PUBLIC NOTICE

Federal Communications Commission
445 12th St., S.W.
Washington, D.C. 20554

News Media Information 202 / 418-0500
Internet: http://www.fcc.gov
TTY: 1-888-835-5322

DA 03-2522
July 29, 2003

AUCTION OF REGIONAL NARROWBAND PCS LICENSES SCHEDULED FOR SEPTEMBER 24, 2003

ROUND RESULTS PROCESS AND RESULTS REPLICATION

Report No. AUC-03-51-C (Auction No. 51)

I. GENERAL INFORMATION

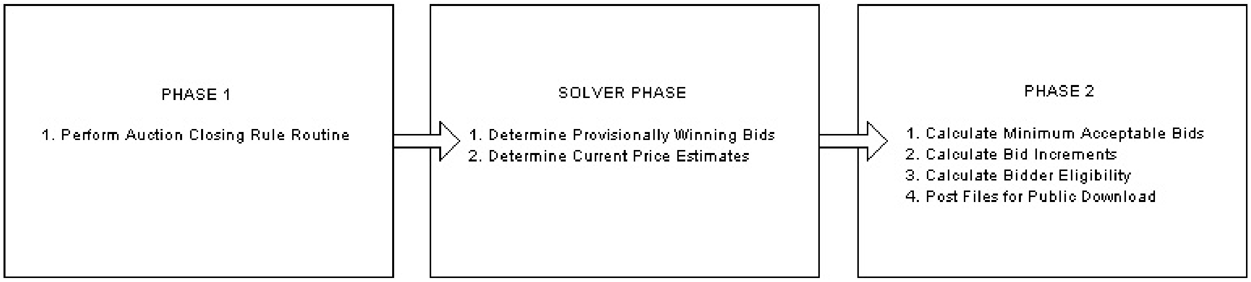
By this public notice, the Wireless Telecommunications Bureau ("Bureau") announces the round results process and results replication procedures for the upcoming auction of six regional narrowband Personal Communications Services ("narrowband PCS") licenses in the 900 MHz band scheduled for September 24, 2003 ("Auction No. 51"). In the *Auction No. 51Procedures Public Notice*, the Bureau noted that it would release a public notice providing details regarding round results formats and locations, to enable participants and interested observers to replicate the FCC's official round results. [1]

II. ROUND RESULTS PROCESS

The *round results* process is a sequence of administrative events that occurs after the close of a round. During this time, bid data collected during the round is processed, downloadable files are compiled, and various indicators of auction status are created. This information can be accessed by the public through the Results and the Bid Reports pages of the Auction 51 Bidding & Results System, which will be available through the Auction No. 51 website at http://wireless.fcc.gov/auctions/51/.

The following subsections describe the events that transpire during the round results process. The flow diagram below depicts this process and its major components.

[1] *See* Auction of Regional Narrowband PCS Licenses Scheduled for September 24, 2003; Notice and Filing Requirements, Minimum Opening Bids, Upfront Payments, Package Bidding and Other Auction Procedures, *Public Notice*, DA 03-1994 (rel. June 18, 2003), ("*Auction No. 51 Procedures Public Notice*").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1. Perform Auction Closing Rule Routine
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2. Calculate Bid Increments
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A. PHASE 1

The auction-closing rule dictates that after two consecutive rounds without new bids the auction will close. The auction software performs a test to see if this condition is met. If the test of the auction-closing rule fails, the auction remains open and the round results process continues. If the auction closes, the round results process does not continue and the provisionally winning bids from the prior round will be declared the winning bids of the auction.

B. SOLVER PHASE

1. Determine Provisionally Winning Bids

Optimization software is used to determine the provisionally winning bids. The determination of provisionally winning bids is a multi-step process that involves solving for the maximum revenue and then choosing among any tied bid sets.

2. Determine Current Price Estimates

Additional optimization procedures are used to obtain a current price estimate of each license in the auction. The current price estimates are, in turn, used to support the calculation of the minimum acceptable bid for licenses and packages in the following round.

C. PHASE 2

1. Calculate Minimum Acceptable Bids

For each bidder, a minimum acceptable bid amount is calculated for each eligible license and constructed package based on the following rule:

The minimum acceptable bid is the *greatest* of:

i. The applicable *minimum opening bid*.

ii. The bidder's own *previous high bid* on the license/package *plus x%*, where the Bureau will specify the value of x in each round.

iii. The *current price estimate* of the license or package *plus z%*, where the Bureau will specify the value of z in each round. The current price estimate for a package will be the sum of the current price estimates of the licenses that comprise the package.

After the first round of the auction, part (iii) of the minimum acceptable bid rule for a "global package" – a package consisting of all six of the licenses available in the auction – will always be the revenue generated by the provisionally winning bid set in the previous round *plus w%*, where the Bureau will specify the value of *w* in each round.

The resulting amount will be rounded using our standard rounding procedure.[2]

2. Calculate Bid Increments

For each minimum acceptable bid, multi-increment bid amounts are calculated. An increment is calculated as v% of the minimum acceptable bid amount, where the Bureau will specify the value of v in each round. The k^{th} increment bid amount is calculated as the minimum acceptable bid amount plus *k* times the increment, rounded where the value of *k* ranges from 1 to 8.

3. Calculate Bidder Eligibility

For each eligible bidder, *current eligibility* for the next round is calculated. Current eligibility is defined as the *lesser* of (i) the bidder's *eligibility activity*[3] in the round and (ii) the reciprocal of the *activity requirement* (1/activity requirement) multiplied by the bidder's eligibility activity in the round. If the bidder's eligibility activity does not meet its eligibility activity requirement and the bidder has a waiver available, the waiver will be automatically applied and the bidder's current eligibility for the next round remains unchanged. If a bidder has no remaining waivers and fails to meet its eligibility activity requirement, the bidder's current eligibility for the next round is calculated as described above – i.e., its eligibility is permanently reduced.

4. Post Files for Public Download

A number of files will be posted and made available for public download on the Results page of the Auction 51 Bidding & Results System and on the Results section of the Auction No. 51 website. These files contain information about all bids made in that round, the set of bids considered when determining the current provisionally winning bids, current price estimates, new minimum acceptable bids for all bidders, current provisionally winning bids, and bidder eligibility status (bidding eligibility and activity rule waivers). Section III describes the downloadable files.

III. ROUND RESULTS DOWNLOADS

Round results files will be available for downloading upon completion of the round results process after each round. The files will supply bidders with more complete auction data than is available in the summary tables and charts on the Results page. They are available in two formats: Microsoft® Access database (.mdb) and text (.txt). In general, the database formatted files provide current round information necessary to verify the provisionally winning bids, while the text files contain more complete information, including data from previous rounds of the auction. They are described in detail below.

A. Database Files

A Microsoft® Access database containing the data needed to verify the provisional winners for the round will be made available for download upon completion of the round results process. Additional tables provided within the database contain information to facilitate the correlation of bidder, package, and

[2] Results above $10,000 are rounded to the nearest $1,000; results below $10,000 but above $1,000 are rounded to the nearest $100; and results below $1,000 are rounded to the nearest $10.

[3] We define a bidder's *eligibility activity* in a round to be the greater of (i) its bidding activity in the round and (ii) the bidding units associated with the bidder's provisionally winning bids from the prior round. *See Auction No. 51 Procedures Public Notice* at 22.

license identification numbers to detailed information about these entities. Data is also provided to verify the selection number generated for each bid considered in the round.

The tables below detail the structure of the Microsoft® Access database and the corresponding data description for each field.

The CONSIDERED_BIDS table contains a single record for every bid that is considered in the current round.[4]

Table Name: **CONSIDERED_BIDS**		
Field Name	**Data Type**	**Data Element Description**
bidder_id	int	Bidder Identification Number
bid_id	int	Bid Identification Number
package_id	int	Package Identification Number
bid_amount	double	Bid Amount
scaled_bid_amount	double	Bid Amount with the decimal point shifted six digits to the left (see Attachments A and B)
bid_round_number	int	Bid Placement Round
selection_number	float	Selection Number

The CONSIDERED_BIDS_DETAIL table contains package construction information for the packages found in the CONSIDERED_BIDS table. This table contains a record for every license in a package. Therefore, a package constructed of five licenses is described by five records, each containing an identical package identification number (package_id) but a different license identification number (license_id).

Table Name: **CONSIDERED_BIDS_DETAIL**		
Field Name	**Data Type**	**Data Element Description**
package_id	int	Package Identification Number
license_id	int	License Identification Number

The BID_SEED table contains a record for every bid that is considered in the current round. Each record provides the six initial seed numbers used with the Random Number Generator (see Attachment C) for the bid described by the associated bid identification number (bid_id).

Table Name: **BID_SEED**		
Field Name	**Data Type**	**Data Element Description**
bid_id	int	Bid Identification Number
seed1	int	First Seed Number for Random Number Generator
seed2	int	Second Seed Number for Random Number Generator
seed3	int	Third Seed Number for Random Number Generator
seed4	int	Fourth Seed Number for Random Number Generator
seed5	int	Fifth Seed Number for Random Number Generator
seed6	int	Sixth Seed Number for Random Number Generator

[4] When determining provisionally winning bids, all bids made in every round throughout the course of the auction (except for bids that are placed and subsequently removed during the same round) will be considered. In addition, each license is treated as having a bid placed by the FCC at some small amount less than the minimum opening bid. *See Auction No. 51 Procedures Public Notice* at 28 and 29.

The BIDDER_ID_MAP table contains a record for every registered bidder for the auction. Each record associates the bidder identification number with a company name.

Table Name: **BIDDER_ID_MAP**		
Field Name	**Data Type**	**Data Element Description**
bidder_id	int	Bidder Identification Number
company_name	char(50)	Company

The LICENSE_ID_MAP table contains a single record for every license in the auction. Each record contains relevant license detail descriptions for the associated license identification number.

Table Name: **LICENSE_ID_MAP**		
Field Name	**Data Type**	**Data Element Description**
license_id	int	License Identification Number
description	char(30)	License Description
service_code	char(2)	License Service Code
market	char(6)	License Market
frequency_block	char(2)	License Frequency Block
sub_market	int	License Sub Market

The Microsoft® Access database is named "auction51Rnd#.mdb", where # indicates the round of the auction. To facilitate the download of a potentially large database, the file will be compressed within a similarly named Zip file, "auction51Rnd#.zip."

B. Text Files

A collection of tab-delimited text files will be made available for public download separately as well as a single zipped file. These files supplement the data made available in the downloadable .mdb file with more extensive and detailed auction data. Each text file will be named *auction#_rnd#xx.txt,* where *auction#* is the number of the auction (i.e., 51), *rnd#*, is the round number, and *xx* is an abbreviated file descriptor. Specific file names are included in the table definitions below.

1. All Bids File

This file includes all bids considered in the determination of the round's provisionally winning bids, which are, every bid by every bidder throughout the auction and a set of FCC bids on each license for a thousand dollars less than the applicable minimum opening bid. This file contains one row for each license of each package. Therefore, the file contains multiple rows and some duplicated data for packages with multiple licenses. Bids placed but removed within a round will not be included in this file. The table below contains information on the column header, data type, and data description for each column within the file.

File Name: 51_*rnd*#ab.txt (All Bids)/		
Column Header	**Data Type**	**Data Description**
round	int	Round in which this bid was placed
fcc_account_num	char(10)	Bidder-held FCC account number
company	char(50)	Company Name of the bidder
package_id	numeric(10)	Unique package identifier
license_desc	char(30)	Description of the license
service_code	char(2)	Identifies type of service (spectrum) offered
market	char(6)	Identifies the geographic region of the license
freq_block_num	char(2)	Identifies the block of frequency of the license
sub_market	int	Identifies a portion of a market being offered
bid_amt	money	Bid Amount of bid
bid_type	char(1)	Identifies the type of bid: B= new bid; L= renewed non-winning bid; W= renewed winning bid; A= last and best bid;

2. Current Round Bids File

This file includes only those bids placed in the current round. The Current Round Bids file contains one row for each license of each package.

File Name: 51_*rnd*#cb.txt (Current Round Bids)/		
Column Header	**Data Type**	**Data Description**
round	int	Round in which this bid was placed
fcc_account	char(10)	Bidder-held FCC account number
company	char(50)	Company Name of the bidder
package_id	numeric(10)	Unique package identifier
license_desc	char(30)	Description of the license
service_code	char(2)	Identifies type of service (spectrum) offered
market	char(6)	Identifies the geographic region of the license
freq_block_num	char(2)	Identifies the block of frequency of the license
sub_market	int	Identifies a portion of a market being offered
gross_bid_amt	money	Gross amount of the bid placed
net_bid_amount	money	Net amount of the bid placed
last_best	char(1)	Identifies a last and best bid
prov_winner	char(1)	Identifies a provisional winner
renewed_bid	char(1)	Identifies a renewed bid
selection_num	float	Selection Number

3. Minimum Acceptable Bids File

This file includes a row for all licenses applied for and every package that was bid on by each respective bidder. **Packages that have been constructed but not bid on in any round are not included in this report.** The table below contains information on the column header, data type, and data description for each column in the file.

File Name: 51_*rnd*#mb.txt		
Column Header	**Data Type**	**Data Description**
round_num	int	Identifies the round for which the minimum acceptable bid value applies
fcc_account_num	char(10)	Bidder-held FCC account number
company	char(50)	Company Name of the bidder
package_id	numeric(10)	Unique package identifier
min_acceptable_bid_amt	money	Minimum acceptable bid amount
gap_method	int	Rule method used to determine minimum acceptable bid

4. Auction Summary File

This file includes round summary statistics. Each row of the file contains the statistics for each round in the auction. The table below contains information on the column header, data type, and data description for each column in the file.

File Name: 51_*rnd*#as.txt		
Column Header	**Data Type**	**Data Description**
round	int	Round number of auction summary
num_elig_bidders	int	Number of bidders still eligible in auction
fcc_owned_licenses	int	Number of FCC owned licenses
new_bids	int	Number of new bids in the round
gross_rev	money	Gross Revenue for the round
net_rev	money	Net Revenue for the round
net_rev_pct	Float(8)	Percentage of Net Revenue increase in the round

5. Auction Parameters File

This file includes the parameters for the auction. Each row of the file contains the parameters for each round in the auction. The table below contains information on the column header, data type, and data description for each column in the file.

File Name: 51_*rnd*#ap.txt		
Column Header	**Data Type**	**Data Description**
round_num	int	Round number of auction summary
required_activity	float(8)	Fraction of current eligibility a bidder must be active on without risk of eligibility reduction or eligibility activity waiver usage.
min_bid_ pctg_x	float(8)	Fraction of previous high bid to be applied in rule (ii) of minimum acceptable bid calculations
min_bid_ pctg_w	float(8)	Fraction of Gross Revenue to be applied in rule (iii) of minimum acceptable bid calculations for global bids
min_bid_ pctg_z	float(8)	Fraction of current price estimate to be applied in rule (iii) of minimum acceptable bid calculations
min_bid_incr_pctg_v	float(8)	Fraction of minimum acceptable bid to be used as bid increment
weight_factor	float(8)	Exponential smoothing weight factor used when determining current price estimates

6. Bidder Summary File

This file includes summary information for bidders and contains one row per qualified bidder. The table below contains information on the column header, data type, and data description for each column in the file.

File Name: 51_*rnd*#bs.txt		
Column Header	**Data Type**	**Data Description**
current_round	int	Current round number of the auction
fcc_account_number	char(10)	Bidder-held FCC account number
company	char(50)	Company name of the bidder
current_dollar_exposure	money	Dollar exposure of bids placed in the round
overall_dollar_exposure	money	Dollar exposure considering all bids placed
initial_max_elig	money	Bidder’s initial eligibility
current_max_elig	money	Bidder’s current eligibility
eligibility_activity	int	Bidder’s eligibility activity for the round
required_activity	int	Bidder’s required activity level for the round
prov_winning_activity	int	Bidder’s activity from provisionally winning bids in the prior round
remaining_waivers	int	Current number of waivers that the bidder has available
waiver_ind	char(1)	Indicator of waiver being used in the round
last_best	char(1)	Indicator of last and best bids in the round
bidding_credit_pctg	float	Bidding credit percentage that the bidder may apply to its gross bid amount

7. Provisionally Winning Bids File

This file contains information about the provisionally winning bids for the round. It contains one row for each license of each package contained in the set of provisionally winning bids. The file is in the same format as the All Bids File. See Section III.B.1 for file specifications.

File Name: 51_*rnd*#pw.txt (Provisionally Winning Bids)		
Column Header	**Data Type**	**Data Description**
round_num	int	Round in which this bid was placed
fcc_account_num	char(10)	Bidder-held FCC account number
company	char(50)	Company Name of the bidder
package_id	numeric(10)	Unique package identifier
license_desc	char(30)	Description of the license
service_code	char(2)	Identifies type of service (spectrum) offered
market	char(6)	Identifies the geographic region of the license
freq_block_num	char(2)	Identifies the block of frequency of the license
sub_market	int	Identifies a portion of a market being offered
bid_amt	money	Bid Amount of bid
bid_type	char(1)	Identifies the type of bid: B= new bid; L= renewed non-winning bid; W= renewed winning bid; A= last and best bid;

8. Current Price Estimate File

This file contains one row for each license in the auction and displays the current price estimate for each license. Bidders can use this file to determine minimum acceptable bid amounts for packages created between rounds. The table below contains information on the column header, data type, and data description for each column in the file.

File Name: 51_*rnd*#pr.txt		
Column Header	**Data Type**	**Data Description**
current_round	int	Current round number of the auction
lic_id	int	Unique license identifier
license_desc	char(30)	Description of the license
service_code	char(2)	Identifies the service (spectrum) being offered
market	char(6)	Identifies the geographic region of the license
freq_block_num	char(2)	Identifies the frequency block of the license
sub_market	int	Identifies the portion of the market offered
estimated_price	money	Estimate of the license price

IV. REPRODUCING THE PROVISIONALLY WINNING BIDS

At the close of each round, the round results process executes software that determines the provisionally winning bids. This software employs the CPLEX® optimization suite developed by ILOG®, Inc. to determine the maximum revenue and to choose among tied bid sets. Bidders will be provided with the necessary instructions, problem models, and bid data to facilitate the reproduction and verification of these results. Instructions, problem models, and sample bid data will be posted on the Auction No. 51 website by August 31, 2003. Below is a summary of the optimization problems that must be solved and a description of a modeling language tool, OPL Studio™, that can be used to verify the provisionally winning bids for each round. The necessary bid data will be provided each round in a Microsoft® Access database format, as described in Section III.A.

A. Summary of How Provisionally Winning Bids Are Determined

Provisionally winning bids are the set of consistent bids that maximize total revenue in a particular round. Consistent bids are those that (i) do not contain overlapping licenses and (ii) are made or renewed by an individual bidder in the same round. Attachment A contains a mathematical formulation of the optimization problem used to determine the maximum revenue.

Since there can be more than one set of consistent bids that produces the maximum revenue, a procedure for randomly selecting among these tied sets is used when determining the provisionally winning bids. This tie breaking procedure involves two steps, the assignment of a *selection number* to each considered bid[5] and the determination of the set that produces the *maximum sum of selection numbers* among all tied bid sets.

A bid's *selection number* is the sum of *n* pseudo-random numbers where *n* is the number of licenses comprising the package being bid. Section IV.B describes the process of assigning selection numbers and explains how a bidder can reproduce selection numbers.

Once the selection numbers have been generated for each considered bid, the optimization software is used to determine the set that produces the maximum sum of selection numbers among any tied bid sets. This set becomes the set of provisionally winning bids. Attachment B contains a mathematical formulation of the optimization problem used to determine this set. Thus, the provisionally winning bids are the set of consistent bids that maximize revenue and maximize the sum of selection numbers.

B. Assigning and Reproducing Selection Numbers

As part of the round results process, each considered bid is assigned a new selection number.[6] A bid's selection number is the sum of *n* pseudo-random numbers where *n* is the number of licenses comprising the package. Each pseudo-random number is generated by invoking a well-documented and well-tested routine developed by Pierre L'Ecuyer. This routine uses two multiple recursive random number generators of order three with period length 2^{219}. The description of the algorithms can be found in L'Ecuyer, P. (1999) "Good Parameters and Implementations for Combined Multiple Recursive Random Number Generators" *Operations Research* 47 (1) pp. 159-164. Attachment C provides the C code used for the generation of these random numbers. This code requires six integer seed numbers as input and produces a double precision number between zero and one.

The CPLEX® optimization software is capable of finding solutions to a limited number of digits of accuracy; therefore, we perform the following calculations to obtain a selection number for each bid. For a bid on a package consisting of *n* licenses, we sum *n* double-precision pseudo-random numbers. We then round this number to the nearest millionth, i.e., six digits of accuracy to the right of the decimal point.

[5] When determining provisionally winning bids, all bids made in every round throughout the course of the auction (except for bids that are placed and subsequently removed during the same round) will be considered. In addition, each license is treated as having a bid placed by the FCC at $1000 less than the minimum opening bid. *See Auction No. 51 Procedures Public Notice* at 28 and 29.

[6] The order by which considered bids are assigned selection numbers is determined by ascending bid_id.

A bid's selection number as well as the beginning seed numbers used to produce it will be released upon completion of round results process for each round within the Microsoft® Access data file as described in Section III.A. This data along with the random number generator routine can be used to reproduce the selection number associated with each considered bid.

C. Using OPL Studio™ to Reproduce Results

Formulations of the maximum revenue problem and the maximum sum of selection numbers problem, written in an algebraic modeling language, will be made available to bidders. Algebraic modeling languages provide a mechanism for taking the algebraic description of the problem and feeding this information along with the data to any of the standard optimization software packages. Such modeling languages have the advantage of keeping the problem in a concise representation common to modelers while the software automatically generates the individual constraints of the problem. Thus, these languages keep the model in a simpler, more easily readable form while the modeling software manages communication with an appropriate solver. Once optimal solutions have been found, they are expressed in terms of the modeler's notation so that they can be viewed and analyzed.

These models, stored as Revenue.mod and SelectionSum.mod, will be available for download from the Auction No. 51 website. They are written in the Optimization Programming Language (OPL) and used in the application OPL Studio™.[7] Most other algebraic modeling languages have similar syntax so that a translation from OPL to any other commercial modeling language is relatively straightforward.[8] Since the generic model is separate from any data instances of a given auction, the same two model formulations can be used at the end of any round of the auction to verify the results reported by the Bureau.

Along with the ".mod" files is an OPL Script file, stored as RoundSolver.osc, which contains a meta-script for executing the two OPL models and outputting information about the results. The OPL Script file first executes the revenue model. Once the maximum revenue has been determined, the OPL Script dynamically fixes the revenue constraint in the selection sum model. This constraint ensures that bids in the solution set produce the maximum revenue. The selection sum model is then executed and information about the bids that compose the winning set is written to the terminal output.

Once round results have been posted, bidders will be able to download the Microsoft® Access database containing the necessary information about the considered bids for that round. With this data the OPL Script can be executed in order to replicate the results obtained by the Bureau. Instructions for setting-up the OPL Studio™ environment and reproducing the provisionally winning bids each round are available with the OPL files found on the Auction No. 51 website.

There is a rare chance that more than one set of bids can satisfy the conditions of provisionally winning bids (i.e., there is more than one set of bids that produces the maximum revenue and the maximum selection sum). If such a case exists, the round results process will generate all such sets and assign each a random number. The set with the largest associated random number will be the set of provisionally winning bids. If this unlikely event occurs, the set of bids that are found when the OPL Script has been

[7] OPL Studio™, developed by ILOG, Inc. (www.ilog.com), is an integrated development environment for combinatorial optimization applications. OPL Studio™ can be purchased at a reduced price for use during the auction by contacting Mark Steidel (775) 881-2813 msteidel@ilog.com.

[8] A description of a number of algebraic modeling languages can be found on the web at http://www-fp.mcs.anl.gov/otc/Guide/SoftwareGuide/Categories/optsysmodlang.html.

executed may not be the provisionally winning bids for the round but will be a set of bids that produces both the maximum revenue and the maximum selection sum.

V. POINT OF CONTACT

For questions regarding round results, download files, and reproducing the provisionally winning bids contact Karen Wrege at (202) 418-0660.

VI. ATTACHMENT A

Formulation of the Maximum Revenue Problem

$$\alpha = \max \quad \sum_{i \in B} b_i \quad x_i$$

$$subject \quad to$$

$$\sum_{i \in B} a_{il}\, x_i \le 1, \quad for\, all\, l \in L \qquad (1)$$

$$\sum_{i \in R_{jk}} x_i - \left|R_{jk}\right| y_{jk} \le 0, \quad for\, all\, k \in AR_j, \;\; j \in BR \qquad (2)$$

$$\sum_{k \in AR_j} y_{jk} \le 1, \quad for\, all\, j \in BR \qquad (3)$$

$$x_i, \;\; y_{jk} \;\; \in \{0,1\}, \quad for\, all\, i \in B, \;\; k \in AR_j, \;\; j \in BR$$

where B is the set of considered bids (we use i as the index related to bids*)*,
b_i is the bid amount of bid i,
L is the set of licenses (we use l as the index related to licenses),
$a_{il} = \begin{Bmatrix} 1 & if\ bid\ i\ contains\ license\ l \\ 0 & otherwise \end{Bmatrix},$
BR is the set of bidders in the current round,
(we use j as the index related to bidders),
AR_j is the set of active rounds of bidder j (we use k as the index related to rounds),
R_{jk} is the set of bids in active round k of bidder j, and $|R_{jk}|$ is the cardinality of the set
$x_i = \begin{Bmatrix} 1 & if\ bid\ i\ is\ in\ the\ winning\ set \\ 0 & otherwise \end{Bmatrix},$
$y_{jk} = \begin{Bmatrix} 1 & if\ bids\ from\ bidder\ j\ in\ active\ round\ k\ are\ used \\ 0 & otherwise \end{Bmatrix}$

Note that an FCC bid on each license is included in the set of considered bids. Therefore, the first set of constraints can be formulated as equality.

Constraints (1) assure that each license is awarded exactly once. The variables y_{jk} are indicator variables that, when equal to one, allow bids of bidder j from round k to be part of an optimal solution. Constraints (3) allow at most one such y_{jk} for each bidder to equal one and therefore at most one round of a bidder's bids can be part of an optimal solution. When y_{jk} is equal to zero, constraints (2) assure that all bids in the round for bidder j in round k (x_i $i \in R_{jk}$) equal zero. Similarly, when y_{jk} equals one, constraints (2) allow all bids in that round to be considered for inclusion in the optimal set.[9]

[9] We acknowledge also that there is an alternative formulation for constraints (2) of the form $x_j \le y_{ij}$ for all i,j that would expand the number of constraints but make the linear program solution value tighter. We have left the formulation as is since optimization packages often add these tightening constraints as needed.

The objective function finds a set of bids that maximize the sum of the bid amounts while simultaneously satisfying the constraints. Thus, the result α is the maximum revenue obtainable for this round.

Note that in order to avoid numerical difficulties in the selection sum problem, bid amounts are "scaled" by shifting the decimal place six units to the left.[10] Thus, the value of α will be scaled similarly. Bidders will be provided with both the scaled and unscaled bid amounts in the Microsoft® Access database. For details on why scaling is important in the selection sum problem, see Attachment B.

[10] No accuracy is lost in altering the data in this manner since the shift results in no loss of significant digits.

VII. ATTACHMENT B

Formulation of the Maximum Selection Sum Problem

$$\beta = \max \quad \sum_{i \in B} s_i x_i$$

$$\textit{subject} \quad \textit{to}$$

$$\sum_{i \in B} a_{il} x_i \leq 1, \quad \textit{for all } l \in L \qquad (1)$$

$$\sum_{i \in R_{jk}} x_i - \left| R_{jk} \right| y_{jk} \leq 0, \quad \textit{for all } k \in AR_j, \; j \in BR \qquad (2)$$

$$\sum_{k \in AR_j} y_{jk} \leq 1, \quad \textit{for all } j \in BR \qquad (3)$$

$$\sum_{i \in B} b_i x_i = \alpha \qquad (4)$$

$$x_i, \; y_{jk} \; \in \{0,1\}, \quad \forall i \in B, \; k \in AR_j, \; j \in BR$$

where:

B is the set of considered bids (we use *i* as the index related to bids*)*,
s_i is the selection number of bid *i*,
L is the set of licenses (we use *l* as the index related to licenses),

$$a_{il} = \begin{Bmatrix} 1 & \textit{if bid i contains license l} \\ 0 & \textit{otherwise} \end{Bmatrix},$$

BR is the set of bidders in the current round,
(we use *j* as the index related to bidders),
AR_j is the set of active rounds of bidder *j* (we use *k* as the index related to rounds),
R_{jk} is the set of bids in active round *k* of bidder *j*, and $|R_{jk}|$ is the cardinality of the set
b_i is the bid amount of bid *i*,
α is the maximum revenue (see Attachment A),

$$x_i = \begin{Bmatrix} 1 & \textit{if bid i is in the winning set} \\ 0 & \textit{otherwise} \end{Bmatrix}$$

$$y_{jk} = \begin{Bmatrix} 1 & \textit{if bids from bidder j in active round k are used} \\ 0 & \textit{otherwise} \end{Bmatrix}$$

Constraints (1)-(3) are identical to the constraints in the maximum revenue problem. Constraints (4) assure that the optimal set will yield the maximum revenue. The objective function maximizes the sum of the selection numbers.

As presented above, the selection sum problem has both very large and very small numbers in the constraint matrix, which can result in numerical difficulties. In order to avoid this, constraint (4) uses scaled bid amounts and the associated scaled maximum revenue value, α, as discussed in Attachment A. Bid amounts are scaled by shifting the decimal place six units to the left. No accuracy is lost in altering the data in this manner since the shift results in no loss of significant digits. Bidders will be provided with both the scaled and unscaled bid amounts in the Microsoft® Access database.

VIII. ATTACHMENT C

C Code for the Random Number Generator

```
#define norm  1.0842021724855052e-19
 #define m1    9223372036854769163
 #define m2    9223372036854754679
 #define a12   1754669720
 #define q12   5256471877
 #define r12   251304723
 #define a13n  3182104042
 #define q13   2898513661
 #define r13   394451401
 #define a21   31387477935
 #define q21   293855150
 #define r21   143639429
 #define a23n  6199136374
 #define q23   1487847900
 #define r23   985240079

 long long s10, s11, s12, s20, s21, s22;

 double MRG63k3a ()
     {
    long long h, p12, p13, p21, p23;
    /* Component 1 */
    h = s10 / q13;   p13 = a13n * (s10 - h * q13) - h * r13;
    h = s11 / q12;   p12 = a12  * (s11 - h * q12) - h * r12;
    if (p13 < 0) p13 += m1;
    if (p12 < 0) p12 += m1 - p13;  else p12 -= p13;
    if (p12 < 0) p12 += m1;
    s10 = s11;  s11 = s12;  s12 = p12;
    /* Component 2 */
    h = s20 / q23;   p23 = a23n * (s20 - h * q23) - h * r23;
    h = s22 / q21;   p21 = a21  * (s22 - h * q21) - h * r21;
    if (p23 < 0) p23 += m2;
    if (p21 < 0) p21 += m2 - p23;  else p21 -= p23;
    if (p21 < 0) p21 += m2;
    s20 = s21;  s21 = s22;  s22 = p21;
    /* Combination */
    if (p12 > p21) return ((p12 - p21) * norm);
    else return ((p12 - p21 + m1) * norm);
    }
```