VERTICALLY INTEGRATED ARMINGTON

Ross Hallren

David Riker

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Ross Hallren and David Riker
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Abstract

In this paper, we derive two vertically integrated CES Armington industry-specific models, one with endogenous producer prices and one with exogenous producer prices. The models incorporate the use of both domestic and foreign intermediate inputs in domestic production. We describe the process for deriving an N-level vertically integrated Armington CES model. We present several examples that demonstrate how results differ from the standard Armington industry-specific model once the model accounts for shocks within the supply chain.

Ross Hallren, Research Division, Office of Economics
ross.hallren@usitc.gov

David Riker, Research Division, Office of Economics
david.riker@usitc.gov

1   Introduction

The industry-specific Armington CES partial equilibrium model provides a nice test kitchen for capturing intricate supply chain linkages, utilizing both statistical and qualitative information. In this paper, we derive vertically integrated industry-specific models that demonstrate how including supply chain features can affect the qualitative and quantitative predictions of the models by capturing feedback loops between final demand actors and upstream suppliers. Moreover, given that production is increasingly vertically and internationally integrated, capturing these linkages is key to explaining growth in trade.[1] Models must include these vertical trade linkages to capture the direct effects of changes in trade policy in one country and also how the effects of a policy shock proliferates and compounds throughout global value chains.[2] Including these features in the model allows researchers to run more nuanced policy experiments.

   The paper proceeds as follows. In section 2, we expand the standard endogenous price Armington CES model to allow for direct, vertical linkages between downstream consumers and upstream suppliers. In section 3, we use simulations to illustrate how quantity shocks and price shocks pass through the supply chain. Section 4 concludes.

2   Vertically Integrated Armington CES Model

2.1   Basic Model

We derive the non-linear Armington CES partial equilibrium modeling following the derivations in Armington (1969) and Hallren and Riker (2017). We then incorporate a supply

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[1]Hummels, Ishii and Yi (2001), Kose and Yi (2001), Yi (2003), Yi (2011), Ng (2010), Bridgman (2012).
[2]Kose and Yi (2001), Bems, Johnson and Yi (2011), Johnson and Noguera (2012), Rouzet and Miroudot (2013), Liao and Santacreu (2015), Duval, Li, Sarafan and Seneviratne (2016), U.S. International Trade Commission (2017).

chain and discuss the general demand function for a good at any stage in the supply chain.

The basic industry-specific Armington CES model focuses on a single national market. The final demand agents, utility-maximizing consumers with CES utility functions or profit-maximizing firms with CES production functions, solve their optimization problem by choosing between a domestic variety and imported varieties. In the basic model there are three varieties: the domestic variety, subject imports, and non-subject imports. The subject variety includes imports that are subject to a change in trade policy. The consumer price of variety i is p_i. The producer price of the domestic variety is p_d, while the producer prices of the imports from i are equal to $\frac{p_i}{1+\tau_i}$ for $i \in \{s, n\}$.

Producers of the three varieties operate in a perfectly competitive market and face the following supply curves:

$$q_i = a_i \left(\frac{p_i}{1 + \tau_i} \right)^{\epsilon_i} \tag{1}$$

The ϵ_i parameter is a constant price elasticities of supply, and the a_i parameter represents factors that shift the supply curves. The equation for the supply curve assumes a specific functional form (in this case, it is log-linear), and it is tailored to the industry by fitting the supply shift parameter to industry data. The calibrated values of the supply shifters reflect a variety of factors, including the level of production capacity and input costs.

Equation (2) represents total demand in the industry, Q.

$$Q = Y P^\theta \tag{2}$$

The variable Y is national aggregate industry expenditure, which we treat as exogenous. The variable P is a price index for the products of the industry in the national market. The parameter $\theta < 0$ is the price elasticity of total demand in the industry. Equation (3) represents the Constant Elasticity of Substitution (CES) demand curve for the variety from

country i.

$$q_i = Y \, b_i \, P^\theta \left(\frac{P}{p_i}\right)^\sigma \tag{3}$$

The parameter b_i represents factors that shift the demand curves. When the model is calibrated to initial equilibrium prices and quantities, with initial prices normalized to one, b_i is set equal to the initial market share of variety i. (The market shares for the three varieties of products in the industry sum to one.) The CES price index in equation (3) is:

$$P = \sum_i \left(b_i \, p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{4}$$

The equation for the demand curve also assumes a specific functional form (in this case, it is log-linear in prices and the price index, and the price index has a CES functional form). This equation is also tailored to the industry by fitting the demand shift parameter to industry data. The calibrated values of the demand shifters reflect a variety of factors, including prices in other industries.

To solve the non-linear version of the model, we use an iterative algorithm to find the set of prices that ensures that quantity supplied of each variety equals quantity demanded in the market. Therefore, the following condition is satisfied for all varieties i:

$$a_i \left(\frac{p_i}{1 + \tau_i}\right)^{\epsilon_i} = Y \, b_i \, P^{\sigma + \theta} \, p_i^{-\sigma} \tag{5}$$

The model is calibrated to the initial equilibrium by normalizing all prices to one and adjusting the model parameters to match the initial values of sales of the different varieties. As explained in Francois and Hall (1997), after calibration $a_i = Y \, b_i \, (1 + \tau_{i0})^{\epsilon_i}$ at the initial equilibrium for all varieties so the market clearing condition in (5) can be simplified to:

$$\left(p_i \left(\frac{1 + \tau_{i0}}{1 + \tau_i} \right) \right)^{\epsilon_i} = P^{\sigma + \theta} p_i^{-\sigma} \tag{6}$$

τ_{i0} is the initial ad valorem equivalent rate of the tariff and international transport costs on imports, and τ_i is the final ad valorem equivalent rate.

2.2 Extension to Include Multiple Stages of Production

Expanding this basic industry-specific CES Armington model to a vertically integrated model is relatively straightforward. The mathematics for vertically integrated demand systems and production chains are well established.[3]

Since the derivations for the equations can be tedious with a large number of stages of production, we focus on the intuition and comparative statics for a two-level nested case. We consider a case in which a consumer at the final demand level chooses between domestic and imported varieties of a final product. To produce this final product, domestic firms use intermediate inputs either sourced domestically or source from abroad. In this case, the cost shares of other factors of production are zero.

Using the nomenclature of Keller (1976), the most downstream sector, the final demand portion, is the Nth level. The next most downstream sector, where firms produce final consumption goods is the $N - 1$ level. The most upstream sector is the 0th level. At each $(N - i)^{th}$ level there can be multiple branches, indicating each input variety (e.g. domestic, subject, non-subject). Consequently, the demand $q_{k,l}$ for each node k in the tree for any level l, is a function of relevant prices along the path between node k and final demand.

At the $N - 1$ level, the notation for quantity demanded is $q_{i,N-1}$. At the $N - 2$ level, demand for variety j that is purchased to produce variety i is $q_{i,j,N-2}$. The index follows the pattern of buyer, seller, level. This notation adds clarity in cases when domestic firms in the

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[3]Sato (1969) derives a two-tiered supply chain, and Keller (1976) formulates a demand equation for an N-level nested CES utility function.

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downstream sector sell to both domestic and foreign firms.

Therefore, the demand functions at the $(N-1)$ and $(N-2)$ levels are:

$$q_{i,N-1} = Y P_N^{\theta} \, b_{i,N-1} \left(\frac{P_N}{p_{(i,N-1)}} \right)^{\sigma_N} \tag{7}$$

$$q_{i,N-2} = Y P_N^{\theta} \, b_{i,N-1} \, b_{j,N-2} \left(\frac{P_N}{p_{i,N-1}} \right)^{\sigma_N} \left(\frac{p_{i,N-1}}{p_{j,N-2}} \right)^{\sigma_{N-1}} \tag{8}$$

The level N price index is similar to equation (5), but now it takes into account the supply chain. We assume that only the domestic variety has a vertically integrated supply chain, so the final demand CES price index is:

$$P_N = \left(\sum_i b_{i,N-1} \, p_{i,N-1}^{1-\sigma_N} \right)^{\frac{1}{1-\sigma_N}} \tag{9}$$

The price of the domestic variety at level $N-1$ is the following index of the prices of $N-2$ level inputs:

$$p_{d,N-1} = \left(\sum_i b_{i,N-2} \, p_{i,N-2}^{1-\sigma_{N-1}} \right)^{\frac{1}{1-\sigma_{N-1}}} \tag{10}$$

Equation (10) is a unit cost function. Since we assume perfect competition, the market prices are determined by unit costs. For the remaining varieties at the $N-1$ level and all varieties at the $N-2$ level, the supply functions have the generic constant elasticity forms in equations (11) and (12).

$$q_{i,N-1} = a_{i,N-1} \left(\frac{p_{i,N-1}}{1 + \tau_{i,N-1}} \right)^{\epsilon_{i,N-1}} \tag{11}$$

$$q_{i,j,N-2} = a_{i,N-2} \left(\frac{p_{i,N-2}}{1 + \tau_{i,j,N-2}} \right)^{\epsilon_{i,N-2}} \tag{12}$$

$\tau_{i,N-1}$ and $\tau_{i,j,N-2}$ are tariff rates at the $N-1$ and $N-2$ levels, respectively.

Given the assumption of perfect competition in the market and the structure of the model's supply chain, the prices of the final goods and the domestic variety at the $N-1$ level are determined by their unit cost functions. We use numerical optimization to solve for import prices at the $N-1$ level and all prices at the $N-2$ level. Specifically, we solve for the set of prices that clears these markets.

Equation (13) applies for all varieties i.

$$P_N^\theta (\frac{P_N}{p_{i,N-1}})^{\sigma_N} = \left(p_{i,N-1} \left(\frac{1 + \tau_{i0,N-1}}{1 + \tau_{i,N-1}} \right) \right)^{\epsilon_{j,N-1}} \tag{13}$$

Equation (14) applies for domestic producer d and all varieties i.

$$P_N^\theta \left(\frac{P_N}{p_{d,N-1}} \right)^{\sigma_N} \left(\frac{p_{d,N-1}}{p_{j,N-2}} \right)^{\sigma_{N-1}} = \left(p_{d,j,N-2} \left(\frac{(1 + \tau_{d0,j,N-2})}{1 + \tau_{d,j,N-2}} \right) \right)^{\epsilon_{j,N-2}} \tag{14}$$

We also analyze an exogenous price model that is based on equations (13) and (14), but with infinite values for the supply elasticities $\epsilon_{j,N-1}$ and $\epsilon_{j,N-2}$.

3   A Simple Example

In this example, we use the $N = 2$ level vertically integrated Armington model, derived above, to show the price and quantity responses to the imposition of a new 10% ad-valorem tariff on subject imports. We consider three scenarios: (i) only impose the tariff on the downstream sector, (ii) only impose the tariff on the upstream sector, and (iii) impose the tariff on both simultaneously. As a baseline comparison, we also estimate the effects on prices and quantities of applying a 10% tariff in the standard ($N = 1$) Armington model.

Table 1 presents the policy shocks in each scenario in the top panel. The middle panel summarizes how the models are parameterized in each scenario. For maximum comparability,

we do not adjust the model parameterization across experiments, only the policy shocks. Additionally, we parameterize both the upstream and downstream portions symmetrically. The bottom panel summarizes the market shares and total industry expenditure.

In table 2, we show the price and quantity effects for domestic and subject varieties resulting from the four policy shocks in the endogenous price model. Scenario 1 is a new 10% ad-valorem tariff on subject imports in a non-integrated, standard Armington model. This scenario serves as our base of comparison for the outcomes of the vertically integrated Armington scenarios (scenarios 2-4). In scenario 1, the shock increases subject prices by 7.9%, domestic prices by 2.1%, the overall CPI by 3.6%, decreases subject import quantities by 17.9%, and increases domestic output by 2.1%.

Applying the same shock to a two-stage vertically integrated Armington model (scenario 2) generates larger price increases for the domestic variety and the CPI and smaller quantity effects for all varieties. In this model, the new tariff on subject imports shifts demand towards the domestic variety. This increases demand for upstream intermediate inputs causing input prices to rise. This further increases the price of the domestic variety, partially offsetting the demand effect of the tariff. Consequently, the effect of the tariff on demand for the downstream final good is mitigated by the increased cost of intermediate inputs for the domestic variety.

Imposing a tariff on imported intermediate inputs increases domestic firms' overall intermediate input costs by 2.7%. This in turn increases domestic downstream prices by 2.7% and has a small price effect on downstream subject (0.4%) and on the final demand CPI (1.8%). The tariff reduces subject imports of intermediate inputs by 20.9% but only increases domestic output of intermediates by 1.1%. The increased product costs causes demand for the domestic final goods variety to decline by 5.4% and the demand for imported final products to rise by 3.8%.

In scenario 4, we shock the model with a 10% tariff on subject imports of both final

demand goods and intermediate goods. The tariffs reduce the volume of subject imports of both types of goods by 15.5%, increase the CPI of final demand goods by 5.0%, and increase the CPI of intermediates by 4.2%. Output of domestic intermediates increases by 3.0%, but the output of domestic final goods decreases slightly (1.9%).

We present the results of the same experiments using the exogenous price Armington model in table 3. In the exogenous price model, shocks to the downstream portion of the model do not affect market prices of intermediate inputs. Rather, all of the adjustments occur via percent changes in quantity demanded and the percent changes in composite prices. However, shocks to the upstream sector (intermediate inputs) are passed downstream to the final demand sector.[4] Consequently, when we look at the results as a whole and compare them with the endogenous price Armington results, the main differences are that the percent change in quantity demanded for the subject variety is always larger in the exogenous price case. Additionally, the subject price always increases by the full amount of the tariff. The price of the domestic variety of final demand goods only increases in scenarios 3 and 4 when a tariff is levied on the upstream intermediate goods. Here the price of the domestic variety final demand good increases, because it uses imported subject intermediate inputs.

4   Conclusions and Areas for Future Research

In this paper we derive both endogenous price and exogenous price two-tiered vertically integrated Armington CES industry-specific models. The derivation serves as a guide for writing an N-tiered vertically integrated Armington CES model.

Additionally, we compare the predictions of these models for a variety of ad-valorem tariff shocks. We show that tariffs on subject imports cause spillover effects on the domestic variety. Specifically, the price of the domestic variety increases in all cases. When a tariff

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[4]Technically, only the prices of products with supply curves (rather than cost functions composed of intermediate inputs) are exogenous in the model.

is placed on imported intermediate inputs, the prices of the domestic varieties of both final demand goods and intermediate inputs increase, and the demand for the final demand good falls.

These spillover effects are more muted in the case of the exogenous price model. However, a tariff on subject intermediate inputs still causes the price of the domestic variety to rise by 2.6% and the quantity demanded of the domestic variety final good to fall by 5.6%.

This is in sharp contrast to the base case (scenario 1) where a tariff on subject imports increases the quantity demanded of the domestic final good and leaves its price unchanged. These results indicate the importance of considering global supply chains when modeling changes to import tariff rates.

The next step in this line of model development is to explicitly include both intermediate and value added inputs. Doing so allows us to see the effect of policy changes on employment and returns to capital at each stage of production. Additionally, modeling the value added portion of the supply chain enables us to add both short-term and long-term adjustments in our static model. This is the first step in transitioning to a dynamic, industry-specific partial equilibrium model.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Tariff Shocks				
Downstream Imports	10%	10%	0%	10%
Upstream Imports		0%	10%	10%
Parameter Inputs				
Industry Price Elasticity	-1	-1	-1	-1
Downstream				
Domestic Supply Elasticity				
Subject Supply Elasticity	10	10	10	10
Non-Subject Supply Elasticity	10	10	10	10
Armington Elasticity	4	4	4	4
Upstream				
Domestic Supply Elasticity		1	1	1
Subject Supply Elasticity		10	10	10
Non-Subject Supply Elasticity		10	10	10
Armington Elasticity		4	4	4
Initial Market Data				
Industry Expenditure	$100	$100	$100	$100
Downstream Market Shares				
Domestic	60%	60%	60%	60%
Subject	30%	30%	30%	30%
Non-Subject	10%	10%	10%	10%
Upstream Market Shares				
Domestic		60%	60%	60%
Subject		30%	30%	30%
Non-Subject		10%	10%	10%

Table 2: Endogenous Price Armington Results

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Price Effects (Pct Chg)				
Downstream				
Domestic	2.1%	1.3%	2.7%	4.2%
Subject Imports	7.9%	7.7%	0.4%	8.2%
Price Index (N)	3.6%	3.0%	1.8%	5.0%
Upstream				
Domestic		1.8%	1.1%	3.0%
Subject Imports		0.6%	7.4%	8.2%
Price Index (N-1)		1.3%	2.7%	4.2%
Quantity Effects (Pct Chg)				
Downstream				
Domestic	2.1%	3.7%	-5.4%	-1.9%
Subject Imports	-17.9%	-18.8%	3.8%	-15.5%
Upstream				
Domestic		1.8%	1.1%	3.0%
Subject Imports		6.6%	-20.9%	-15.5%

Table 3: Exogenous Price Armington Results

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Price Effects (Pct Chg)				
Downstream				
Domestic	0.0%	0.0%	2.6%	2.6%
Subject Imports	10.0%	10.0%	0.0%	10.0%
Price Index (N)	2.6%	2.6%	1.5%	4.3%
Upstream				
Domestic		0.0%	0.0%	0.0%
Subject Imports		0.0%	10.0%	10.0%
Price Index (N-1)		0.0%	2.6%	2.6%
Quantity Effects (Pct Chg)				
Downstream				
Domestic	8.1%	8.1%	-5.6%	2.4%
Subject Imports	-26.2%	-26.2%	4.7%	-22.4%
Upstream				
Domestic		8.1%	4.7%	13.6%
Subject Imports		8.1%	-28.5%	-22.4%

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