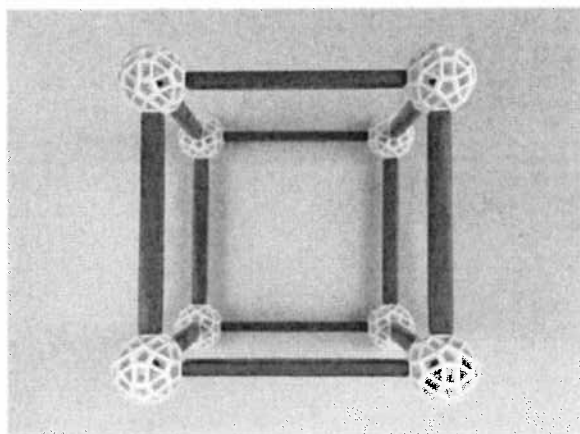


FIG. 41. Cube constructed using Zometool. The construction violates four-fold rotation symmetry about axes passing through the centers of opposing faces.

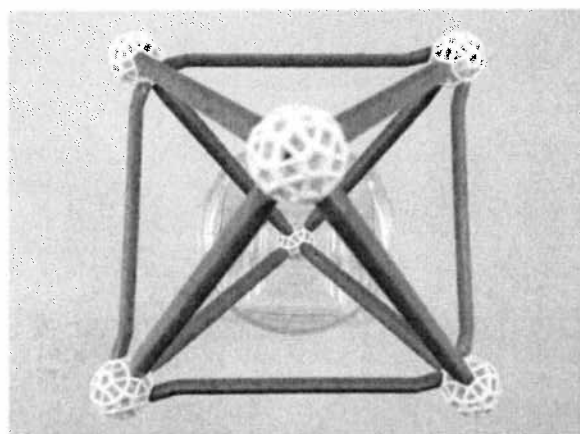


FIG. 42. Octahedron constructed using Zometool. The construction violates four-fold rotation symmetry about axes passing through the centers of opposing nodes.

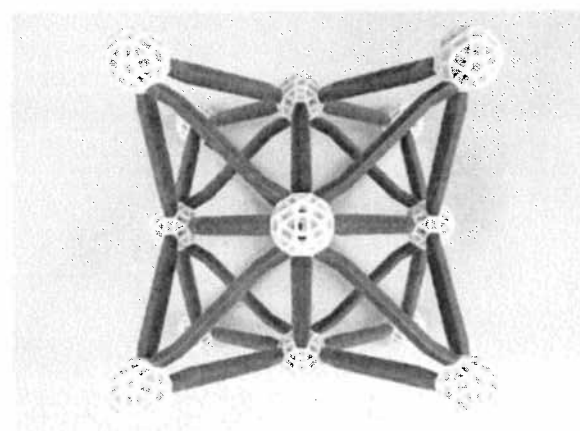


FIG. 43. Face-centered cubic lattice constructed using Zometool. The construction violates four-fold rotation symmetry about the axis perpendicular to the photographic plane and passing through the center node.

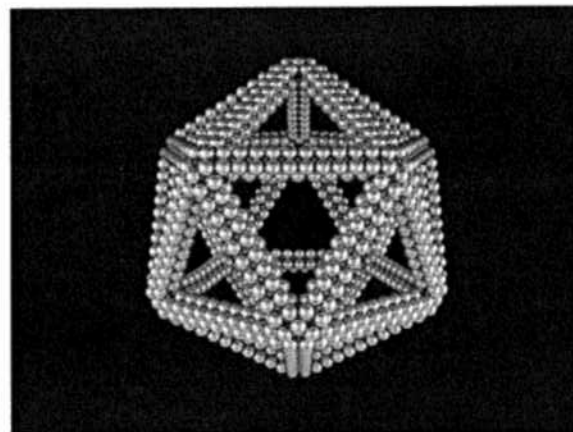


FIG. 44. Icosahedron constructed in 7 minutes and 22 seconds using 1260 Zen Magnets. Calvin Grover, age 11, built this shape in various sizes (Fig. 50).

of the ASTM International Standard 963-11, *Standard Consumer Safety Specification for Toy Safety*, (the “Toy Standard,” [57]).

In this section, I consider the educational utility of sets of magnets that would not qualify as hazardous magnets under the Toy Standard. According to section 3.1.37 of the Toy Standard, a hazardous magnet is one with a flux index greater than 50, and that is a small object.

Section 8.24 of the Toy Standard defines the flux index as

$$I = B^2 A, \quad (7)$$

where  $I$  denotes the flux index,  $B$  denotes the magnitude of the polar magnetic field, measured in kiloGauss, and  $A$  denotes the largest magnet cross-sectional area that is perpendicular to the axis of the magnetic poles, measured in square millimeters. This magnetic field is to be measured by a probe whose sensor is located  $0.38 \pm 0.13$  mm from the probe surface.

Section 3.1.37 of the Toy Standard refers to section 4.6 and Fig. 3 of the Toy Standard to define a small object as one that fits into the small parts cylinder. According to Fig. 3 of the Toy Standard, the small parts cylinder has an inner diameter of 1.25 inches and has one square end and one slanted end, in such a way that the cylinder length is 1 inch on one side and 2.25 inches on the other.

Paragraphs 9-12 of the Second Amended Complaint report diameter and flux-index measurements that lead to the conclusion that Zen Magnets and Neoballs qualify as hazardous magnets.

My own measurements confirm that Zen Magnets qualify as hazardous magnets according to the Toy Standard. I measured flux indexes between  $527$  and  $564 \text{ kG}^2\text{mm}^2$  for standard Zen Magnets that I purchased in July 2014 (Table I). These values are somewhat smaller than the range  $577.1$  to  $581.4 \text{ kG}^2\text{mm}^2$  obtained by the CPSC for these magnets (paragraph 11, Second Amended Complaint).

The difference between the CPSC measurements and my measurements could be due to differences in the gauss meters used to measure the magnetic fields, uncertainties in the distance between the sensor and the probe surface, or variations in the magnets themselves. Whatever the reason, the differences are unimportant because of the agreement that the flux index exceeds  $50 \text{ kG}^2\text{mm}^2$ .

One way to meet the Toy Standard is to replace small (5 mm) neodymium magnet spheres with large neodymium magnet spheres that do not fit into the small parts cylinder. Such large magnets attract each other with forces sufficient to splinter themselves, causing an eye hazard, and to crush human tissues caught between two magnets. Such dangers would render such large magnets to be unsuitable for education.

Another way to meet the Toy Standard is to keep the magnet size comparable to Zen Magnets (5 mm diameter) and to reduce the magnetization so that the flux index is 50 or less. At my request, Shihan Qu worked with his factory to build some 5 mm magnets with flux indexes that are close to 50 in order to evaluate the educational utility of such weak magnets. The factory used the same neodymium alloys and the same nickel coatings as Zen Magnets, but used weaker magnetizing fields. Table I shows my measurements of the flux index for four such magnets, together with my measurements for standard Zen Magnets in my possession. My measurements of the flux indexes for the four weak magnets ranged from 57 to  $145 \text{ kG}^2\text{mm}^2$  - all four exceeding a flux index of 50. Since the CPSC measurements of flux indexes for Zen Magnets exceed my measurements, it seems likely that their measurements for these weak magnets would also exceed a flux index of 50.

According to my experiments, despite exceeding a flux index of 50 (though not exceeding this value by as much as standard Zen Magnets), these weak magnets fail at building even the simplest of shapes, including the 180-magnet icosahedron (Fig. 29), the 60-magnet rhombicosidodecahedron (Fig. 45), and the  $6 \times 6 \times 6$  cube (Fig. 6, Video “Lattices.Weak.mp4”). Although weak magnets can be used to demonstrate lattice defect formation (Video “Lattice\_Defects\_Weak\*.mp4”, see Fig. 23), they lack the strength needed to build robust structures, especially those with antiparallel connections. Weak magnets can be used to build the hexagonal close-packed lattice (Fig. 11), but the hexagonal lattice (Fig. 8) and the standard  $2 \times 2$  strut (Fig. 2) are so flimsy as to be impractical (Fig. 46, Video “Connections.Weak.mp4”, see Figs. 2 and 4). Magnets with a flux index of 50 or below would be even less capable of forming sculptures of educational interest.

In conclusion, magnets that comply with the Toy Standard would fail to fill the educational niche occupied by Zen Magnets.

**Magnetic flux measurements:** Table I shows magnetic field measurements made using a hand-held Alpha-Lab Gaussmeter Model GM2 with an ST universal probe. Imbedded within this probe is a Hall-effect magnetic sen-

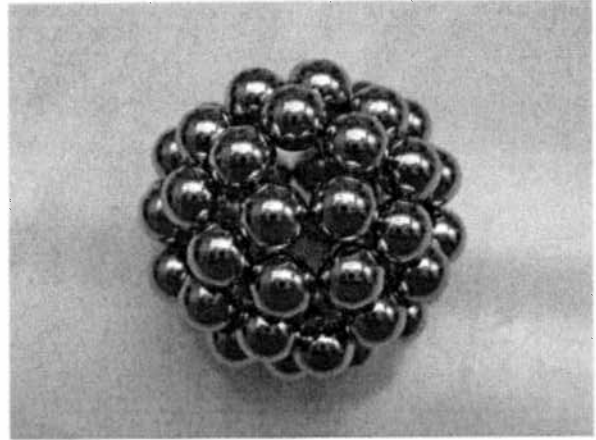


FIG. 45. Photograph of a rhombicosidodecahedron, an Archimedean solid with 12 pentagonal faces, 20 triangular faces, and 30 square faces, built using 60 Zen Magnets [58].

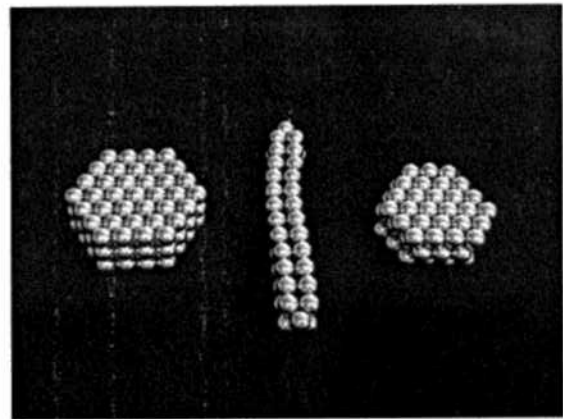


FIG. 46. Structures built using a set of weak magnets whose flux indexes are close to 50, showing a hexagonal lattice (left), a  $2 \times 2$  strut, and a hexagonally close-packed lattice.

sor of dimensions  $0.2 \text{ mm} \times 0.2 \text{ mm}$  located  $0.85 \text{ mm}$  above the flat probe surface [59, 60]. Such gauss meters are often employed in advanced physics laboratory courses. Using such devices to measure the magnetic fields of neodymium magnet spheres of diameter 5 mm presents an instructive challenge for undergraduate students because the magnetic fields are highly concentrated at the north and south poles of the magnets, and the sensor area is very small, requiring care in positioning the sensor directly over the pole.

In making the measurements in Table I, I took advantage of the polar alignment of magnet chains to position the gauss meter sensor over the probe. Magnets in a single-strand chain line up with the north pole of one magnet attached to the south pole of its neighbor (Fig. 3), hence the axis of the chain passes through the poles of all of the magnets. With magnets lined up in such a chain, I grasped two magnets in the chain be-

TABLE I. FLUX INDEX MEASUREMENTS

N	BN1 (G)	BN2 (G)	BS1 (G)	BS2 (G)	B (kG)	B' (kG)	m (A m <sup>2</sup> )	I	Comments
1	3189.7	3185.3	3168.7	3168.6	3.18	5.00	0.0596	491	older magnet from my collection
2	3140.3	3146.7	3135.9	3126.6	3.14	4.94	0.0588	479	older magnet from my collection
3	3064.4	3047.9	3020.1	3027.3	3.04	4.78	0.0570	449	older magnet from my collection
4	2993.3	2991.4	3011.6	3007.0	3.00	4.72	0.0563	438	older magnet from my collection
5	3001.2	3001.7	3000.8	3007.8	3.00	4.73	0.0563	439	older magnet from my collection
6	3141.0	3153.2	3164.6	3146.8	3.15	4.96	0.0591	483	older magnet from my collection
7	3040.9	3021.6	3014.9	3034.5	3.03	4.77	0.0568	446	older magnet from my collection
8	3134.4	3159.7	3174.5	3170.7	3.16	4.97	0.0592	486	older magnet from my collection
9	3042.5	3028.7	3023.1	3052.1	3.04	4.78	0.0569	448	older magnet from my collection
10	2998.1	2987.5	2929.6	2954.8	2.97	4.67	0.0556	428	older magnet from my collection
11	3143.9	3142.1	3143.2	3129.7	3.14	4.94	0.0589	479	older magnet from my collection
12	3236.1	3231.5	3172.5	3201.8	3.21	5.05	0.0602	501	older magnet from my collection
13	3333.3	3333.7	3332.4	3336.5	3.33	5.25	0.0625	541	new magnet, received July 2014
14	3322.0	3322.8	3293.9	3321.4	3.32	5.22	0.0621	534	new magnet, received July 2014
15	3438.1	3409.2	3387.8	3387.1	3.41	5.36	0.0638	564	new magnet, received July 2014
16	3290.3	3292.8	3279.3	3301.3	3.29	5.18	0.0617	527	new magnet, received July 2014
17	1110.7	1114.0	1275.7	1279.6	1.20	1.88	0.0224	69	weak magnet
18	1627.0	1622.5	1825.9	1832.2	1.73	2.72	0.0324	145	weak magnet
19	1359.8	1353.6	1460.4	1459.9	1.41	2.22	0.0264	96	weak magnet
20	1051.2	1054.3	1113.9	1108.4	1.08	1.70	0.0203	57	weak magnet

## LEGEND:

N = magnet number

BN1 = measurement 1 of the magnitude of the north pole magnetic field (G), measured by the Alphalab gauss meter

BN2 = measurement 2 of the magnitude of the north pole magnetic field (G), measured by the Alphalab gauss meter

BS1 = measurement 1 of the magnitude of the south pole magnetic field (G), measured by the Alphalab gauss meter

BS2 = measurement 2 of the magnitude of the south pole magnetic field (G), measured by the Alphalab gauss meter

B = average of magnetic field measurements = estimated magnetic field in kG at a distance of 0.85 mm from the polar surface

B' =  $B \cdot r^3 / r'^3$  = magnetic field in kG at a distance of 0.38 mm from the polar surface, the field used to calculate the flux index $r = 3.35 \text{ mm}$  (2.5 mm + 0.85 mm) $r' = 2.88 \text{ mm}$  (2.5 mm + 0.38 mm)I =  $B'^2 \pi \cdot 2.5^2$  = flux index, the product of the square of B' and the cross sectional area of the sphere, in kG<sup>2</sup> mm<sup>2</sup> $m = 2 \cdot \pi \cdot r^3 \cdot B / \mu_0$  = magnetic dipole moment, in A mm<sup>2</sup> $\mu_0 = 1\text{E-}06 \text{ N/A}^2$  permeability of free spaceTABLE I. This table shows the values of the polar magnetic field  $B$  and flux index  $I$  measured by the author for several magnet spheres.

tween the thumb and forefinger of one hand, used the other hand to place the tip of a felt marker in the valley between the two magnets, and rolled the pair between the thumb and forefinger in order to scribe circles around the poles of both magnets. I then separated these magnets from the chain and, for each magnet, measured its north and south pole fields, sliding the probe over the pole and making subtle changes in its position until the maximum magnetic field reading is obtained, indicating that the probe sensor is directly over the pole.

The Gaussmeter shows a positive magnetic field reading when its probe is placed on a north pole and a negative reading when placed on a south pole. I typically made four readings for each magnet, two at the north pole and two at the south, alternating between poles between each of the four measurements to ensure a fresh search for the pole in each measurement. Magnetic field

readings for the standard Zen Magnets (numbers 1-16) typically ranged from 3000 G to 3400 G, where G represents Gauss, a unit of measurement of magnetic fields (Table I). If two north-pole readings differed by more than 30 G, I made new north pole readings until I had two readings that differed by no more than 30 G, yielding a measurement error of about 1%. I did similarly if two south-pole readings differed by more than 30 G. In Table I, values of  $B$  are the average of the magnetic field measurements for each magnet, with  $B$  measured in kG =  $10^3$  G. Magnetic field readings for the weak magnets were in the range of 1000 G to 1600 G, and measurements were carried out on each pole until two measurements agreed to within 10 G to ensure a measurement error of about 1%.

Equation (4) can be used to determine the magnetic dipole moment  $m$  from the measurements in Table I. Zen

Magnets have radii of approximately  $a = 2.5$  mm and cross-sectional area  $A = \pi a^2$ . The AlphaLab Gaussmeter sensor is a distance  $d = 0.85$  mm from the probe surface, so the distance between the sphere center and this sensor is  $r = a + d$ . Combining this with values of  $B$  from Table I and the value of the magnetic permeability  $\mu_0 = 4\pi \times 10^{-7}$  N/A<sup>2</sup> of free space (measured in Newtons per square Ampere), and converting to Teslas ( $1 \text{ T} = 10^4 \text{ G}$ ) and meters ( $1 \text{ m} = 10^3 \text{ mm}$ ), yields the values of  $m$  listed in Table I.

The flux index requires a magnetic field  $B' = \mu_0 m / 2\pi r'^3$  evaluated at a distance of  $d' = 0.38$  mm from the sphere surface, that is, a distance of  $r' = a + d'$  from the center of the sphere. And finally, the flux index follows as  $I = B'^2 A^2$ , as shown in Table I.

## IX. LEARNING COMMUNITY

The grassroots magnet sphere community is dedicated to learning about and sharing construction techniques for magnetic sculptures. YouTube is an active forum for this community, where magnet-sphere videos have generated over 140 million views (Appendix D). Other active forums include the Zen Magnets Gallery (Appendix B, [53]) and the Neodymium Magnet Sphere Commons gallery [61]. Computer simulations of interactions between magnet spheres have been written to aid in the understanding of these interactions [62]. The magnet sphere learning community is driven by intellectual curiosity, creative design, and appreciation for symmetry and beauty. The participants in this community aren't learning for a grade in a class, or for pay. They are working together to learn for the sheer love of learning and artistic expression.

In early August 2014, to better understand the educational value of this learning community, I spoke via telephone with **Austin Deschner** [63], author of a popular YouTube channel with 144 magnet sphere videos, 9,255 subscribers, and 3,778,110 views. Mr. Deschner, who posts videos and Flickr photos under the pseudonym "Magnenaut" [64], holds a B.S. in Mechanical and Biomedical Engineering from Duke University. He is a medical student at Baylor College of Medicine in Houston, Texas. He anticipates completing his M.D. in 2015. His clear tutorials and beautiful magnetic sculptures have inspired many to buy and learn about magnet spheres.

As mentioned in Sec. I, I am one of these. As I learned about magnet spheres and contemplated placing my first order for them, I compiled a list of 66 shapes of interest to me, many designed by Magnenaut. His use of correct geometrical names of shapes in his videos motivated me to study about these shapes and their properties. I learned about the dodecahedron, tetrahedron, octahedron, rhombicosidodecahedron, cuboctahedron, deltoidal hexecontahedron, truncated icosahedron, cumu-

lated cuboctahedron, stellated cuboctahedron, great dodecahedron, icosidodecahedron, rhombic dodecahedron, rhombic triacontahedron, and stellated rhombic dodecahedron. I studied Magnenaut's construction techniques. When I had learned enough to produce YouTube videos of my own, I found myself mimicking the style of Magnenaut's videos, including his "Hello, everyone!" starting line.

While an undergraduate student of engineering, Mr. Deschner used his magnets to illustrate classroom topics including sphere packing, magnetism, statics, and entropy to himself and his classmates, and reported that they considered the magnets to be a valuable visual aid. He reported that while he hasn't needed to do math while building structures, he does need to know some statics in order to build strong structures. He found that working with magnets gives him an intuitive, non-rigorous grasp of mathematics, statics, and magnetism that he couldn't get any other way.

I also have experienced the development of such an intuitive grasp through my own experiences with magnets, but unlike Mr. Deschner, I have done some mathematics while building and understanding magnetic structures. In the written description for my Double-Walled Tube YouTube Tutorial, I proved mathematically that forty-column cylinders best match the natural curvature of the double-walled tube [65]. In the written description for my Definitive Diagonal Cube Tutorial, I worked out the mathematics of the layer dimensions needed to build the layers in a diagonal cube of any size [66]. I have filled many sheets of paper with calculations and sketches of shapes that I've built or considered building.

Mr. Deschner said that most of the teaching that he has done has been to himself. I found this statement to be curious in light of the nearly 10,000 individuals who have benefitted enough from his videos to subscribe to his YouTube channel. His videos have taught me many lessons about solid geometry and magnetic design, and have inspired me to begin a hobby that is a continuing source of enjoyment and learning for me and my family.

In early August 2014, I also contacted **Curtis McClive**, who contributes to the magnet sphere learning community primarily through his stunning Flickr photostream, under the pseudonym "Sonyador6" [67]. He holds a 1983 B.S. from Cornell University, with a major in Electrical Engineering and a minor in Neurobiology, and holds a 2004 M.B.A. from the University of Washington, where he was ranked first in his class. Since March 2007, he has worked at Microsoft Corporation, in Redmond, Washington, recently as Hardware Test Manager.

He responded via e-mail with a summary of things that he has learned from magnet spheres:

In the less than two years I've been working with magnet spheres, I have personally learned a tremendous amount. A lot of what I've learned is inferential, i.e. magnet spheres

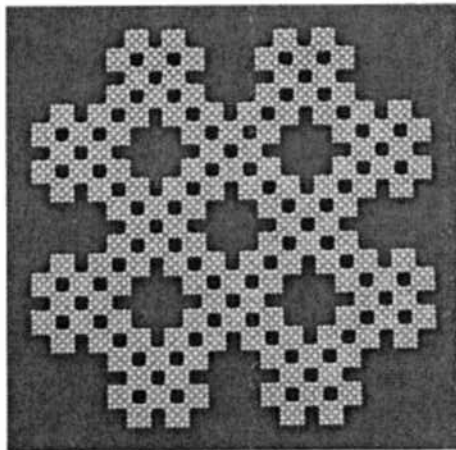


FIG. 47. "Fractal progression 3," by Curtis McClive [68]. This is the third of three photographs showing a fractal progression of replication of smaller units to create a fractal.

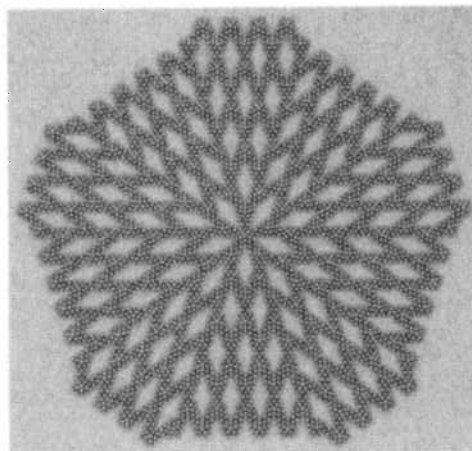


FIG. 48. "Tiling Pattern for 5-Fold Symmetry," by Curtis McClive [69].

are representative of idealized concepts such as mathematical points/lines/surfaces, atoms or materials. Some examples:

1. Geometric shapes and various relationships between different geometric shapes, including polygons, polyhedrons, truncations, stellations, and duals.
2. Crystal lattice structures and the relationship of various crystal lattice structures to macro shapes and structures.
3. Strength of materials, strength of mechanical shapes and how to transfer weight through structures.
4. Photographic techniques, photographic equipment, photographic illumination including the concepts of diffusion and color temperature (spectral content), the physics of photography and pushing the limits of photography.

He later wrote to expand on the educational value of magnet spheres:

I believe the learning potential of spherical magnets is best fulfilled by using them and building shapes with them. I have never known a better medium in which to explore so many different geometrical relationships. To that ideal, I have often tried to have my builds and photographs highlight new build techniques, mathematical relationships, structural qualities and/or photographic qualities. By doing so, I hope to create a sense of curiosity in the observer that motivates them to learn more and try new things themselves. A few notable examples:

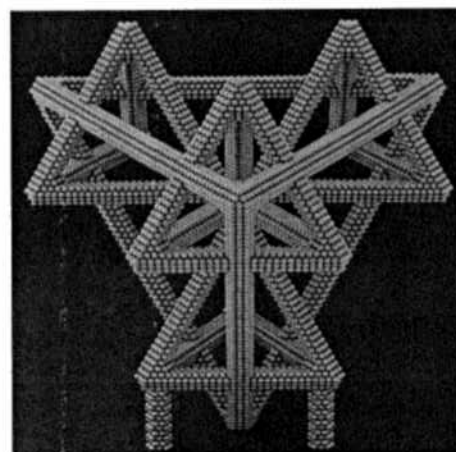


FIG. 49. "Tetrahedrons in the Air," by Curtis McClive [70].

1. I created three builds and photographs that highlight the fractal quality of self-similarity at different scales. The two larger builds use the smaller build as the component or "subunit", while maintaining the same overall pattern (Fig. 47, [68]).
2. I discovered a tiling pattern with 5-fold symmetry that lies flat in a plane. It's probably known to Geometers but it was new for me. By creating several appealing builds and photos of the pattern, I believe I created an awareness in others of the pattern's unique qualities (Fig. 48, [69]).
3. I created a design that incorporates four smaller, equally sized tetrahedrons, each intersecting a larger tetrahedron in one of its four vertices. The four smaller tetrahedrons meet at the geometric center of the larger tetrahedron. I designed the build so the



larger tetrahedron is “standing on its head”. The build is eye-catching in its improbability and explicitly highlights both the geometrical relationships of the tetrahedrons and the structural challenge of transferring the weight of the upper portion to the supporting surface. The photos generated conversation and awareness of these qualities, which is the essence of a learning experience (Fig. 49, [70]).

I greatly appreciate the sharing ethic of the magnet sphere community and have learned many things from the tutorial videos of contributors such as “Mathnetism”, “Magne-naut”, Dimitri Tischenko, Boyd Edwards and Damian O’Connor. I have also contributed with a few tutorials, three on my YouTube channel and one in my Flickr photostream [67].

Like observational astronomy, magnetic construction is a learning hobby. I have told hundreds of students in my introductory astronomy classes at West Virginia University that purchasing a telescope gets you nowhere unless you know where to point it, and unless you know something about the objects of your search. For magnet spheres, it’s not only the beauty and symmetry of magnetic sculptures that fascinate, but the science, magnetic design, and construction procedures behind them that drives people to learn and experiment and create, and that motivates people to share their ideas and creations freely with others. It is fascinating to handle an icosahedron frame built of magnet spheres – 20 equilateral triangles uniting perfectly in groups of five to form pentagons at each of 12 identical vertices, but even more exciting to learn how to harness the magnetic forces needed build it (Fig. 44, Video “Icosahedron\_Zen\*.mp4”). Once he learned how to build the icosahedron frame, my then 11-year-old grandson spent several hours building it in various sizes (Fig. 50). Like astronomy, magnetic construction is a fascinating learning hobby that many share with family and friends, and, through the Internet, with the rest of the world.

## X. PUBLIC STATEMENTS

Appendix E contains 413 statements about the educational, artistic, and creative utility of magnet spheres. These statements were written to protest CPSC’s administrative complaints against magnet companies and CPSC’s proposed ban on these spheres, and were collected from three sources:

- 309 letters written to government officials [71].
- 2,593 public comments received by the CPSC between Sept. 4, 2012 and Nov. 19, 2012 [72]. Of the 2,593 comments, 91% oppose the ban [73, 74].



FIG. 50. Family of icosahedra built by Calvin Grover, age 11.

- 1431 comments made by signers of a petition [75]. This petition has been signed by 5,058 individuals to date [76].

Recurrent themes are:

1. The risk posed by magnet spheres is smaller than the risks posed by other accepted consumer items, such as balloons.
2. Responsible individuals heed warnings and take precautions against these risks.
3. The value of magnet spheres for education, artistic expression, creativity, and stress relief far outweigh their risk.
4. Banning the sale, to adults, of a product whose intended use is not harmful constitutes unacceptable governmental interference in the private sector.

These statements describe a wide variety of educational uses of magnet spheres, only some of which have been discussed above. These statements reflect the passion of a magnet learning community that sees both a strong need for and a great utility for magnet spheres, clearly refuting paragraphs 105 and 106 of the Second Amended Complaint, cited in the first section of this report.

A poll conducted on July 10-11, 2013 by Public Policy Polling supplies further evidence of public opposition to the ban. Of the 755 registered voters who responded to the poll, 88% oppose a sales ban to all ages, 6% support it, and 6% were undecided [77]. The margin of error is  $\pm 3.5\%$ .

## XI. INTERVIEWS

In addition to the interviews described above, I interviewed other individuals to learn about their educational

uses of magnet spheres. These interviews were conducted during August, 2014, and are summarized in this section. Interviewees were selected from the individuals quoted in Appendix E. I approached each of these interviewees with the following introduction and questions:

I have been asked by Zen Magnets LLC to offer my opinion about the educational utility of magnet spheres in connection with an administrative complaint against this company by the United States Consumer Protection Safety Commission (CPSC Docket No. 12-2). I would appreciate receiving your responses to the following:

1. Do you own, or have access to, sets of magnet spheres? If so, how many sets of 216 magnets do you own or have access to, and what brand(s) of magnets do you own or have access to?
2. Do you consider these magnets to have educational utility? If so, please describe.
3. If you have used these magnets to educate yourself or others, please give specific details about the subjects and concepts taught, to whom these concepts have been taught, and where these have been taught (classroom, laboratory, home, etc.). Please include copies of any educational materials, lesson plans, illustrations, photographs, etc., if applicable.

After completing each interview and writing up its summary, I sent the summary to the interviewee for comments and corrections, and received permission from each interviewee to include the summary, and his/her contact information, in this report.

I corresponded via e-mail with **Abdul Ibrahim**, author of statement 114 in Appendix E. In August 2015, Mr. Ibrahim expects to receive his B.S. in Computer Engineering Technology from Wentworth Institute of Technology (WIT) in Boston, Massachusetts [78]. He reports that access to magnet spheres during tutoring sessions that he received as a high school student helped him to develop a passion for engineering and science. As a student at WIT, he was able to turn the tables and use magnet spheres to tutor other students, saying:

In my sophomore year of college, I tutored students in geometry and physics. Buckyballs are convenient in that, with the proper practice, they can be molded into a variety of different geometric shapes. This was a boon in that creating a shape, then deconstructing it and asking the student to replicate it and give the proper equations to find the number of sides or the length of a given line on the shape. This technique was not only stimulating to the student, but also beneficial in that



FIG. 51. Photograph of Alan and Maureen Bayless, with their sons Sam, Jacob, David, and Noah, whose self-directed explorations using magnet spheres fueled their interest in math and science.

they would learn to create the shapes kinesi-  
thetically.

I spoke via telephone with **Dr. Keith Ray**, author of statement 151 in Appendix E [79]. Dr. Ray received a 2013 Ph.D. in Physics from the University of California at Berkeley. He is completing a postdoctoral appointment at that institution, and is preparing to begin employment at Lawrence Livermore National Laboratories in Livermore, California. He said that, while a graduate student at the University of California at Berkeley, he explained two-dimensional lattices to his sister, then a mechanical engineering freshman, using a square lattice and a hexagonal lattice made using magnet spheres (Fig. 32). He mentioned that it's wonderful that we can use such magnets to demonstrate concepts in crystallography, binding, and materials science.

I spoke via telephone with **Maureen Colclough Bayless**, author of statement 174 in Appendix E [80]. Ms. Bayless and her husband, Alan Bayless, have raised four sons whose self-directed investigations with Zen Magnets have fueled their interest and careers in math and science. In an e-mail to me, Ms. Bayless describes:

Neither my husband nor I have a strong background in science. Neither of us studied physics in high school beyond what was in the general science program. My math skills are feeble, although in recent years I've learned many cool things about math and now think mathematically even though I don't know the times table.

All of my sons developed a strong interest in physics and math, particularly physics, largely through exploring with materials and

projects at home. We did not have the money to put them into summer camp or special programs, for the most part, but we would happily wander over to Radio Shack or the electronics store to pick up wire, batteries, LEDs, neodymium magnets, piezoelectric starters and anything else that they wanted for a project that they were building. They would come up with a plan for something that interested them and if their plan didn't work, they'd gather around the white board in the kitchen to figure out what had gone wrong and to devise a new plan. This instilled in all of them not only a love of math and science, but real friendship and respect for each other as inventors. To this day, they consult each other over the projects, even writing projects, and if they are not all at home, they can be found all together online via Skype, creating or solving something. Its a joy to watch.

Small, powerful neodymium magnets were essential to many of their projects. They experimented with magnetic fields and explored many science concepts, often using the magnets to demonstrate an idea. Small neodymium magnets and steel balls were used to build motors and robotics. My youngest son, while still in elementary school, created a Gauss rifle using neodymium magnets, following instructions in Simon Quellen Field's *Gonzo Gizmos* book. This was not a real rifle, but a linear accelerator that demonstrates the transfer of kinetic energy. He later gave this as a gift to his high school science teacher, who used it for classroom demonstrations.

I am very concerned that banning Zen magnets and other small magnets would significantly reduce the opportunities for children to learn science at home: real, hands-on, investigative science, which is very different from the kind of science they can learn from a book. How do magnets work? How does potential energy become kinetic energy? How do magnetic fields affect the way that Zen magnets can be manipulated? These magnets are small, light but very powerful, and therefore they can be used to give robots a grip that enables them to lift objects (for example), or to build mag-lev trains. Without small magnets like these, many projects would be out of the reach of teen inventors using household equipment.

Solely because of their self-directed, discovery-based interest in science, all four of my sons earned medals in Grade 8 at the UBC regional science fairs.

To this day, if you ask Sam, Jacob, David or

Noah a question about physics, you will get a very detailed, enthusiastic answer. If appropriate, they will pull out their magnets to demonstrate a principle. They have read in depth on many science topics as a result of their experiments and projects. I really hope that these magnets are not banned, because they have truly enriched my sons' learning, particularly in physics and math. The magnets were fun, but they also raised countless questions and my children diligently tracked down the answers and explored their properties. (Why is it harder to pull magnets apart when they are in a circle than when they are in a string?) This more than made up for their parents' lack of science background. We could not teach them, but to this day they love teaching us.

The Bayless sons have transformed their interests into careers. Sam Bayless holds a B.S. in Cognitive Studies from the University of British Columbia (UBC), and is nearing completion of his Ph.D. in Computer Science at the same institution. Jacob Bayless holds a Bachelor of Applied Science in Engineering Physics from UBC, a Masters of Engineering from Massachusetts Institute of Technology, and just returned from a six-month study of motors using neodymium magnets at the Tokyo Institute of Technology. David Bayless holds a B.A. in Classical Studies from UBC, and writes fantasy fiction. Noah Bayless has been accepted into the Science One enriched science program at UBC for September 2014, having created his own independent studies course in linear algebra and computer programming through the Vancouver School Board when he was in Grade 11.

Over the telephone, Ms. Bayless mentioned that it would be a mistake to say that magnets can only be used in the classroom, saying that if you only do things that the teacher tells you to do, it is passive learning. She has seen first-hand the effects of active, inquiry-based learning in her sons.

I spoke via telephone with **Lee Walsh**, author of statement 199 in Appendix E, a physics Ph.D. student at the University of Massachusetts at Amherst, with anticipated graduation in 2015 [81]. Mr. Walsh has used magnet spheres to understand energetics and topological defects in crystals. In my telephone conversation with him, he mentioned that it can be difficult to understand these three-dimensional concepts when they are presented on the blackboard or in a textbook, but they become understandable when you demonstrate them by manipulating magnets with your own hands. He mentioned that magnets are easier to manipulate than ball-and-stick molecular models, and said that magnets allow you to feel the energetics that mimic molecular bonds. He said that he has collaborated with other students to figure out how to use magnet spheres to understand such concepts, saying

that the magnets can be used by researchers and students to model defects in a crystal or a thin sheet. This demonstrates on a human scale the types of reactions that can occur on the microscopic scale. He has used magnet spheres in his research on wrinkling in thin, elastic sheets, by building and deforming magnet sheets in a manner reminiscent of defect production in crystals. He said that understanding the way that thin sheets react to external forces helps us learn to control the strength of these sheets, and can lead to applications to manipulate and control thin films used in microscopic technology.

I corresponded via e-mail with **David Nicholaeff**, author of statement 215 in Appendix E. Mr. Nicholaeff holds a B.S. in Physics and Mathematics from the University of California at Los Angeles and an M.S. in Mathematics from Oxford University [82]. He is currently employed as a computational physicist at Los Alamos National Laboratory, in Los Alamos, New Mexico. About his use of magnet spheres, he wrote:

They allow me to prototype low-dimensional projections of complicated higher dimensional lattices. This greatly aids in my ability to study symmetries in higher dimensional structures. As for the necessity of the magnets, the firm structure they induce allows me to force the constraints of the transformations which can be applied, i.e. the transformations which leave the lattices invariant.

In a follow-up discussion via telephone, Mr. Nicholaeff said that magnet spheres are a great way for students to gain an appreciation for symmetry, geometry, and lattice structures, and offer the opportunity to play with really beautiful shapes. He uses magnet spheres in his research in computational meshes, constructing lower-dimensional meshes out of magnet spheres and using these to help him visualize higher-dimensional meshes, including meshes with dimensionalities greater than three. These help him to choose computational cell geometries and to study various tessellations of Euclidean space.

I spoke via telephone with **Adam Love**, author of statement 399 in Appendix E. Mr. Love holds a B.S. in Physics and Cognitive Science from Massachusetts Institute of Technology, and tutors math and science in the New York area, primarily to juniors and seniors in high school [83]. He has used magnet spheres to teach magnetic polarity, magnetic induction, molecular structure, and lattice packings. He finds physical objects such as magnets to be more effective in teaching these concepts than a diagram or a book, saying that if you can touch and feel something, the concepts stick better in the mind. He says that neuroscience research indicates that students retain information better if multiple senses are stimulated simultaneously in the learning process.

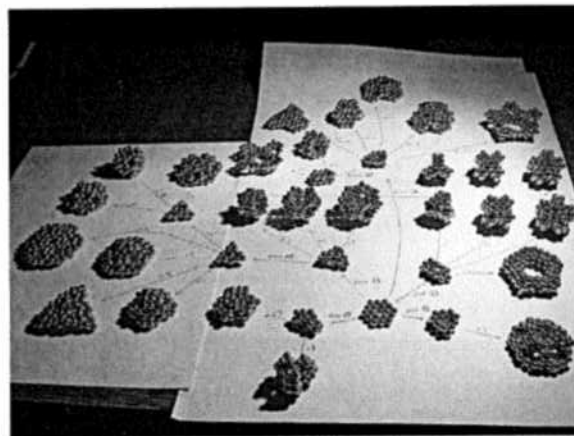


FIG. 52. Photo of subunits made using magnet spheres and used to explore geometry and group theory, by Cale Gibbard. The photo is from his Flickr photostream, <https://www.flickr.com/photos/cgibbard/5168792854/>.

I spoke via telephone with **Cale Gibbard**, author of statement 413 in Appendix E. Mr. Gibbard holds a B.S. in Pure Mathematics from the University of Waterloo, in Waterloo, Ontario, Canada, and currently works as a consultant in functional computer programming [84]. He uses magnet spheres to explore geometry and group theory, and is particularly interested in exploring point symmetry groups. He says that one begins to have an intuitive feel for the magnetic fields just by playing with the magnets. He enjoys sharing his explorations and his magnetic artwork with family and friends, and shares his work globally through his YouTube channel and his Flickr photostream [84]. Shown in Fig. 52 is a photo of his diagrammatic approach to some of these explorations.

These interviews describe many uses of magnet spheres for education, research, and exploration, both in and out of the classroom, and demonstrate the educational utility and consumer need for these products.

## XII. CONCLUSION

My fascination with magnet spheres stems from their magnetic properties, their educational possibilities, and the seemingly endless number of fascinating shapes that can be built from them. These magnets have provided delightful creative, artistic, and educational opportunities for me and my family. Magnet spheres offer a respite from my administrative pressures and a chance to apply critical thinking, physics, and mathematics to build structures of artistic beauty and symmetry. Visitors and residents of my home enjoy building shapes with Zen Magnets while they converse.

Magnet spheres provide a valuable educational tool. They provide a uniquely engaging way of demonstrating

lattice structures such as face-centered-cubic packing and have helped me to understand these structures better. They can be used to build realistic models of molecules and atomic clusters. They teach concepts of magnetism that I did not know before, such as the attraction between parallel chains. They allow a hands-on approach to the study of highly symmetric geometric solids. These topics can become stale if taught solely at the blackboard. Magnet spheres breathe life into these topics.

I have taken many opportunities to use magnet spheres to educate individuals and groups about magnetic properties, Platonic solids, molecular symmetry, and lattice structure. These topics are taught in courses in electricity and magnetism [6], astronomy [47], physical chemistry [22], and solid state physics [85]. As a full-time administrator with no current teaching responsibilities, I look forward to the day when I can return to the university classroom and there exploit the unique educational possibilities of magnet spheres. I am also interested in writing a physics education publication on the properties of

these magnets, pitched at the undergraduate level. Some of these concepts can also be taught at the high-school and junior-high level, where hands-on engagement with magnet spheres can help to make science more relevant and more interesting.

Like all good science, magnet spheres engage both the analytical and the artistic centers of the brain, echoing Henri Poincaré's sentiment, "The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing..." These magnets invite individuals to stretch their imaginations for shapes to build, to use critical thinking to figure out how to build them, and to appreciate the beauty of the shapes thus created.

Banning the sale of neodymium magnet spheres would throttle the grassroots magnet sphere learning community and would squander the opportunity to enrich the classroom and the laboratory with the unique and engaging properties of these magnets.

- 
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- [79] Dr. Keith Ray, [raykeithg@gmail.com](mailto:raykeithg@gmail.com), 408-799-9789.
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