# A Model for Opposed Flow Flame Spread Over Charring Materials

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### INTRODUCTION

Opposed-flow flame spread over the surface of a semi-infinite solid has been a subject of much investigation in the past. Several elegant analytical flame spread models have also appeared in the literature [see for example, Refs. 1-5]. It is indeed fascinating to note how the authors of these and other previous studies reduced the complex flame spread problem with novel approximations to obtain analytical flame spread formulas. It is even more fascinating to see how well these formulas agree with the experimental measurements for materials for which they were designed. Previous studies, however, have focused primarily on the gas phase part of the problem and the solid was assumed to "vaporize" at a known fixed temperature. Yet, a large number of natural and synthetic polymers do not simply "vaporize" - instead, they undergo a complex thermal decomposition process which results in an insulating layer of char on the surface. This char layer thickens with time and gives rise to a two-phase moving boundary problem in the solid. Due to this difficulty, the understanding of flame spread over these solids is lacking (a noteworthy contribution toward developing a model of wind-aided flame spread over charring materials was made by Carrier, et. al [6]). Also, to obtain an analytical solution, it was often necessary to simplify the gas-phase momentum and continuity equations by using a uniform Oseen velocity 'Ug' and the constant pressure condition. Thus, the objective of the present study is to extend the previous work on "vaporizing" solids to charring materials like wood and provide a more realistic description of the gas phase.

The analyses most relevant to the present problem are those of deRis [1] and Wichman and Williams [4]. deRis [1], in his seminal work on opposed-flow flame spread, has used the Oseen approximation in the gas phase, and by assuming infinite reaction rates he developed a flame spread formula relating the flame spread rate to the solid and gas phase properties, which were assumed to be constant. His results indicate that the flame sheet almost lies along the fuel surface behind the flame-inception point. Physically, this will be the case for a flame spreading on the underside of a semi-infinite solid rather than above it.

Wichman and Williams [4] have also employed the Oseen approximation in the gas phase and have utilized deRis's result by hypothesizing at the outset that the combustion heat release occurs along the fuel surface. This hypothesis eliminates the necessity of considering species conservation equations, and thus allows a more detailed analysis of a simplified thermal flame spread model. Their model also yields the same formula for the opposed-flow flame spread rate as deRis's model. Their formula was derived by requiring that steady spread rate must be sufficient to remove the combustion heat release by downstream gas and solid-phase convection (in the flame-fixed coordinates). Heat transfer upstream to the as yet unignited fuel was not used. In fact, at steady state both conditions must be satisfied. This concept of determining the steady flame spread rate is exploited here to obtain a solution for opposed-

flow flame spread on a semi-infinite charring solid. In addition, the Oseen approximation is eliminated and a more realistic description of the gas-phase flame is provided.

## **MODEL FORMULATION**

The physical problem considered here is schematically shown in Figure 1. Familiar examples of this type of flame spread are: lateral or downward flame propagation on a vertical wooden slab or opposed-flow flame propagation on the top or underside of a horizontal wood slab.

In the model problem, the slab is assumed to be semi-infinite (a reasonable approximation for wood slabs > 1" thick) and the diffusion flame is assumed infinity extend to to downstream of the flameinception point (as shown by the dotted line in figure 1 in the plane y > 0 – flame sheet). In the real case, the thickening of the char layer and re-radiation from the char surface will result in extinction of the diffusion



**Figure 1:** Schematic of the physical problem – steady propagation of an opposed-flow diffusion flame on a slab of a charring solid. The dotted lines show the approximations made in the gas and solid phases. The pyrolysis zone is approximated by a pyrolysis surface and the reaction zone by a flame sheet.

flame further downstream of the flame-inception point. This aspect, caused by finite reaction rates, is ignored. The origin of the coordinate system is located at the flame-inception point and the flame is held stationary by feeding the solid into the flame at the steady flame spread rate 'V'. As the solid travels into the flaming zone it decomposes abruptly (endothermically or exothermically) at a specified pyrolysis temperature 'T<sub>P</sub>'; resulting in a pyrolysis surface (as shown by the dotted line in figure 1 in the plane y < 0). Again, in reality, a pyrolysis zone is formed where thermal decomposition occurs over a region of finite thickness. However, this is ignored in the simplified model presented here.

A further simplification is made based on the results of Wichman and Williams [4]. Their results show that for  $\gamma = 1$  (the ratio of solid-phase to gas-phase characteristic heat transfer length scales), no heat exchange occurs between the solid and the gas across the plane y = 0, and x < 0. For representative flame spread rates, induced air velocities and property values for air and wood (a typical charring material; data from Ref. [7]) ' $\gamma$ ' varies between  $0.5 < \gamma < 2$ . For this variation, the total heat exchanged across the plane y = 0 and x < 0 is less than 15% of the heat conducted through the solid across the plane x = 0 and y < 0. Physically, in the region x < 0, with both the solid and the gas initially at T<sub>∞</sub>, significant heat exchange between the two media does not occur until they reach close to the flame-inception point. For high gas velocities, additional heat is transferred to the gas in the region x < 0 via upstream conduction of heat through the solid and vice versa. Thus, most of the heat conducted upstream across x = 0 in both the solid and the gas phases remains within the respective phases. This is an important observation because the temperature gradient in both phases can be approximated by zero on x < 0 & y= 0. This enables the use of orthogonal parabolic cylindrical coordinates both in the solid and in the gas phases. As it will be seen later, deRis' [1] flame spread formula is recovered in the limit of zero char thickness by this analysis - implying that in the classical solution by deRis [1], the Oseen flow assumption, in effect, rendered the heat exchange between the two phases along y = 0 and x < 0irrelevant to determining the flame spread rate.

<u>Gas phase formulation</u>: The conservation of mass, gaseous species, and energy are described by the following equations:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{1}$$

$$\rho u \frac{\partial Z}{\partial x} + \rho v \frac{\partial Z}{\partial y} = \frac{\partial}{\partial x} \left( \rho D \frac{\partial Z}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho D \frac{\partial Z}{\partial y} \right)$$
(2)

Where  $\rho(x, y)$  is the gas density and Z(x, y) is the mixture fraction in a two-dimensional flow field  $\vec{u} = (u, v)$ . The mass conservations equation (1) can be satisfied exactly with the introduction of a stream function  $\varphi(x, y)$  defined as:  $\partial \varphi / \partial y = \rho u$  and  $\partial \varphi / \partial x = -\rho v$ .

These equations are transformed by introducing dimensionless parabolic coordinates as follows:

$$\xi + i\eta = \sqrt{\frac{U_{\infty}}{v_{\infty}}}(x + iy)$$
<sup>(3)</sup>

Here  $U_{\infty}$  is the ambient wind speed in a coordinate system moving with the flame front, and  $v_{\infty}$  is the kinematic viscosity evaluated at ambient temperature. By postulating  $\varphi = \mu_{\infty} \xi f(\eta)$ , defining

$$\zeta = \int_{0}^{\eta} \frac{\rho(\eta)}{\rho_{\infty}} d\eta$$
, and seeking solutions of the form  $Z = Z(\zeta)$ , the gas-phase problem in the burning region

is essentially reduced to the problem solved by Emmons [8]. Upstream of the burning region, the noslip condition is violated, but the upstream influence of variable density flow is retained.

<u>Solid phase formulation</u>: Equations for energy transfer in the pristine solid and char (denoted by subscripts 'w' and 'c' respectively) are needed to describe the solid-phase problem. Constant thermophysical properties are assumed; regression or oxidation of the char surface and thermal contact between the char and fuel gases flowing through it are ignored. This leads to the following set of steady-state equations for the temperature field in the two media.

For the pristine solid :

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$$\rho_{w}VC_{pw}\frac{\partial T_{w}}{\partial x} - \lambda_{w}\left(\frac{\partial^{2}T_{w}}{\partial x^{2}} + \frac{\partial^{2}T_{w}}{\partial y^{2}}\right) = 0 \qquad -\infty < x \le g(y); y \le 0$$
(4)

and for the char matrix :

$$\rho_c V C_{pc} \frac{\partial T_c}{\partial x} - \lambda_c \left( \frac{\partial^2 T_c}{\partial x^2} + \frac{\partial^2 T_c}{\partial y^2} \right) = 0 \qquad g(y) \le x \le \infty; y \le 0 \tag{5}$$

Appropriate boundary conditions require that along the interfaces the temperatures are equal. At the char-gas interface the temperature is assumed to be a constant, i.e.  $T_g(x,\theta) = T_c(x,\theta) = T_s$  for  $x > \theta$  and as mentioned earlier, the boundary conditions along x < 0 are:

$$T_{w}(x,\theta) = T_{g}(x,\theta) \quad and \quad \lambda_{g} \frac{\partial T_{g}}{\partial y}(x,\theta) = \lambda_{w} \frac{\partial T_{w}}{\partial y}(x,\theta) = \theta; \quad for \quad x < \theta$$
(6)

The solid phase equations and boundary conditions again lend themselves to the use of parabolic coordinates defined by the relation:

$$\beta + i\omega = \sqrt{\frac{2V}{\alpha_w}(x + iy)} ; \text{ where } V \text{ is the flame speed } \& \alpha_w = \frac{\lambda_w}{\rho_w} C_{pw}$$
(7)

Solution of the form  $T = T(\omega)$  satisfies all the equations and boundary conditions in the solid phase.

### FLAME SPREAD RATE

The steady flame spread rate is determined by energy balance downstream of the point of flame inception. The net heat released per unit area per unit time ( $\phi$ ) along y = 0;  $x \ge 0$  is given by:

$$\phi = -\lambda g \frac{\partial T_g}{\partial y}(x,\theta) - \lambda c \frac{\partial T_c}{\partial y}(x,\theta) = \frac{q}{\sqrt{x}}$$
(8)

It is important to note that the approximate upstream boundary condition introduced to facilitate parabolic coordinates, may somewhat alter the individual components of ' $\phi$ ', but <u>the net value which</u> <u>determines the flame spread rate remains unchanged</u>. For the special case of Oseen flow, substituting the expressions for temperature gradients and 'q' gives the flame spread rate as:

$$V = U_{\infty} \frac{\lambda_g \rho_g C_{pg}}{\lambda_c \rho_c C_{pc}} \left[ \left( \frac{T_f - T_s}{T_s - T_p} \right) \operatorname{erf} \left( \sqrt{\frac{\delta_c}{2}} \ c \right) \right]^2$$
(9)

Here  $\delta_c$ ,  $T_f$ ,  $T_P$  & c are the ratio of solid to char thermal diffusivities, flame temperature, surface temperature, pyrolysis temperature and a constant defining the location of the parabolic char-solid interface respectively. It is interesting to note that in the limit of zero char thickness (a "vaporizing" solid is obtained; i.e. as  $c \rightarrow 0$  and  $T_p \rightarrow T_s$ ), Equation (9) reduces to deRis's [1] flames spread formula.

The more interesting case with physically realistic gas flow that incorporates the no-slip boundary condition and "blowing" at the burning surface, as well as, expansion due to combustion will be presented at the meeting.

## REFERENCES

- 1. deRis, J. N. "Spread of a Laminar Diffusion Flame," <u>Twelfth Symposium (International) on</u> <u>Combustion</u>, The Combustion Inst., P. 241, (1969).
- 2. Ohki, Y., and Tauge, S. "On Flame Spreading Over a Polymer Surface.' <u>Combustion Science and</u> <u>Technology</u>, v. 9, P. 1, (1974).
- 3. Fernandez-Pello, A.C., and Williams, F.A. "A Theory of Laminar Flame Spread Over Flat Surfaces of Solid Combustibles." <u>Combustion and Flame</u>, v. 28, P. 251, (1977).
- 4. Wichman, I.S. and Williams, F.A. "A Simplified Model of Flame Spread in an Opposed Flow along a Flat Surface of a Semi-infinite Solids," <u>Combustion Science and Technology</u>, v. 32, P. 91, (1983).
- 5. Quintiere, J.G. "A Simplified Theory for Generalizing Results from a Radiant Panel Rate of Flame Spread Apparatus." Fire and Materials, v. 5, P. 52, (1981).
- 6. Carrier, G. F., Fendell, F.E., and Fink, S. Combustion Science and Technology, 32, P. 161, (1983).
- 7. Atreya, A. "Pyrolysis, Ignition and Fire Spread on Horizontal Surfaces of Wood." Ph.D. Thesis, Harvard University, Cambridge, MA, (1983).
- 8. Emmons, H.W., "The Film Combustion of Liquid Fuel", Z. Angew. Math. Mech., Vol. 36, p. 60 (1956).