Mine Inspectors Mathematics Workbook

U.S. Department of Labor

MSHA

Mine Safety and Health Administration

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INTRODUCTION – PAGE 1

The rationale for teaching mathematics to mine inspectors and for preparing this workbook is to prepare inspectors to solve mathematical problems which occur during regular mine inspections; accident investigations; or mine inspector training courses in the following areas: industrial hygiene, ventilation, electricity, hoisting, and haulage.

The approach of this workbook is practical, not theoretical. The emphasis is on the "how" of problem solving, not the "why". Not only does the workbook use a skills approach, but a very direct skills approach. Since the final objective is to enable inspectors to solve problems in the diverse mining areas mentioned above, only those skills which relate directly to those specific problems will be addressed – namely, computation with decimals, fractions, percents, and solution of formulas.

The focus will be on three different methods of problem solving: pencil and paper computation, calculator computation, and chart interpretation. Each of these methods has advantages and disadvantages. Obviously, a hand-held calculator is more accurate and faster than pencil and paper computation when solving complex formulas. Charts can also be used to save time and produce more accurate results than pencil and paper computation. In some instances chart interpretation actually saves more time than calculator computation, because of the complexity of the problem. Yet it would be foolish to depend on calculators and charts to perform all calculations, particularly the simple ones.

Under problems with relying on calculators are: (1) occasionally they need to be recharged – at inopportune times, (2) occasionally they malfunction, and (3) they are not allowed in the face areas of gassy mines, unless they are "permissible".

WORKSHOP GOALS

You should be able to:

(1) Solve problems in the area of industrial hygiene, ventilation, electricity, hoisting, and haulage.

(2) Perform specific mathematical skills such as computation with decimals, fractions, percents, and solution of formulas.

(3) Solve problems by using one of the following methods: pencil and paper computation, calculator computation, and chart interpretation.

DECIMALS

DECIMALS - PAGE 2

The system we use when we write whole numbers is called the *decimal system.* In this system, the position of each numeral determines its value.

Example: 423 the 3 represents 3 ones or 3

531 the 3 represents 3 tens or 30

368 the 3 represents 3 hundreds or 300

It is also possible to use the decimal system to represent fractions. A decimal fraction is indicated by placing a decimal point before a numeral. The position of the decimal point shows how many parts the whole has been divided into (tenths, hundredths, thousandths, etc.), and the numeral shows how many of the equal parts you actually have.

Example: .2 of a foot means a foot is divided into ten equal parts, and you have two of them.

CHAPTER OBJECTIVES

Using pencil and paper only, you should be able to do each of the following:

(1) Add decimal numbers

(2) Subtract decimal numbers

(3) Multiply decimal numbers

(4) Divide decimal numbers

CRITERIA: 70% accuracy

DECIMALS - PAGE 3

This table indicates the positions in the decimal system.

Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Ones	***Decimal Point***	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths

In this system, zeros before the whole numbers and zeros after the decimal fractions do not affect the values and therefore they can be eliminated or added.

Example: 00234.5700

~~00~~234.57~~00~~ = 234.57

Example: 34.857 = 0034.857000

Also, any whole number is understood to have a decimal point after it.

Example: 21 = 21.0

314 = 314.0

DECIMALS

<u>Addition and Subtraction of Decimal Numbers - PAGE 4</u>

To add or subtract decimals **<u>keep the decimal points in line</u>** and then add or subtract like whole numbers.

Example: **34 + 21.5 + .45 =**

```
  34.
  21.5
+   .45
  55.95
```

Example: 24 – 13.67 =

```
  24.       =     24.00
! 13.67     =  !  13.67
                  10.33
```

Exercises

Add each of the following decimal numbers.

1. 43.625 + 17.911 + 8.65 + 4.31 =
2. 39.4 + 2.79 + .089 + 1.8 =
3. 23.89 + 123.5 + 34.965 + 3 =
4. 4563 + 37.987 + 312.23 + 234 =
5. 4.005 + 37.1 + .0002 + 42.23 =

<u>OBJECTIVES</u>

You should be able to add and subtract decimal numbers using pencil and paper.

CRITERIA: 70% accuracy

Exercises

1.

2.

3.

4.

5.

DECIMALS

Exercises - PAGE 5

6.
7.
8.
9.
10.
11.
12.
13.
14.
15.
16.
17.
18.
19.
20.

6. 53 + 231 + 131.9 + .002 =

7. .0062 + 3.04 + .007 + .006 =

8. 42 + .34 + 1.9 + 342.66 =

9. .074 + .9 + 3 + 23.007 =

10. 34.8 + 31.2 + .008 + .9 =

Subtract each of the following decimal numbers.

11.	46.37 - 18.163 =	12.	364.07 - 42.68 =
13.	397.14 - 43.875 =	14.	67.46 - .003 =
15.	.826 - .0003 =	16.	.004 - .0039 =
17.	208.65 - 99 =	18.	201 - 53.87 =
19.	1.74 - .799 =	20.	.0023 - .00045 =

DECIMALS

<u>Multiplication of Decimal Numbers - PAGE 6</u>

To multiply decimal numbers, first ignore the decimal points and multiply them as if they were whole numbers. Place the decimal point in the answer so that the number of decimal places is equal to the total number of decimal places in the numbers being multiplied.

Example: 23.4 x .45 =

23.4	1	decimal place
x .45	+ 2	decimal places
1170		
936		
10530		
10.530	3	decimal places

Example: .007 x 4

.007	3	decimal places
x 4	+ 0	decimal places
28		
.028	3	decimal places

In this example we have to add a 0 to the left of the 28 so that we will have three places behind the decimal point.

<u>OBJECTIVE</u>

You should be able to multiply decimal numbers using pencil and paper.

<u>CRITERIA: **70% accuracy**</u>

Exercises – PAGE 7

1.
2.
3.
4.
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6.
7.
8.
9.
10.

Exercises

Multiply each of the following decimal numbers.

1. 46.83 x 2.07 =
2. 53.86 x 5.07 =
3. .496 x 3.8 =
4. .636 x .21 =
5. 6.2 x .008 =
6. 24.9 x .01 =
7. .0064 x .008 =
8. 1.1 x 23.88 =
9. 5.002 x 1.003 =
10. 3.6 x .301 =

<u>Division of Decimal Numbers - PAGE 8</u>

When dividing decimal numbers, first find the decimal point in the answer, and then divide as you would with whole numbers.

To locate the decimal point in the answer, do the following. Make the divisor a whole number by moving the decimal point to the right as many places as necessary. Move the decimal point in the dividend the same number of places to the right. Now place the decimal point in the answer directly above the point in the dividend.

<u>Example</u>: 2.034 ÷ .03

Dividend Divisor

```
.03 ) 2.034      move the decimal point
                 2 places to the right

       67.8
  3 ) 203.4
      18
       23
       21
        24
        24
         0
```

<u>OBJECTIVE</u>

You should be able to divide decimal numbers using pencil and paper.

CRITERIA: 70% accuracy

Exercises - PAGE 9

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.

13.

14.

Example: 213.2 ÷ .052 =

$$.052\overline{)213.200}$$

So that you can move the decimal point 3 places to the right, you must add 2 zeros to the right side of the dividend.

$$\begin{array}{r} 4100. \\ 52\overline{)213200.} \\ \underline{208} \\ 52 \\ \underline{52} \\ 00 \end{array}$$

Exercises

Divide each of the following decimal numbers.

1.	47.28 ÷ 3.7 =	2.	6.387 ÷ .42 =
3.	.83216 ÷ .301 =	4.	.79003 ÷ 52.1 =
5.	807.9 ÷ .502 =	6.	.04638 ÷ 27.1 =
7.	0.76 ÷ 4 =	8.	91.74 ÷ .834 =
9.	1.001 ÷ .101 =	10.	18.8888 ÷ .099 =
11.	.428 ÷ 100 =	12.	67.8 ÷ 10 =
13.	346.77 ÷ 1000 =	14.	55 ÷ 100 =

SELF-TEST ITEMS- PAGE 10

Solve each of the following problems using pencil and paper.

1.	$40.6 + 8.06 =$	2.	$21.21 + 46.19 =$
3.	$3.105 + 4.005 =$	4.	$18.05 + 18.05 =$
5.	$24.3 + 36.3 =$	6.	$166.99 - 58.33 =$
7.	$1.765 - 1.334 =$	8.	$25 - 14.3 =$
9.	$4.10 - 3.22 =$	10.	$32.8 - 31.8 =$
11.	$2.1 \times 4.3 =$	12.	$.2 \times 1.5 =$
13.	$71.3 \times .02 =$	14.	$.03 \times .04 =$
15.	$.012 \times 5.23 =$	16.	$4.32 \div .02 =$
17.	$52.2 \div 3 =$	18.	$42 \div 1.5 =$
19.	$1.35 \div .3 =$	20.	$.015 \div .07 =$

SELF-TEST ITEMS

(CRITERIA 14/20)

1.	2.
3.	4.
5.	6.
7.	8.
9.	10.
11.	12.
13.	14.
15.	16.
17.	18.
19.	20.

(Answers on page 11)

SELF-TEST ITEMS - PAGE 11

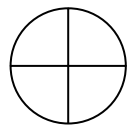
(Answers)

1.	48.66	11.	9.03
2.	67.4	12.	.3
3.	7.11	13.	1.426
4.	36.1	14.	.0012
5.	60.6	15.	.06276
6.	108.66	16.	216
7.	.431	17.	17.4
8.	10.7	18.	28
9.	.88	19.	4.5
10.	1	20.	.214

FRACTIONS

FRACTIONS - <u>PAGE 12</u>

A fraction is a part of a whole number. For example, if we take a circle and divide it into equal parts,



each part is a fraction of the circle. In this example, each part is one-fourth (1/4) of the whole.

Fractions are written with one number beside the other and separated by a slash (5/9), or one number above another and separated by a line $\frac{5}{9}$.

The bottom number (*denominator*) shows how many equal parts the whole has been divided into. The top number *(numerator)* shows how many of the equal parts you actually have.

Five-ninths (5/9) of a foot means a foot is divided into nine equal parts, and you have five of them.

<u>CHAPTER OBJECTIVES</u>

You should be able to multiply, divide, add, and subtract fractions using only pencil and paper by one of the following methods:

a) Changing the fraction problem to a decimal problem and then solving.

b) Solving the problem in fraction form.

Criteria: 70% accuracy

FRACTIONS - PAGE 13

Methods of Solving Fraction Problems

One method of solving fraction problems is to change each fraction into a decimal and then follow the rules that apply to the decimal system. This is probably the easiest method to use, especially if you plan to use a calculator. Fractions must be converted to decimals before they can be entered in a calculator.

For the sake of conversation, and also since some answers are more accurate if left in fractional form, the traditional method of solving fractions is also shown in this workbook.

You may use either method that you prefer.

Changing Fractions to Decimals – PAGE 14

To change a fraction to a decimal, divide the top number (numerator) by the bottom number (denominator).

Example: Change $\frac{4}{9}$ to a decimal number.

$\frac{4}{9}$ means $4 \div 9$

$$\begin{array}{r} .44 \\ 9 \overline{)4.00} \\ \underline{3\,6} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

Exercises

Change the following fractions to decimals using pencil and paper.

1. $\frac{16}{55}$
2. $\frac{3}{8}$
3. $\frac{17}{50}$
4. $\frac{3}{25}$
5. $\frac{17}{125}$
6. $\frac{19}{5}$
7. $\frac{16}{64}$
8. $\frac{3}{13}$
9. $\frac{30}{7}$
10. $\frac{2}{7}$
11. $\frac{5}{4}$
12. $\frac{147}{3125}$

SUBOBJECTIVE

You should be able to change a fraction to a decimal using pencil and paper.

Exercises

1.	2.	3.
4.	5.	6.
7.	8.	9.
10.	11.	12.

OBJECTIVE – PAGE 15

Using only pencil and paper you should be able to add, subtract, multiply, and divide fraction problems by changing them to decimal form.

CRITERIA: 70% accuracy

Solving Problems by Changing Fractions to Decimals

To solve fraction problems by changing them to decimals, you must first change every fraction in the problem to decimal form. Then follow the rules of operation that apply to the decimal system.

Example: $\frac{3}{4} + \frac{4}{5}$

First change each fraction to a decimal:

$$\frac{3}{4} + \frac{4}{5}$$

$$.75 + .80$$

Then add:

$$\begin{array}{r} .75 \\ +\ .80 \\ \hline 1.55 \end{array}$$

<u>Exercises - PAGE 16</u>

Using pencil and paper solve each of the following fraction problems by changing them into decimal form.

1. $\frac{3}{8} + \frac{1}{3} =$

2. $\frac{1}{2} + \frac{3}{8} + \frac{3}{10} =$

3. $19\frac{7}{10} + 86\frac{19}{10} =$

4. $62\frac{1}{8} + 27\frac{9}{10} + 16\frac{3}{8} =$

5. $\frac{3}{5} - \frac{1}{4} =$

6. $74\frac{3}{5} - 42\frac{3}{8} =$

7. $\frac{3}{4} \times \frac{2}{9} =$

8. $\frac{3}{25} \times \frac{2}{3} \times 8 =$

9. $5\frac{1}{2} \times 3\frac{1}{2} =$

10. $31\frac{1}{2} \times 2\frac{3}{8} \times 7\frac{1}{4} =$

11. $\frac{3}{4} \div \frac{2}{9} =$

12. $\frac{3}{25} \div \frac{3}{5} =$

13. $7\frac{1}{2} \div 2\frac{1}{4} =$

14. $(3\frac{1}{2} + 7\frac{1}{8}) - (2\frac{2}{3} \times 1\frac{1}{3})$

15. $(22\frac{3}{5} \div 7\frac{1}{4}) \times (5\frac{1}{6} + 2\frac{3}{7})$

Exercises

1.	2.
3.	4.
5.	6.
7.	8.
9.	10.
11.	12.
13.	14.
15.	

FRACTIONS

<u>OBJECTIVE - PAGE 17</u>

Using pencil and paper only, you should be able to add, subtract, multiply, and divide fractions without changing them into decimal form.

CRITERIA: 70% accuracy

<u>Solving Fraction Problems Using the Traditional Approach</u>

Before attempting to solve fraction problems you need to learn a few more mathematical terms and the fundamental principle underlying all computations with fractions. Terms that will be defined as you go through this chapter are: numerator, denominator, proper fraction, improper fraction, mixed number, and lowest terms.

The fundamental principle basic to all operations with fractions is:

> *The value of a fraction remains the same if the numerator and denominator are both multiplied (or divided) by the same number (not zero).*

FRACTIONS

Equivalent Fractions – PAGE 18

We can change the form of a fraction without changing its value. This is done by multiplying (or dividing) both the numerator and the denominator by the same number.

Examples: $\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ $\qquad \frac{24}{28} \div \frac{4}{4} = \frac{6}{7}$

A fraction is said to be in *lowest terms* when the numerator and the denominator cannot be divided evenly by the same number other than one.

Examples of fractions in lowest terms: $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{15}$

Exercises

Change the following to equivalent fractions with the given denominators.

1. $\frac{1}{3} = \frac{\_}{6}$ 2. $\frac{3}{8} = \frac{\_\_}{24}$ 3. $\frac{4}{9} = \frac{\_\_\_}{36}$

4. $\frac{16}{20} = \frac{\_\_\_}{10}$ 5. $\frac{24}{27} = \frac{\_\_\_}{9}$ 6. $\frac{44}{48} = \frac{\_\_\_}{12}$

Change the following fractions to lowest terms.

7. $\frac{4}{6}$ 8. $\frac{9}{12}$ 9. $\frac{16}{48}$

10. $\frac{75}{216}$ 11. $\frac{540}{720}$ 12. $\frac{215}{335}$

SUBOBJECTIVE

Using pencil and paper only, you should be able to:

a) Change fractions to equivalent fractions

b) Reduce fractions to lowest terms

Exercises

1.	2.	3.
4.	5.	6.
7.	8.	9.
10.	11.	12.

SUBOBJECTIVE – PAGE 19

Using pencil and paper only, you should be able to:

a) Change improper fractions to mixed numbers

b) Change mixed numbers to improper fractions

Improper Fractions and Mixed Numbers

An *improper fraction* is one in which the numerator is larger than the denominator.

Examples: $\frac{9}{7}$, $\frac{3}{2}$, $\frac{7}{4}$, $\frac{12}{10}$

A *mixed number* is one, which has a whole number part and a fraction part.

Examples: $4\frac{1}{2}$, $2\frac{2}{3}$, $9\frac{4}{7}$, $7\frac{3}{5}$

Often we must change an improper fraction to a mixed number or vice versa. It is helpful to remember when changing forms of fractions that the <u>bar separating the numerator from the denominator means division.</u>

Examples: $\frac{6}{5}$ means $6 \div 5$ $\qquad$ $\frac{2}{3}$ means $2 \div 3$

Therefore, to change an improper fraction to a mixed number, divide the numerator by the denominator and express the remainder in fractional form.

Examples: $\frac{9}{7} = 9 \div 7 = 1\frac{2}{7}$

$$\frac{147}{5} = 147 \div 5 = 29\frac{2}{5}$$

To change a mixed number to an improper fraction, multiply the denominator of the fraction by the whole number and add the numerator. This result is put over the original denominator.

Examples: $2\frac{3}{8} = \frac{(8 \text{ x } 2) + 3}{8} = \frac{19}{8}$

$$4\frac{1}{2} = \frac{(2 \text{ x } 4) + 1}{2} = \frac{9}{2}$$

Exercises – PAGE 20

Change the following improper fractions to mixed numbers.

1.	$\frac{36}{7}$	2.	$\frac{29}{4}$	3.	$\frac{42}{10}$
4.	$\frac{80}{8}$	5.	$\frac{162}{32}$	6.	$\frac{331}{12}$

Change the following mixed numbers to improper fractions.

7.	$3\frac{2}{3}$	8.	$4\frac{1}{8}$	9.	$7\frac{6}{11}$
10.	$14\frac{1}{2}$	11.	$26\frac{3}{5}$	12.	$9\frac{1}{9}$

Exercises

1.

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12.

SUBOBJECTIVE – PAGE 21

Using pencil and paper only, you should be able to find the least common denominator of a series of fractions.

Common Denominators

When dealing with several fractions it is often necessary to change each one into an equivalent fraction with a common denominator. A *common denominator* is a number that is evenly divisible by the denominator of the given fractions.

Example: Find a common denominator for the fractions

$\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{6}$.

A common denominator is 12, since 12 is evenly divisible by 3, 4, and 6.

Another common denominator is 24, since 24 is evenly divisible by 3, 4, and 6.

Usually it is easier to deal with several fractions when we know their *least common denominator*. This is the smallest number, which is evenly divisible by the denominators of the given fractions.

Example: Find the least common denominator of the fractions $\frac{5}{14}$, $\frac{9}{18}$, and $\frac{7}{21}$.

Step 1 Set the denominators in a line.

$$\underline{\quad 14 \quad 18 \quad 21 \quad}$$

Step 2 Choose the smallest number which will divide evenly into two or more of the denominators. (In this case it is 2)

$$2 \,\underline{\big|\; 14 \quad 18 \quad 21 \quad}$$

<u>CONTINUATION – PAGE 21</u>

Step 3	Divide each of the evenly divisible denominators by this number, and put the answer on a line below them. If a denominator is not evenly divisible by the number, do not divide. Just bring down the same denominator.	2	14	18	21
			7	9	21
Step 4	Repeat steps 2 and 3 until no two numbers left on the bottom line can be evenly divided by the same number (except 1)	2	14	18	21
		3	7	9	21
		7	7	3	7
			1	3	1
Step 5	To find the least common denominator, multiply all the numbers down the side and across the bottom.				

In this example, we have: 2 x 3 x 7 x 1 x 3 x 1 = 126.

Exercises – PAGE 22

1.

2.

3.

4.

5.

6.

7.

Exercises

Find the least common denominator of the following fractions by inspection.

1. $\frac{3}{5}$, $\frac{2}{3}$, $\frac{1}{2}$

2. $\frac{1}{3}$, $\frac{5}{6}$, $\frac{3}{4}$

3. $\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{10}$

Find the least common denominator of the following fractions by calculation.

4. $\frac{3}{4}$, $\frac{6}{9}$, $\frac{3}{8}$, $\frac{2}{3}$

5. $\frac{4}{15}$, $\frac{9}{7}$, $\frac{5}{3}$, $\frac{7}{10}$

6. $\frac{4}{9}$, $\frac{7}{12}$, $\frac{5}{18}$

7. $\frac{4}{22}$, $\frac{16}{5}$, $\frac{12}{7}$, $\frac{2}{11}$

FRACTIONS

Addition of Fractions – PAGE 23

Fractions can be added only when they have common denominators. To add any fractions, first change each of the fractions to equivalent fractions with a common denominator. Then add the numerators together and place that answer over the common denominator. If the answer is in improper fraction form, you may change it to a mixed number form.

Example: $\frac{2}{3} + \frac{3}{4} =$

$$\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}$$

If the fractions to be added are in mixed number form, first add the whole number part and then add the fraction part.

Example: $1\frac{4}{7} + 2\frac{3}{5} =$

$$1\frac{20}{35} + 2\frac{21}{35} = 3\frac{41}{35} = 4\frac{6}{35}$$

OBJECTIVE

You should be able to add fractions using only pencil and paper.

CRITERIA: **70% accuracy**

Exercises – PAGE 24

1.
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7.
8.
9.
10.
11.
12.

Exercises

Add each of the following fractions.

1. $\frac{3}{8} + \frac{1}{3} =$

2. $\frac{1}{2} + \frac{3}{8} =$

3. $\frac{7}{8} + \frac{1}{6} =$

4. $\frac{1}{3} + \frac{1}{4} + \frac{1}{6} =$

5. $\frac{5}{8} + \frac{1}{6} + \frac{7}{12} =$

6. $\frac{5}{6} + \frac{1}{2} + \frac{3}{10} =$

7. $$\begin{array}{r} 8\frac{1}{4} \\ 17\frac{3}{8} \\ 9\frac{5}{12} \\ \hline \end{array}$$

8. $$\begin{array}{r} 14\frac{1}{3} \\ 17\frac{5}{6} \\ 21\frac{1}{12} \\ \hline \end{array}$$

9. $$\begin{array}{r} 62\frac{1}{8} \\ 27\frac{13}{16} \\ 29\frac{3}{4} \\ \hline \end{array}$$

10. $$\begin{array}{r} 47\frac{7}{10} \\ 86\frac{19}{100} \\ 37\frac{3}{4} \\ \hline \end{array}$$

11. $$\begin{array}{r} 19\frac{17}{45} \\ 64 \\ 37\frac{8}{27} \\ \hline \end{array}$$

12. $$\begin{array}{r} 44\frac{15}{44} \\ 36\frac{11}{36} \\ 18\frac{7}{8} \\ \hline \end{array}$$

<u>Subtraction of Fractions – PAGE 25</u>

Fractions can be subtracted only when they have common denominators. To subtract any fractions, first change each of the fractions to an equivalent fraction with a common denominator. Then subtract the second fraction from the first and place the answer over the common denominator.

Example:

$$\frac{7}{8} - \frac{2}{3} =$$

$$\frac{21}{24} - \frac{16}{24} = \frac{5}{24}$$

If the fractions to be subtracted are in mixed number form, first subtract the fraction part and then subtract the whole number part.

Example:

$$3\frac{5}{6} - 1\frac{1}{3} =$$

$$3\frac{5}{6} - 1\frac{2}{6} = 2\frac{3}{6} = 2\frac{1}{2}$$

If the fractions to be subtracted are in mixed number form, and if the fraction part of the second number is larger than the fraction part of the first number, do the following: Borrow 1 from the whole number part of the first. Change the 1 to a fraction having the same denominator as the fraction part. Add this fraction to the fraction part of the first number. Then continue with the subtraction.

<u>OBJECTIVE</u>

Using only pencil and paper you should be able to subtract fractions.

CRITERIA: 70% accuracy

<u>Exercises – PAGE 26</u>

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.
11.
12.

Example:

$$5\frac{1}{4} - 2\frac{3}{5} =$$

$$5\frac{5}{20} - 2\frac{12}{20} =$$

$$\left(4\ \frac{20}{20} + \frac{5}{20}\right) - 2\frac{12}{20} = 4\frac{25}{20} - 2\frac{12}{20} = 2\frac{13}{20}$$

<u>Exercises</u>

Subtract each of the following fractions.

1. $\frac{3}{5} - \frac{1}{4} =$

2. $\frac{3}{4} - \frac{1}{8} =$

3. $\frac{13}{80} - \frac{1}{16} =$

4. $\frac{17}{18} - \frac{20}{27} =$

5. $\frac{3}{11} - \frac{4}{19} =$

6. $\frac{19}{35} - \frac{17}{42} =$

7. $\begin{array}{r} 18\frac{2}{3} \\ -\ 12\frac{1}{2} \\ \hline \end{array}$

8. $\begin{array}{r} 17\frac{3}{4} \\ -\ 12\frac{1}{3} \\ \hline \end{array}$

9. $\begin{array}{r} 74\frac{3}{5} \\ -\ 42\frac{3}{8} \\ \hline \end{array}$

10. $\begin{array}{r} 639\frac{3}{8} \\ -\ 428\frac{3}{4} \\ \hline \end{array}$

11. $\begin{array}{r} 40\frac{1}{5} \\ -\ 17\frac{19}{20} \\ \hline \end{array}$

12. $\begin{array}{r} 600\frac{3}{7} \\ -\ 199\frac{5}{8} \\ \hline \end{array}$

<u>Combining Addition and Subtraction of Fractions – PAGE 27</u>

When solving fraction problems, which contain both addition and subtraction, you must solve only one operation at a time. The result is then used to perform the next operation. This procedure is followed until all operations are performed.

Example: $3\frac{7}{8} + 2\frac{2}{3} - 3\frac{1}{2} - 1\frac{3}{4}$

First add: $3\frac{7}{8} + 2\frac{2}{3} - 3\frac{1}{2} - 1\frac{3}{4}$

(Common Denominator) $3\frac{21}{24} + 2\frac{16}{24} - 3\frac{1}{2} - 1\frac{3}{4}$

$6\frac{13}{24} - 3\frac{1}{2} - 1\frac{3}{4}$

Then subtract: $6\frac{13}{24} - 3\frac{1}{2} - 1\frac{3}{4}$

(Common Denominator) $6\frac{13}{24} - 3\frac{12}{24} - 1\frac{18}{24}$

$3\frac{1}{24} - 1\frac{18}{24}$

OBJECTIVE

Using only pencil and paper you should be able to solve fraction problems, which contain both addition and subtraction.

CRITERIA: 70% accuracy

OBJECTIVE

Using only pencil and paper you should be able to:

a) multiply fractions

b) simplify multiplication problems by canceling

Multiplication of Fractions – PAGE 28

To multiply numbers expressed as fractions, multiply the numerators together and multiply the denominators together.

Example: $\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12} = \frac{1}{2}$

To multiply mixed numbers, first express each mixed number as an improper fraction and then multiply as described above.

Example: $2\frac{1}{2} \times 3\frac{2}{5}$

$$\frac{5}{2} \times \frac{17}{5} = \frac{5 \times 17}{2 \times 5} = \frac{85}{10} = 8\frac{5}{10} = 8\frac{1}{2}$$

Cancellation

Multiplication of fractions can sometimes be simplified by dividing (*canceling*) a numerator and a denominator by the same number.

Example: $\frac{2}{3} \times \frac{3}{8}$

The numerator of the first fraction and the denominator of the second fraction can both be evenly divided by 2. Also the denominator of the first fraction and the numerator of the second fraction can both be evenly divided by 3.

$$\frac{{}^{1}\cancel{2}}{{}_{1}\cancel{3}} \times \frac{\cancel{3}^{1}}{\cancel{8}_{4}} = \frac{1 \times 1}{1 \times 4} = \frac{1}{4}$$

Exercises – PAGE 29

1.

2.

3.

4.

Then subtract again:

$$3\frac{1}{24} - 1\frac{18}{24}$$

$$\downarrow$$

(Borrowing) $$2\frac{25}{24} - 1\frac{18}{24}$$

$$1\frac{7}{24}$$

Exercises

Compute each of the following.

1. $\frac{3}{5} + 6\frac{3}{7} - \frac{18}{35} =$

2. $\frac{3}{4} - \frac{5}{8} + \frac{1}{2} =$

3. $3\frac{2}{5} - 1\frac{2}{3} + \frac{13}{15} =$

4. $7\frac{1}{4} - 2\frac{1}{3} - 4\frac{3}{8} =$

Exercises – PAGE 30

1. 11.
2. 12.
3. 13.
4. 14.
5.
6.
7.
8.
9.
10.

BONUS PROBLEMS

1.

2.

Exercises

Multiply each of the following fractions canceling when possible.

1. $\frac{3}{4} \times \frac{2}{9} =$

2. $\frac{3}{5} \times \frac{1}{3} =$

3. $\frac{5}{7} \times 2 =$

4. $\frac{3}{25} \times \frac{2}{3} \times 8 =$

5. $\frac{3}{4} \times 2 \times \frac{1}{2} =$

6. $\frac{12}{35} \times \frac{3}{4} \times \frac{7}{10} =$

7. $\frac{3}{4} \times \frac{2}{3} \times 8 =$

8. $\frac{3}{2} \times \frac{5}{7} \times \frac{11}{8} =$

9. $2\frac{1}{3} \times 3\frac{1}{5} =$

10. $8 \times 4\frac{3}{16} =$

11. $2 \times 4\frac{3}{4} \times 0 =$

12. $2\frac{1}{4} \times \frac{3}{4} \times 2\frac{8}{9} =$

13. $\frac{8}{35} \times \frac{2}{9} \times \frac{3}{16} \times \frac{7}{3} =$

14. $3\frac{1}{5} \times 4\frac{1}{2} \times 3\frac{5}{12} \times 4 =$

BONUS PROBLEMS:

1. $1\frac{2}{3} \times \frac{18}{21} \times \frac{6}{9} \times \frac{15}{24} \times 6 \times 3\frac{1}{3} \times \frac{7}{25} \times \frac{3}{10}$

2. $4\frac{1}{5} \times 16\frac{2}{3} \times 9 \times \frac{7}{45} \times \frac{17}{100} \times 15 \times \frac{2}{51} \times \frac{5}{7}$

Division of Fractions – PAGE 31

To divide numbers expressed as fractions, first invert (turn upside down) the second number, and then multiply following the rules of multiplication.

Example:

$$\frac{4}{9} \div \frac{5}{6} =$$

$$\frac{4}{9} \times \frac{6}{5} = \frac{4 \times 6}{9 \times 5} = \frac{24}{45} = \frac{8}{15}$$

Example:

$$3\frac{2}{5} \div 5\frac{1}{4} =$$

$$\frac{17}{5} \div \frac{21}{4} =$$

$$\frac{17}{5} \times \frac{4}{21} = \frac{17 \times 4}{5 \times 21} = \frac{68}{105}$$

OBJECTIVE

You should be able to divide fractions using only pencil and paper.

CRITERIA: 70% accuracy

Exercises – PAGE 32

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.
11.
12.
13.
14.
15.
16.

Exercises

Divide each of the following fractions, canceling when possible.

1. $\frac{3}{4}$) $\frac{2}{5}$ =
2. $\frac{3}{5}$) $\frac{4}{5}$ =
3. $\frac{3}{4}$) $\frac{1}{3}$ =
4. $\frac{7}{22}$) $\frac{2}{11}$ =
5. $\frac{2}{3}$) 6 =
6. 8) $\frac{3}{4}$ =
7. $\frac{7}{36}$) $\frac{14}{27}$ =
8. $\frac{24}{7}$) $\frac{2}{3}$ =
9. 17) $\frac{34}{35}$ =
10. $1\frac{2}{3}$) $6\frac{2}{3}$ =
11. $6\frac{3}{4}$) $9\frac{1}{3}$ =
12. $4\frac{8}{19}$) $5\frac{1}{4}$ =
13. $9\frac{1}{3}$) $4\frac{2}{3}$ =
14. $6\frac{3}{5}$) 11 =
15. 51) $3\frac{2}{5}$ =
16. $14\frac{2}{7}$) $7\frac{1}{2}$ =

SELF-TEST ITEMS – PAGE 33

Using pencil and paper solve each of the following fraction problems using either method you prefer.

1. $\frac{1}{2} + \frac{1}{8} =$

2. $\frac{9}{10}$) $\frac{2}{3} =$

3. $\frac{2}{3} \times \frac{3}{8} =$

4. $\frac{1}{8} \div \frac{1}{4} =$

5. $2\frac{1}{3} + 3\frac{5}{6} =$

6. $14\frac{1}{2} - 12\frac{3}{4} =$

7. $7\frac{1}{3} \times 1\frac{1}{11} =$

8. $2\frac{1}{2} \div 6\frac{2}{3} =$

9. $41\frac{7}{10} + 71\frac{85}{100} + 16\frac{1}{4} =$

10. $35\frac{3}{8} - 22\frac{7}{9} =$

11. $3\frac{1}{8} \times 2\frac{5}{6} \times 10\frac{4}{7} =$

12. $17\frac{2}{3} \div 5\frac{2}{5} =$

SELF-TEST ITEMS
(CRITERIA 11/12)

1.	2.
3.	4.
5.	6.
7.	8.
9.	10.
11.	12.

(Answers on page 34)

SELF-TEST ITEMS – PAGE 34

(Answers)

1. 5/8 or .625
2. 7/30 or $.\overline{23}$
3. 1/4 or .25
4. 1/2 or .5
5. 6 1/6 or $6.\overline{16}$
6. 1 3/4 or 1.75
7. 8
8. 3/8 or .375
9. 129 4/5 or 129.8
10. $12\ \frac{43}{72}$ or 12.59
11. $93\ \frac{101}{168}$ or 93.60
12. $3\ \frac{22}{81}$ or 3.27

PERCENTS

PERCENTS – PAGE 35

Percent (%) is a term we use to mean "per hundred," "out of 100," or "compared to 100."

Examples:

	57%	means 57 out of 100
	4.3%	means 4.3 out of 100
	.03%	means .03 out of 100
	8%	means 8 out of 100
	438%	means 438 compared to 100

Problems involving percent often arise in our work. Normally, in solving them we do not use the percent expression, but replace it with a decimal or fractional equivalent.

CHAPTER OBJECTIVE

Using any method you prefer you should be able to interchange the following:

a) percents and decimals

b) percents and parts per million (ppm)

CRITERIA: 90% accuracy

PERCENTS

OBJECTIVE – PAGE 36

Using any method you prefer you should be able to interchange decimals and percents.

CRITERIA: 90% accuracy

Interchanging Percents to Decimals

Method 1

To change a percent to a decimal, move the decimal point two places to the left and drop the percent sign.

Examples: 37% = .37

21.4% = .214

To change a decimal to a percent, move the decimal point two places to the right and add a percent sign.

Examples: .56 = 56%

.314 = 31.4%

Method 2

To change a percent to a decimal, divide the percent by 100 and drop the percent sign.

Examples: 3% = 3 ÷ 100 = .03

.05% = .05 ÷ 100 = .0005

To change a decimal to a percent multiply the decimal by 100 and add the percent sign.

Examples: 1.8 = 1.8 x 100% = 180%

25 = 25 x 100% = 2500%

<u>The aid shown below may help you to remember these rules. – PAGE 37</u>

$$\text{decimal} \underset{\div 100}{\overset{\times 100}{\rightleftarrows}} \text{percent}$$

Exercises

Interchange each of the following percents and decimals.

	Percent		Decimal
1.	19%	=	______
2.	_____	=	.35
3.	5%	=	______
4.	_____	=	.02
5.	.5%	=	______
6.	_____	=	.058
7.	150%	=	______
8.	_____	=	4.6

<u>Exercises</u>

1.

2.

3.

4.

5.

6.

7.

8.

PERCENTS

OBJECTIVE – PAGE 38

Using any method you prefer you should be able to interchange percents and parts per million (ppm).

CRITERIA: 90% accuracy

Interchanging Percents and Parts Per Million

Method 1: To change percent to parts per million, move the decimal point four places to the right and change the percent sign to ppm.

Examples: 4% = 40000 ppm

.51% = 5100 ppm

To change parts per million to decimals, move the decimal point four places to the left and change the ppm to a percent sign.

Examples: 20,000 ppm = 2%

153 ppm = .0153%

Method 2: To change percent to parts per million, multiply the percent by 10,000 and change the percent sign to ppm.

Examples: 15% = 15 x 10,000 = 150, 000 ppm

4.8% = 4.8 x 10,000 = 48,000 ppm

To change parts per million to percent, divide the parts per million by 10,000 and change the ppm to a percent sign.

Examples: 5,000 ppm = 5000) 10,000 = .5%

75,000 ppm = 75,000) 10,000 = 7.5%

The aid shown below may help you remember these rules.

x

percent 10,000 parts per million

÷

<u>Exercises – PAGE 39</u>

Interchange each of the following percents and parts per million.

	Percent	Parts per million
1.	_________	250,000
2	83%	_________
3.	_________	98,000
4.	42%	_________
5.	_________	4,200
6.	4.9%	_________
7.	_________	100
8.	.03%	_________

<u>Exercises</u>

1.

2.

3.

4.

5.

6.

7.

8.

SELF-TEST ITEMS
(Answers)

1.	58%	580,000 ppm
2.	.42	420,000 ppm
3.	.31	31%
4.	430%	4,300,000 ppm
5.	.07	70,000 ppm
6.	.071	7.1%
7.	.5%	5,000 ppm
8.	1.5	1,500,000 ppm
9.	**.0015**	**.15%**

SELF-TEST ITEMS – PAGE 40

CRITERIA: 16/18

1.
2.
3.
4.
5.
6.
7.
8.
9.

SELF TEST

Interchange the following decimals, percents, and parts per million.

	Decimal		Percent		Parts Per Million
1.	.58	=	______	=	______________
2.	_________	=	42%	=	______________
3.	_________	=	______	=	310,000 ppm
4.	4.3	=	______	=	______________
5.	_________	=	7%	=	______________
6.	_________	=	______	=	71,000 ppm
7.	.005	=	______	=	______________
8.	_________	=	150%	=	______________
9.	_________	=	______	=	1,500 ppm

(Answers on page 39)

FORMULAS – PAGE 41

A formula is *a rule which, in mathematical language, gives directions for finding an unknown value.*

Example: The rule "the area of a rectangular entry is equal to its width multiplied by its height" may be stated mathematically by the formula

$$\mathbf{A = w \times h}$$

CHAPTER OBJECTIVES

Using any method you prefer you should be able to select the appropriate formula from those listed below and solve it.

a) Area of a rectangular passage A = wh

b) Area of a circular passage $A = \pi r^2$

c) Area of a trapezoidal passage
$$A = \left(\frac{B + b}{2}\right) \times h$$

d) Perimeter of a 4-sided passage

e) Circumference of a circular passage
$C = 2\pi r$ or $C = \pi d$

SUBOBJECTIVE - PAGE 42

You should be able to write a mathematical formula from a given rule.

Exercises

1.

2.

3.

4.

5.

Exercises

Write the formulas for each of the following rules.

1. The electrical power (P) is equal to the voltage (E) multiplied by the current (I).
2. The area of a triangle (A) is equal to ½ times the base (b) of the triangle times the height (h) of the triangle.
3. The diagonal of a square (d) is equal to 1.414 times the side (s) of the square.
4. The perimeter (P) of a rectangular airway is equal to twice the width (w) plus twice the height (h).
5. The total resistance (R_T) of a series circuit is equal to the sum of the individual resistances (R_1 , R_2 , and R_3).

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6. The total ventilating pressure (P) is equal to the unit ventilating pressure (p) times the cross sectional area (A). 6.

7. The unit ventilating pressure (p) is equal to the inches of water gage (i) times 5.2. 7.

8. The quantity of air (Q) passing through an airway in a certain length of time is the product of the velocity (v) and the cross sectional area of the airway (A). 8.

9. The voltage (E) in a circuit is equal to the current (I) times the resistance (R). 9.

10. The area (A) of a circular shaft is equal to 3.14 times the radius (r) squared. 10.

11. The volume (V) that a rectangular entry will contain is equal to the product of the width (w) of the entry, the height (h) of the entry, and the length (l) of the entry. 11.

12. The circumference (C) of a circuit is equal to 3.14 times the diameter (d) of the shaft. 12.

13. The power factor (PF) of an ac circuit is equal to the total resistance (R_T) divided by the impedance (Z). 13.

14. The capacitive reactance (X_c) of a capacitor is equal to 159,000 divided by the product of the frequency (f) and the capacitance (C). 14.

15. The square of the hypotenuse (c) of a right triangle is equal to the sum of the squares of the sides (a and b). 15.

PAGE 44

16.

17.

18.

19.

20.

16. The perimeter (P) of a square is equal to the sum of the sides (s).

17. The area of a square airway (A) is equal to the side of the airway (s) squared.

18. The number of inches indicated on the water gage (i) is equal to the unit ventilating pressure (p) divided by 5.2.

19. Total ventilating pressure (P) is equal to the product of the coefficient of fraction (k), the length of the airway (l), the perimeter of the airway (o), and the velocity of air (v) squared.

20. The unit ventilating pressure (p) is equal to the product of the coefficient of friction (k), the length of the airway (l), the perimeter of the airway (o), and the velocity of air (v) squared, divided by the cross sectional area of the airway (A).

Substituting Values into Formulas – PAGE 45

That which makes a problem a problem is the existence of an unknown value. The way we solve a problem is by gathering enough known facts to allow us to determine the unknown value.

Example: Substitute the known values into the formula A = wh to find the cross sectional area of a rectangular entry 10 feet wide by 8 feet high.

$A = w \times h$

$A = 10 \times 8$

SUBOBJECTIVE

You should be able to substitute number values into formulas.

Exercises

In each of the following problems, substitute the known value into the given formulas. DO NOT SOLVE.

1. If $P = 2w + 2h$, what is the perimeter of a rectangular entry whose width is 2.5 feet and whose height is 14 feet?
2. If $A = wh$, find the area of a rectangular entry whose width is 2.5 feet and whose height is 14 feet.
3. If $P = a + b + c$, find the perimeter of a triangle whose sides are 75, 166, and 152 feet.
4. If $P = 2\pi r$, what is the circumference of a circular shaft with a radius of 16 inches? ($\pi = 3.14$)

Exercises

1.

2.

3.

4.

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5.

6.

7.

8.

9.

10.

11.

12.

13.

5. If $A = 1/2bh$, what is the area of a triangle with a base of 40 feet and a height of 60 feet?

6. If $P = 2w + 2h$, what is the width of a rectangular entry whose perimeter is 50 feet and whose height is 5 feet?

7. If $A = wh$, find the height of a rectangular airway whose area is 50 square feet and whose width is 20 feet.

8 If $P = a + b + c$, find the missing side of a triangle whose perimeter is 48 inches and whose other sides are 12 inches and 18 inches.

9. If $C = 2\pi r$, find the radius of a circular airway whose circumference is 36 feet. ($\pi = 3.14$)

10. If $A = 1/2bh$, find the base of a triangle whose area is 132 square feet and whose height is 12 feet.

11. If $Q = Av$, find the velocity of air in an airway if the cross sectional area of the airway is 48 square feet and if the quantity of air passing in the airway is 20,000 cubic feet per minute.

12. If $^{o}F = 9/5\ (^{o}C + 40) - 40$, find the temperature in degrees Fahrenheit if the centigrade reading is 22^{o}.

13. If $p = 5.2i$, find the water gage reading, if the unit ventilating pressure is 12 pounds per square foot.

PAGE 47

14. If $P = klov^2$, find the total ventilating pressure if the coefficient of friction is .00000002, the length of the airway is 36 feet, the perimeter of the airway is 24 feet, and the velocity of air is 2400 feet per minute. 14.

15. If $p = \frac{klov^2}{A}$, find the cross-sectional area of an airway if the unit ventilating pressure is 10 pounds per square foot, the coefficient of friction is .00000002, the length of the airway is 320 feet, the perimeter of the airway is 36 feet, and the velocity of air is 1600 feet per minute. 15.

OBJECTIVE – PAGE 48

Using any method you prefer you should be able to select the appropriate formula and solve area problems for the following types of passages:

a) Rectangular
$A = wh$

b) Trapezoidal

$A = \frac{B + b}{2} \times h$

c) Circular
$A = \pi r^2$

Solution of Formulas

To solve a formula means to find the value of an unknown (represented by a letter). This value is found by getting the letter to stand alone on one side of the formula.

In some of the formulas you need to solve, the unknown value is already alone on one side.

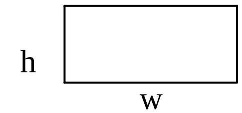
Example: If A = wh, find the area of a rectangular entry whose width is 10 feet and whose height is 5 feet.

	$A = wh$
substituting values	$A = 10 \times 5$
solving	$A = 50$ sq ft

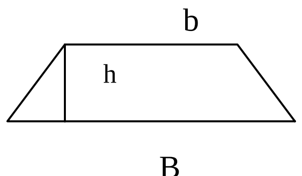
Area Formulas

There are several area formulas you need to be able to solve:

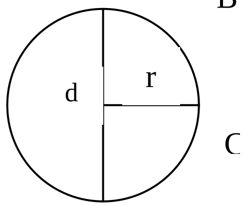
Rectangular passage $A = wh$



Trapezoidal passage $A = \frac{B + b}{2} \times h$



Circular passage $A = \pi r^2$
$(\pi = 3.14)$
or $A = \frac{\pi}{4} (d^2)$
or $A = \frac{C^2}{4\pi}$



CONTINUATION – PAGE 48

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Example: **Find the area of a rectangular airway 4 feet wide and 3 feet high.**

Use the formula $A = wh$

Substitute values $A = 4 \times 3$

Solve $A = 12$ sq. ft

Example: Find the area of a passage 8 feet wide at the bottom, 6 feet wide at the top, and 5 feet high.

Use the formula $A = \left(\frac{B + b}{2}\right) \times h$

Substitute values $A = \left(\frac{8 + 6}{2}\right) \times 5$

Solve $A = \frac{14}{2} \times 5$

$A = 7 \times 5$

$A = 35$ sq. ft

Example: Find the cross-sectional area of a shaft 5 feet in radius.

Use the formula $A = \pi r^2$

Substitute values $A = 3.14 \times 5^2$

Solve: $A = 3.14 \times 25$

$A = 78.5$ sq. ft.

$A = 78.5$ sq. ft

Exercises – PAGE 50

1.

2.

3.

4.

5.

6.

7.

Exercises

1. A passage is 5 feet high and 9 feet wide. What is its area?

2. What is the area of a crosscut 16 feet wide and 6 feet high?

3. Find the area of an airway 5 feet 6 inches high and 14 feet wide.

4. A passage is 10 feet wide at the bottom, 6 feet wide at the top, and 7 feet high. What is the area?

5. An airway is 12 feet wide at the bottom, 9 feet 6 inches wide at the top, and 6 feet high. What is the area?

6. What is the cross-sectional area of a pipe 5 feet in diameter?

7. What is the cross-sectional area of a shaft 6 feet 4 inches in diameter?

Perimeter Formulas – PAGE 51

There are also several perimeter formulas you need to be able to solve.

Perimeter of a 4-sided passage	or	P = Sum of the sides
		P = 2w + 2h (Rectangle)
Perimeter of a circular passage		$P = \pi \times d$ ($\pi = 3.14$)

Example: Find the perimeter of an airway 8 feet wide and 6 feet high.

Use the formula	P = 2w + 2h
Substitute values	P = (2 x 8) + (2 x 6)
Solve	P = 16 + 12
	P = 28 feet

Example: Find the perimeter of a shaft 12 feet in diameter.

Use the formula	$P = \pi \times d$
Substitute values	P = 3.14 x 12
Solve	P = 37.68 feet

OBJECTIVE

Using any method you prefer you should be able to select the appropriate formula and solve perimeter problems for the following types of passages:

a) four-sided

b) circular

Exercises – PAGE 52

1.

2.

3.

4.

Exercises

1. What is the circumference (perimeter) of a circular shaft 14 feet in diameter?

2. Find the perimeter of a rectangular entry that measures 10 feet by 15 feet.

3. Determine the circumference of a circular shaft with a radius of 5 feet.

4. What is the perimeter of a crosscut that is 5 feet 6 inches high and 8 feet wide?

SELF-TEST ITEMS – PAGE 53

1(a) What is the area of a circular shaft 13.5 feet in diameter?

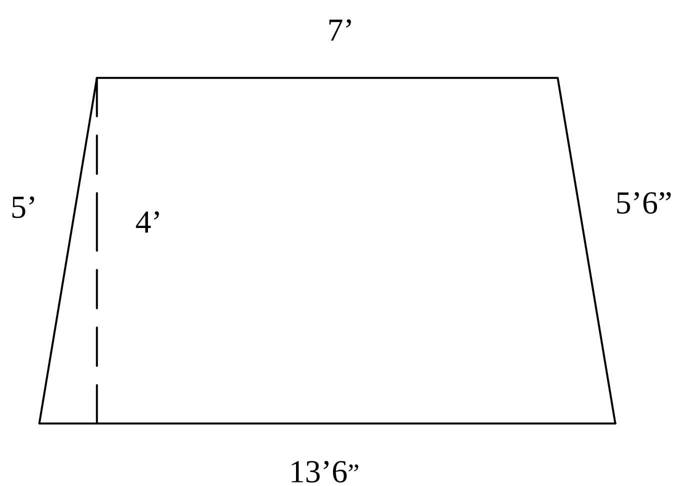
(b) What is the circumference?

2(a) What is the perimeter of a crosscut that is 6 feet 6 inches high and 7 feet wide?

(b) What is the cross-sectional area?

3(a) What is the cross-sectional area with dimensions shown on the figure below?

(b) What is the perimeter?



SELF-TEST ITEMS CRITERIA:

1 (a)
(b)

2 (a)
(b)

3 (a)
(b)

(Answers on page 54)

SELF-TEST ITEMS - PAGE 54

(Answers)

1 (a) 143.139 sq. ft.
(b) 42.412

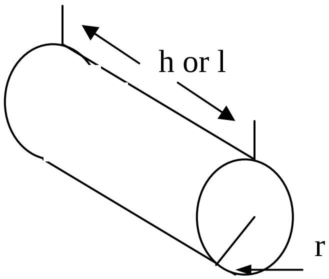
2 (a) 27 ft.
(b) 45 ½ sq. ft.
or
45.5 sq. ft.

3 (a) 41 sq. ft.
(b) 31 ft.

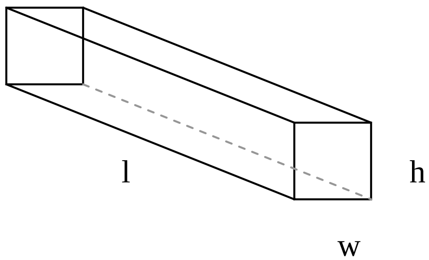
Volume Formulas – PAGE 55

There are times when it is necessary, particularly when working with mine ventilation problems, to solve for the volume of an airway or piece of tubing. These volumes are typically in two geometric shapes:

a)



b)



To solve for the volume of these figures, first find the area of the face and multiply that by the length.

Or, the following formulas may be used:

Cylinder or Circular shaft $V = \pi r^2 h$

or $V = \frac{\pi}{4} d^2 l$

If the radius (or diameter) and the height (or length) are given in feet, then the answer will be in cubic feet (or ft^3). To change cubic feet to gallons, multiply by 7.48.

Rectangular Airway $V = lwh$

SUBOBJECTIVE

You should be able to solve for the volume of the following types of figures:

a) Cylinder or Circular Shaft

$V = \pi r^2 h$

or

$V = \frac{\pi}{4} d^2 l$

b) Rectangular Airway

$V = lwh$

Example: Find the volume of a piece of tubing 12” in diameter and 50’ in length.

$V = \pi r^2 h$

Since the diameter is 12”, the radius is 6” or ½’ or .5’

$V = 3.14 \text{ x } (.5')^2 \text{ x } 50'$

V = 3.14 x .25 sq ft x 50 ft

V = .785 sq ft x 50 ft

V = 39.25 cu ft

Example: Find the volume of a rectangular airway that is approximately 5.5’ high, 16’ wide, and 200’ long.

V = lwh

V = 200’ x 16’ x 5.5’

V = 3200 sq ft x 5.5 ft

V = 17,600 cu ft

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Exercises

1. A passage is 5 feet high, 12 feet wide, and 50 feet long. What is its volume?

2. What is the volume of an airway that is 7.5 feet high, 21.3 feet wide, and 250 feet long?

3. What is the volume of a circular shaft that is 7 feet in diameter and 85 feet deep?

4. **A piece of tubing is 1.5 feet in diameter and 43 feet long. What is its volume?**

Exercises

1.

2.

3.

4.

SUBOBJECTIVE – PAGE 58

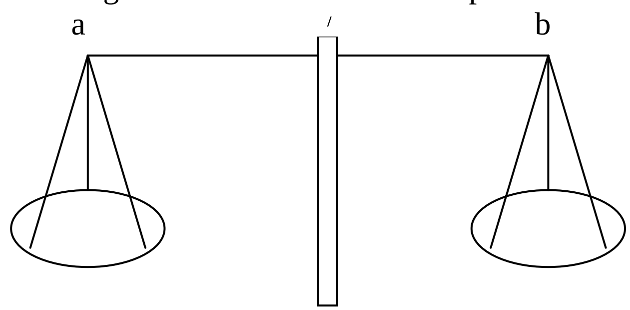
You should be able to solve for the value of an unknown letter in a simple formula, involving only a single step of addition, subtraction, multiplication, or division.

Other Formulas

Sometimes when solving formulas, the unknown letter is not alone on one side of the equal side. This makes solving the formula a little more difficult. To solve the formula we must eliminate all the numbers and letters which appear on the same side of the equal sign as the unknown letter. This can be done by applying the following rule:

> *Any mathematical operation performed on one side of an equal sign must also be performed on the other side.*

A formula can be thought of as a balance. with the point of balance being the equal sign.



It is obvious using this comparison that whatever weight is added to side *a* of the balance must also be added to side *b* in order for it to remain in balance. The same is true for subtraction, multiplication, or division.

In other words, *whatever we do to one side of the formula must also be done to the other side.*

Solving Other Formulas

Solving a formula means to get the unknown letter all alone on one side of the equal sign, with its value on the other side. We can get rid of a number of letters on one side of the formula if we do the opposite operation.

PAGE 59

To get rid of a value *added* to an unknown, subtract it.

$A + 6 - 6 = A$

To get rid of a value subtracted from an unknown, add it.

$B - 3 + 3 = B$

To get rid of a value multiplied by an unknown, divide by it.

$5C \div 5 = C$

To get rid of a value that is divided into an unknown, multiply by it.

$D / 2 \times 2 = D$

Example: Solve $X + 5 = 8$

We know we can solve for X if we can get rid of the 5. Since 5 is added to X, we can get rid of it by subtracting 5 from both sides of the formula.

$X + 5 - 5 = 8 - 5$

$X + 0 = 3$

$X = 3$

Example: Solve $Y - 8 = 3$

We can solve this formula for Y if we can get rid of the 8. Since 8 is subtracted from Y, we can get rid of it by adding 8 to both sides.

$Y - 8 + 8 = 3 + 8$

$Y + 0 = 11$

$Y = 11$

Exercises – PAGE 60

1.

2.

3.

4.

5.

6.

7.

8.

FORMULAS

Exercises

Solve each of the following formulas for the value of the unknown letter.

1. $E + 3 = 9$
2. $R - 5 = 12$
3. $10 + P = 80$
4. $2 + R = 10$
5. $I - 4 = 18$
6. $110 = E + 60$
7. $R + 7 - 2.4 + 3.6 = 19.68$
8. $X + 2.6 + 1.05 + 3.0 = 9.4$

PAGE 61

The following examples show how to solve formulas, which involve multiplication and division.

Example: Solve $2R = 10$

Since R is multiplied by 2, we can get rid of the 2 by dividing both sides by 2

$$\frac{2R}{2} = \frac{10}{2}$$

Then, if we cancel

$$\frac{\overset{1}{\cancel{2}}R}{\underset{1}{\cancel{2}}} = \frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{2}}}$$

We are left with

$R = 5$

Example: Solve $\frac{Q}{20} = 13$

Since Q is divided by 20, we can get rid of the 20 by multiplying both sides of the formula by 20

$$20 \times \frac{Q}{20} = 13 \times 20$$

Then, if we cancel on the left and multiply on the right

$$^{1}\cancel{20} \times \frac{Q}{\cancel{20}_1} = 260$$

We are left with $Q = 260$

PAGE 62

1.

2.

3.

4.

5.

6.

FORMULAS

Exercises

Solve each of the following formulas for the value of the unknown letter.

1. $3R = 15$
2. $\frac{E}{10} = 3$
3. $8X = 12$
4. $\frac{Y}{3} = 10.5$
5. $\frac{A}{2} = 11.4$
6. $1.5B = 25.5$

FORMULAS

Ratio and Proportion – PAGE 63

A *ratio* is a comparison of two numbers by division. For example, $3 \div 4$ or $\frac{3}{4}$ is a ratio.

A *proportion* is a statement that two ratios are equal. For example, $\frac{1}{2} = \frac{3}{6}$ is a proportion.

Given a proportion $\frac{a}{b} = \frac{c}{d}$, b and c are defined as the *mean* terms, and *a* and *d* are defined as the *extreme* terms.

Means *Extremes*

$$\frac{a}{b} = \frac{c}{d}$$

Many formulas related to mining can be solved by applying the following rule of proportions:

The product of the means is equal to the product of the extremes.

In other words, the proportion $\frac{a}{b} = \frac{c}{d}$ can also be written in **the form: b x c = a x d**

SUBOBJECTIVE

You should be able to solve a proportion for an unknown value.

FORMULAS

<u>Ratio and Proportion - Page 63</u>

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<u>SUBOBJECTIVE</u>

You should be able to solve a proportion for an unknown value.

PAGE 64

Exercises

1.

2.

3.

4.

5.

6.

7.

8.

FORMULAS

Example: Solve the proportion $\frac{E}{2} = \frac{5}{20}$

By applying our rule, *the product of the means is equal to the product of the extremes,* we get

$$2 \times 5 = E \times 20$$

$$10 = E \times 20$$

$$\frac{10}{20} = \frac{E \times 20}{20}$$

$$\frac{{}^{1}\cancel{10}}{{}_{2}\cancel{20}} = \frac{E \times \cancel{20}^{1}}{\cancel{20}_{1}}$$

$$\frac{1}{2} = E$$

Exercises

Solve each of the following proportions for the unknown value.

1. $\frac{A}{5} = \frac{6}{3}$

2. $\frac{10}{4} = \frac{C}{8}$

3. $\frac{7}{B} = \frac{2}{4}$

4. $\frac{3}{2} = \frac{9}{D}$

5. $\frac{24}{2} = \frac{6M}{5}$

6. $\frac{3P}{.04} = \frac{12}{2}$

7. $\frac{3}{12} = \frac{4}{3T}$

8. $\frac{1}{2R} = \frac{13}{26}$

PAGES 4-5

(1)	74.496	(2)	44.079	(3)	185.355
(4)	5147.217	(5)	83.3352	(6)	415.902
(7)	3.0538	(8)	386.9	(9)	26.981
(10)	66.908	(11)	28.207	(12)	321.39
(13)	353.265	(14)	67.457	(15)	.8257
(16)	.0001	(17)	109.65	(18)	147.13
(19)	.941	(20)	.00185		

PAGE - 7

(1)	96.9381	(2)	273.0702	(3)	1.8848
(4)	.13356	(5)	.0496	(6)	.249
(7)	.0000512	(8)	26.268	(9)	5.017006
(10)	1.0836				

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(1)	12.77	(2)	15.20	(3)	2.76
(4)	.0151	(5)	1609.36	(6)	.0017
(7)	.19	(8)	110	(9)	9.91
(10)	190.79	(11)	.004	(12)	6.78
(13)	**.34**	**(14)**	**.55**		

ANSWERS

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(1) .29	(2) .37	(3) .34	(4) .12
(5) .14	(6) 3.8	(7) .25	(8) .23
(9) 4.29	(10) . 29	(11) 1.25	(12) .05

PAGE - 16 (answers are rounded to 3 places)

(1) .705	(2) 1.175	(3) 107.6	(4) 106.4
(5) .35	(6) 32.225	(7) .167	(8) .640
(9) 19.25	(10) 542.390	(11) 3.375	(12) .2
(13) 3.33	(14) 7.069	(15) 23.676	

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(1) 2/6	(2) 9/24	(3) 16/36	(4) 8/10
(5) 8/9	(6) 11/12	(7) 2/3	(8) 3/4
(9) 1/3	(10) 25/72	(11) 3/4	(12) 43/67

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(1) 5 1/7	(2) 7 1/4	(3) 4 1/5	(4) 10
(5) 5 1/16	(6) 27 7/12	(7) 11/3	(8) 33/8
(9) 83/11	(10) 29/2	(11) 133/5	(12) 82/9

ANSWERS

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(1)	30	(2)	12	(3)	10
(4)	72	(5)	210	(6)	36
(7)	770				

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(1)	17/24	(2)	7/8	(3)	25/24 = 1 1/24
(4)	9/12 = 3/4	(5)	33/24 = 1 3/8	(6)	49/30 = 1 19/30
(7)	35 1/24	(8)	53 3/12 = 53 1/4	(9)	119 11/16
(10)	171 64/100 = 171 16/25				
(11)	120 91/135	(12)	99 413/792		

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(1)	7/20	(2)	5/8	(3)	8/80 = 1/10
(4)	11/54	(5)	13/209	(6)	29/210
(7)	6 1/6	(8)	5 5/12	(9)	32 9/40
(10)	210 5/8	(11)	22 5/20 = 22 1/4	(12)	400 45/56

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(1)	6 18/35	(2)	5/8	(3)	2 9/15 = 2 3/5
(4)	13/24				

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ANSWERS

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(1) 1/6	(2) 1/5	(3) 10/7 = 1 – 3/7
(4) 16/25	(5) 3 / 4	(6) 63/350 = 9/50
(7) 4	(8) 165/112 = 1 53/112	(9) 112/15 = 7 7/15
(10) 67/2 = 33 1/2	(11) 0	(12) 39/8 = 4 7/8
(13) 1/45	(14) 984/5 = 196-4/5	
Bonus (1) 1	**Bonus** (2) 7	

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(1) 15/8 = 1 7/8	(2) 3 / 4	(3) 9/4 = 2 1 / 4
(4) 7/4 = 1 3 / 4	(5) 1/9	(6) 32/3 = 10 2/3
(7) 3/8	(8) 36/7 = 5 1/7	(9) 35/2 = 17 1/2
(9) 35/2 = 17 1 / 2	(10) 1 / 4	(11) 81/112
(12) 16/19	(13) 2	(14) 3/5
(15) 15		(16) 40/21 = 1 19 /21

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(1) .19	(2) 35%	(3) .05	(4) 2%
(5) .005	(6) 5.8%	(7) 1.5	(8) 460%

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(1) 25%	(2) 830,000	(3) 9.8%
(4) 420,000	(5) .42%	(6) 49,000
(7) .01%	(8) 300	

ANSWERS

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PAGES 42 - 44

(1)	$P = IE$	(2)	$A = \frac{1}{2} bh$	(3)	$d = 1.414s$
(4)	$P = 2w + 2h$	(5)	$R_T = R_1 + R_2 + R_3$	(6)	$P = pA$
(7)	$p = 5.2I$	(8)	$Q = Av$	(9)	$E = IR$
(10)	$A = 3.14r^2$	(11)	$V = whl$	(12)	$P = 3.14d$
(13)	$PF = \frac{RT}{\mathbf{Z}}$	(14)	$x_c = \frac{159{,}000}{\mathbf{fc}}$	(15)	$c^2 = a^2 + b^2$
(16)	$p = s + s + s + s$	(17)	$A = s2$	(18)	$I = \frac{p}{5.2}$
(19)	$P = klov^2$	(20)	$p = \frac{klov^2}{A}$		

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(1)	$P = 2\ (2.5) + 2\ (14)$	(2)	$A = 2.5 \times 14$
(3)	$P = 75 + 166 + 152$	(4)	$P = 2\ (3.14)\ (16)$
(5)	$A = \frac{1}{2}\ (40)\ (60)$	(6)	$50 = 2w + 2\ (5)$
(7)	$50 = 20\ h$	(8)	$48 = 12 + 18 + c$
(9)	$36 = 2\ (3.14)\ \pi$	10)	$132 = \frac{1}{2}\ b\ (12)$
(11)	$20{,}000 = 48v$	(12)	$^{o}F = 9/5\ (22+40) - 40$
(13)	$12 = 5.2i$	(14)	$P = (.00000002)(36)(24)(2400)^2$
(15)	$10 = \frac{(.00000002)(320)(36)(1600)^2}{A}$		

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(1)	45 sq. ft.	(2)	96 sq. ft.	(3)	77 sq. ft.
(4)	56 sq. ft.	(5)	64.5 sq. ft.	(6)	19.63 sq. ft.
(7)	31.50 sq. ft.				

ANSWERS

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(1) 43.98 ft. (2) 50 ft. (3) 31.42 ft. (4) 27 ft.

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(1) 3000 cu. ft. (2) 39937.5 cu. ft. (3) 3269.53 cu. ft.
(4) 75.95 cu. ft.

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(1) E = 6 (2) R = 17 (3) P = 70 (4) R = 8
(5) I = 22 (6) E = 50 (7) R = 11.48 (8) X = 2.75

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(1) R = 5 (2) E = 30 (3) X = 1.5 (4) Y = 31.5
(5) A = 22.8 (6) B = 17

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(1) A = 10 (2) C = 20 (3) B = 14
(4) D = 6 (5) M = 10 (6) P = .08
(7) T = 5.3 (8) R = 1

ANSWERS

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