THE PERSISTENCE OF LEARNING AND ACQUISITION STRATEGIES

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The acquisition strategies implied by two theories of learning—learning curve theory and knowledge depreciation theory—are quite different. This article reexamines empirical data for land-based weapon systems to determine if knowledge depreciation theory can be confirmed. Results fail to confirm knowledge depreciation theory and support learning curve theory. The author concludes that acquisition managers should continue to use learning curve theory to model their acquisition strategies.

The twin analytic foundations for acquisition strategy are the theory of markets (from modern microeconomics) and the learning curve (from industrial engineering). Argot, Beckman, and Epple (1990) argue that learning does not persist in industrial settings. They have developed a theory of knowledge depreciation that calls into question learning curve theory. In this theory learning is an attempt to keep up with changing circumstances.

To explore the differences in and implications of these theories, and as an empirical test of these theories, we reexamine data from two land-based weapon systems, the M-113 and the M-60, and we present new data on the Abrams Tank. The data provide little support for knowledge depreciation theory and confirm the usefulness of learning curve theory. We conclude that acquisition managers should continue to use analysis based on learning curve theory when evaluating alternative acquisition strategies.

THE CONTEXT OF ACQUISITION STRATEGY ANALYSIS

The twin foundations of modern acquisition strategy analysis are the micro-economic theory of markets and the learning curve. The premise of existing law and regulation is that competitive markets produce goods for the government at the lowest prices and best quality available. Where competition exists, the government routinely expects to avail itself of its benefits. The opposite extreme

in a market is monopoly, where one firm maximizes profits by exploiting the marginal revenue derived from demand. The Congress and the military departments have addressed the problem of monopoly in military acquisition through the Truth in Negotiations Act, the Cost Accounting Standards Act, and implementing regulations.

The economic theory of the firm was developed extensively during the earlier part of this century. The monopolistic case was expanded to include duopoly, where two firms control supply, and oligopoly, where multiple firms control supply. These cases were also expanded to include nonprice competition. These analyses are now considered standard treatments and are covered in most micro-economics textbooks. (An excellent graduate level text that requires only college algebra is Ferguson and Gould [1975].) Where clear analogies to the classical cases exist, little analysis is required. For new or emerging commercial markets, proper market research is usually sufficient.

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Economists also have attempted to study the case where one buyer and one seller constituted the market. This case became known as bilateral monopoly and was studied as early as 1931. As Ferguson and Gould note, the analysis based on classical models is indeterminate. Fouraker and Siegel (1963) studied bilateral monopoly from the point of view of bargaining. This analysis provides significant insight into the behavior of participants in poorly formed markets. The outcome is for participants to divide a total profit pool based on their relative market power. While powerful, this analysis provides no new tools that might help a buyer bring additional suppliers into a market.

Acquisition strategy analysis was developed to fill this gap in economic theory. For a variety of military weapon systems there are only a small number of potential producers. Some problems are rooted in the historical practice of production in government-owned facilities or with government-owned production and research property. Some are characterized by low production rates that discourage capital investments. Some companies possess carefully guarded trade secrets that are used to produce weapon systems. Other barriers to competition exist. Historically, the military departments have used competition strategy analysis in those cases where acquisition managers had some reason to believe that competition could be introduced for a weapon system that either was currently being produced or was expected to be produced on a sole source basis. The analysis usually examines the particulars of a given case to determine what barriers exist to competition and to estimate the expected returns from competition.

The very act of inducing a competition may be costly. Where government-owned production equipment is used by a sole source, there may be costs associated with changing plant management from one firm to another, duplicating equipment in a second source plant, paying for investigations and studies, paying competitors to develop proposals, and leader-follower arrangements or educational buys. Up-front investment for a program can be substantial, reaching into the tens of millions of

dollars. Potential competitors are likely to invest their own money in the campaign, and that spending can mitigate the government's investment.

The output of a competitive strategy analysis is an assessment of alternatives to induce competitors to bid on weapon system production. It usually takes the form of a comparison of alternatives for competition with the sole source case, and may include an assessment of one or more dual sourcing strategies. The results of the analysis may be presented as point estimates of expected savings, as sophisticated probability assessments, or in hybrid formats.

CURRENT PRACTICE

The model used to estimate the effects of competition on weapon systems or component prices has been developing for more than 30 years. Washington (1997) provides a good historical overview. Typically, the analysis begins with an existing sole source production program. The analysis assumes that costs will be a function of quantity produced according to learning curve theory. Effects of competition are modeled as either shifts or rotations in the learning curve. Unique contracting arrangements such as multiyear contracting with economic order quantity funding may be modeled as well, usually as a shift in the curve. The models typically assume that once a program reverts to a sole source status, the remaining incumbent will exploit its position and cost will revert to near-precompetitive levels. Analysis may proceed either by analyzing the functions and generating discrete values for competitive savings, or by

probabilistic modeling, generating ranges of outcomes and sensitivity points. Watkins (1982) summarized existing knowledge of commodity learning curves and their known probability distributions.

Traditional economic analysis does not recognize learning curves. Rather, quantity produced is said to be a function of inputs, usually labor and capital. Various forms of the production function have been investigated, typically providing for diminishing returns to inputs consistent with the marginal productivity theory of microeconomics. Perhaps the best known is the Cobb-Douglas production function $Q = L^\alpha K^\beta$, where quantity produced is an exponential function of labor and capital. The modern innovation, explored by Argot et al., is the addition of a knowledge input to the equation.

The learning curve arises from industrial engineering observations. As summarized in various places, most notably in Asher (1956), observations of plant experience in a variety of industries, and particularly in the airframe industry, led to the conclusion that

> **"Traditional economic analysis does not recognize learning curves."**

labor hours and material costs decline with each doubling of quantity produced. Costs are log-linear functions of the form, $c = Ax^b$, where labor hours or costs (c) are an exponential function of quantity produced (x) and first unit cost (A). This type of analysis usually assumes a given state of tooling and capital equipment. (Note the similarity to the Cobb-Douglas production function.) Analysts modify functions for changes in tooling, capital equipment, and configuration.

The classical theory assumes that learning arises from repeated physical motion and the application of both physical and conceptual learning to physical processes. The theory posits that learning persists through time and is a function of quantity produced.

The learning curve leads us to several conclusions about competitive acquisition strategy:

- Incumbents with experience have a competitive advantage. Lack of experience producing a product or service can be a significant barrier to market entry.

- Educational quantities can boost the competitive position of secondary or tertiary sources.

- Lost learning may override the effects of competition and make it uneconomical to sustain multiple sources.

- Changes in production disrupt learning, which is recovered with additional quantity produced.

KNOWLEDGE DEPRECIATION

In contrast, Argot et al. argue that knowledge depreciates rapidly. Testing the Liberty Ship data from World War II, the authors concluded that learning depreciated at a rate of 25 percent per month (97 percent per year) and that there was little, if any, transfer of learning between shipyards. What little transfer did occur came from improvements in design. Unit labor hours were approximately constant after design introduction. (The Liberty Ship data is a classic public database used extensively in learning curve studies.)

The knowledge depreciation hypothesis reflects the more recent conception of learning as innovation. In this view the business environment is ever changing and requires firms to change (adapt), change in turn demands innovation, learning produces that innovation, and innovation makes existing knowledge obsolete. The processes used to weld steel are no longer applicable when new grades of steel and new welding methods come into use.

The method of analysis used is similar to learning curve analysis. The basic equation is log linear in labor hours, capital inputs, and knowledge, with dummy variables used to capture unique inputs. Knowledge in turn is modeled as a linear combination of current period quantity and last period quantity. A constant parameter applied to prior period quantity is used to capture depreciation of knowledge from the prior period.

The implications are quite different for competitive acquisition strategy:

- Incumbents gain no competitive advantage from experience.

- Competition can be effectively introduced at any time, provided there are no other barriers.

- Dual sourcing can be sustained indefinitely.

- Changes in production must be accompanied by new learning if labor efficiency is to be maintained.

COMPARISON OF RECOMMENDED ACQUISITION STRATEGIES

As we can see from the previous discussion, the contrast between the strategies produced by these two theories is marked.

INCUMBENT ADVANTAGE

Learning curve theory predicts that incumbents will have a competitive advantage from experience. The longer an incumbent has produced a product or service and the steeper the learning curve, the larger the incumbent's advantage. New entrants must produce substantial improvements in first unit cost *and* equal or steeper learning to be competitive. The knowledge depreciation theory predicts that incumbents will have no competitive advantage. New entrants will be on equal footing with incumbents to the extent that they can produce innovations.

Stated another way, learning curve theory posits that knowledge (particularly tacit knowledge) is a prime input to the production process while knowledge depreciation theory says that, at best, knowledge is like public infrastructure, a necessary precondition to production.

TIMING OF A COMPETITION

Learning curve theory leads one to introduce competition as early as possible in a production program to mitigate any potential incumbent advantage. Similarly, educational buys or other pilot production contracts for second sources reduce incum-bent advantages in two ways: first, by increasing the quantity produced by competitors, thereby increasing opportunities for learning and improvements, and

second, by reducing the quantity available to the incumbent, limiting such opportunities.

Knowledge depreciation theory predicts that competition will be successful at any time, provided competitors are available. Educational buys and other pilot programs are of little value.

DUAL SOURCING

Learning curve theory predicts that there is an opportunity cost associated with dual or multiple sourcing. Provided the incumbent has sufficient capacity, any quantity given to competing sources reduces the quantity available to the incumbent and corresponding-

"Learning curve theory predicts that there is an opportunity cost associated with dual or multiple sourcing."

ly limits the opportunities for cost reductions through learning. If lost learning is significant, the corresponding opportunity cost can exceed the gains from competition.

Knowledge depreciation theory predicts that there are no opportunity costs associated with lost-incumbent production quantities, and therefore any gains from competition are pure gains.

CHANGES IN PRODUCTION

The weakness of learning curve theory is that it predicts continued improvements even after major changes in production. The Boeing "S" curve was an early attempt to estimate the impact of production changes on learning (Asher, 1956). Yet, for all attempts to incorporate

additional techniques, learning curves do not predict the types of problems cited by Argot on the Lockheed L-1011 or more recently, on the Boeing 757 (Boeing, 1999). In these cases increasing production led to *increases* in both unit and total costs. Profitability disappeared where it should have increased.

The theory of knowledge depreciation predicts not only that new knowledge must accompany changes in production, but that the production of new knowledge is itself likely to disrupt the process of renewing existing knowledge. In this regard the two theories stand in stark contrast, learning curve theory holding that learning is a continuous function of quantity produced, and knowledge depreciation theory holding that learning is essentially memoryless, or Markovian.

> **"Real organizational life is more complicated than either theory."**

Real organizational life is more complicated than either theory. Learning curve theory was developed at a time when standard industrial engineering techniques could be applied to simplified, labor-intensive processes. Once laid out, the assembly line could be balanced and maintained at peak efficiency using standard analytic tools. Today, more often than not, production lines include many more operations than just assembly, are made up of mostly automated processes, incorporate active control programs, may include artificial intelligence, and are more complicated to manage. Learning curve theory rests on a view of labor that is inherently self-contained. Management's role is compliance and little more.

Knowledge depreciation theory is similarly one-dimensional, resting on the assumption that all that is needed is continuous innovation to renew knowledge of production processes. In fact, modern production systems are more complex both in their management and in the interactions among component processes. While such complexity makes prediction more difficult, it need not render it random. In this regard, knowledge depreciation theory captures part of the problem of implementing changes in production in a complex system, while learning curve theory does not.

In sum, learning curve theory calls for a careful analysis of the advantages and disadvantages of introducing competition for limited production quantities and, when the decision is made to compete, to do so at the earliest possible moment. Lost-learning opportunities may drive a decision for a competitive down-select to a single source. Knowledge depreciation theory predicts competition can be successful at any time and need never be discontinued. Most important, learning curve theory predicts competition will be difficult to induce when an incumbent has significant experience, while knowledge depreciation theory predicts competitors will always be available provided that there are no capital or other market barriers.

RESULTS FROM M-113 AND M-60 DATA

Watkins (1982) presented data from several land-based weapon systems, both sole source and competitive. For this study I selected two programs with extensive data, the M-113 armored personnel carrier and M-60 main battle tank. The M-113 is an

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The M-113 (Armored Personnel Carrier)

ideal exemplar for learning curve theory. The competitive history yields a 91-percent learning curve that persists even after the market reverts to sole source procurements. The M-60 tank ought to be a fine exemplar for knowledge depreciation theory. The M-60 had one competition followed by an entire life cycle of sole source contracts. It exhibits essentially no learning (99.9-percent learning curve). Both began production in 1959–1960, ramped up production for the Vietnam War, ramped down production after the war, and went through successive model changes.

For each program, I followed the methodology described in the technical appendix. Model changes or significant engineering change proposals were treated as

shifts in the learning curve. In the case of the M-60, competition is modeled by a one-time shift. No attempt is made to control for capital inputs as these are unknown. We know that the M-113 was produced in two plants by FMC from 1966 to 1971. We know that the M-60 was produced in the Lenape, DE, Tank Plant in 1959 and then in the newly renovated Detroit Arsenal Tank Plant from 1960 onward, the shift coinciding with the one and only competition for M-60 production. Using Argot's model, multiple regression will capture changes in capital in the dummy variable regression. In fact, the change in capital does not show up in the M-113 data and is inseparable from competition effects in the M-60 data. In

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both cases, capital inputs do not influence either the slope of the learning curve or the knowledge depreciation parameter and were not pursued further.

These two systems provide an essential test of the knowledge depreciation theory. Knowledge depreciation theory predicts rising prices with falling lot size. Learning curve theory predicts continued improvements with quantity produced. Both systems experienced falling production quantities in the 1970s, with the end of the Vietnam War.

The key results are as follows:

- There was no support for the prediction that knowledge decays. For the M-113 the best fit was with the knowledge depreciation factor set to "one," that is, full retention. For the M-60 the best fit was with the factor set to "zero" (no retention). Both results are consistent with the learning curve theory.

- Model changes accounted for the majority of the price variance. The knowledge depreciation equation accounted for less than 5 percent of the price variance.

- Price changes were predicted by learning curve theory, but not by the knowledge depreciation theory. (See "Forecast Accuracy" below.)

ABRAMS MAIN BATTLE TANK EXPERIENCE

I also examined the price history on the Abrams Main Battle Tank (M-1 Series). Initial production costs trends are difficult to evaluate because of component breakout prior to 1983. The Army stabilized the work share for the prime contractor in 1983. The Army bought 8,038 M-1 or M-1A1 tanks with the last unit produced in fiscal year 1991. The Abrams series has been sole source from initial production. Beginning in 1981 the Army applied should-cost techniques to reduce the cost of the prime contractor's content.

One might expect the M-1 to be very similar to the M-60. The M-1 was produced at two plants, along side the M-60 at the Detroit Arsenal and by itself in the Lima Army Tank Plant. General Dynamics bought Chrysler Defense in 1982 and operated both plants for the Army, producing both the M-1 and the M-60. The M-1 fabrication processes made significant use of robotic welding, which in theory flattens learning curves. In fact, the M-1 and the M-60 were quite different.

Key results were:

- A learning curve with a 91.4-percent slope fit the data adjusted for model changes.

- As with the competitive M-113 data, there is no knowledge decay. The line of best fit was when the knowledge depreciation factor is "one"—that is, full retention.

- Price changes were predicted well by the learning curve while the knowledge depreciation theory had only modest success, as discussed immediately below.

FORECAST ACCURACY

Using the equations derived from the initial regression analysis, one can make predictions of prices for the ensuing production lots. Price changes can be

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The M-1Abrams Tank

computed directly from these predictions. The forecast price changes are then compared with the actual price changes using regression analysis to measure the variance explained by each equation.

This element of the analysis measures a key difference between the two theories. Learning curve theory predicts prices will continue to decline with quantity produced even when lot sizes shrink. Knowledge depreciation theory predicts prices will increase with shrinking lot size and decrease when lot size increases, regardless of cumulative quantity produced. All three weapon systems had periods of significant change in lot sizes. The results

confirmed learning curve theory and failed to confirm knowledge depreciation theory (Table 1).

A NOTE ABOUT LABOR EXPERIENCE

We have also gleaned extensive data and knowledge from detailed observation of operations in Army plants. While much of this data is proprietary, some of the conclusions we have drawn can be presented in this public forum.

First, if we define process knowledge as the "why" of processes, then it is our observation that knowledge usually enhances physical learning. Operator

Program	Learning Curve	Knowledge Depreciation Equation
M-113	.6107	.2477
M-60	.6084	−.0324
M-1	.8450	.3480

Table 1.
Correlation of Predicted Price Changes with Actual Price Changes

experience with welding, machining, grinding, or assembly operations benefits from conceptual knowledge of operations, visual models, plans of operations, and a variety of experiences. The classic example is minor engineering changes. Changing the location of a hole, its size, and even the material may have little impact on the time it takes an experienced operator to drill a hole, given machine factors. Similar types of changes make no difference in assembly operations with experienced assemblers. By contrast, minor changes of any kind are likely to disrupt the pace and output of inexperienced workers. To the extent that learning is taking place, variety actually helps workers anticipate changes and adapt to them without loss of productivity.

Second, changes in processes can be graded by their effects on learning. While such a system relies to a large degree on engineering judgment, observation has born out the general principle that changes can be grouped by the degree of impact the changes have on learning.

Third, in forecasting labor hours, the assumption that learning is always retained, even under severe production disruption, has been effective for us. We have modeled changes in labor hours due to disruption by assuming the labor hour *changes* decay exponentially with the passage of time. This model has consistently predicted aggregate labor hours for a disruptive event with moderate accuracy and predicted the point of disappearance of effects to within one production lot.

These observations are in sharp contrast to the knowledge depreciation theory. That theory would predict that engineering changes, strikes, material shortages, or similar disruptions would result in lost learning with no recovery. Our experience is otherwise.

EXTENDING RESULTS TO OTHER INDUSTRIES

A journal referee noted that only two industries were represented in these studies and asked whether the results extended to other industries. I appreciate the referee raising this important question. We can say several things about that issue.

First, Argot et al. did not directly test the predictive value of their equations. I selected automotive commodities because they display the least learning of all

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products studied for competitive acquisition strategies (Watkins, 1982). We can expect the effects of knowledge depreciation theory to be most prominent in this industry, especially with those programs like the M-60 that display no learning. Automotive commodities should give us our best test of the predictive power of the knowledge depreciation theory. Given that other industries display greater learning, one can logically expect that learning curve theory will be more effective at predicting costs and prices than knowledge depreciation theory in such industries.

Second, learning curve theory is a robust estimating tool across a variety of industries. During the past two decades, competitive acquisition strategies using learning curve theory have been successfully developed for aircraft, munitions, electronics, and combat vehicles. Considering the current lack of support for knowledge depreciation theory as an estimating tool and the robustness of learning curve theory, it is likely that learning curve theory will retain an advantage in predictive power across industries.

Finally, the Liberty Ship program may itself be unique. Thompson recently analyzed new data from the National Archives on the Liberty Ship program. He found that certain proxies for capital investment that Argot used were in fact inadequate. The new data showed that major capital spending on capacity improvements at the shipyards continued through the first two years of production. Thompson attributes 50 percent of the productivity gains in the Liberty Ship program to capital improvements, 44 percent to learning, 5 percent to relaxed quality standards, and the remaining percent being error (Thompson, in press). Argot's results may be due to

the rapidly changing nature of the yards' capital equipment and production processes. If so, then knowledge depreciation theory may only extend to those situations where process change is a significant factor. The cases examined in this paper all used relatively stable production technologies.

It is plausible that learning curve theory is a better estimating tool for plants and products with relatively stable production processes and that knowledge depreciation theory is a better estimating tool where production processes are changing rapidly. It is also plausible that learning curve theory has an advantage as an estimating tool regardless of the industry. However plausible, these are hypotheses that remain to be tested.

A NOTE ABOUT TECHNOLOGICAL LEAPS

Common sense tells us that when radical shifts in technology occur, little of our accumulated experience will be useful. Firms that have extensive accumulated experience can be vulnerable to competitors who lower their first unit production costs with new technology. Knowledge depreciation theory seems to fit well with this scenario.

On the other hand, firms with adaptive managers often learn quickly how to incorporate new technologies into their production processes, which in turn allows these firms to build on their accumulated experience and gain even greater competitive advantages than new rivals. Accumulated managerial and production line experience, or their combination, may contribute significantly to the introduction of new technologies.

The treatment of model changes in this paper suggests that technological leaps could be modeled using learning curves. The evidence suggests that once a technological leap is made, further improvements will follow a learning curve. One can then model a technological leap as either a new curve or as a shift in an existing curve, possibly with a curve rotation (change in slope).

Where technological leaps are anticipated, acquisition managers would do well to consider using both knowledge depreciation theory and learning curve theory to assess acquisition alternatives.

CONCLUSION

These results suggest that the most effective means we have for modeling the potential impact of competition on acquisition strategies is the analysis of market power combined with the learning curve. It is useful to note that in the cases examined here the majority of the variance is captured by product model changes and by changes in market position. While learning was important, it was not as significant as either model changes or market power. Therefore, it is important to view learning curve theory as a supplement to market analysis—not the other way around. Each alternative acquisition strategy needs to be examined in light of the particular details of the specific market as well as the experience of market participants.

Knowledge depreciation theory failed to improve on current models of cost behavior under varying competitive conditions for land-based systems. It merely confirmed the results from established analysis. However, as knowledge depreciation theory is plausible under some scenarios, such as rapidly changing production processes or leaps in technology, decision makers need to consider the possibility that accumulated experience may have less impact where these conditions are present.

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TECHNICAL APPENDIX

SOURCES OF DATA

The data for the M-113 and M-60 are previously published data taken from Watkins (1982). This data was also published in Williams (1982). The contract data are vehicle quantities and final unit prices. Final unit prices include all price adjustments and engineering changes. Unit prices were then adjusted to 1980 dollars using a composite inflation index consisting of weighted values for industrial commodities and average hourly earnings. Weights were assigned for each system

that reflected the approximate material and labor content.

M-1 Abrams data are presented below. Unit prices are initial unit prices except for fiscal year 1979 and fiscal year 1980 where they are final prices. Unit prices are then adjusted to constant 1997 dollars. The key difference from the M-113 and M-60 data is that all Abrams prices from fiscal year 1981 forward are firm-fixed prices. The only missing component is negotiated engineering changes incorporated after contract award. However, the

Table 2. Abrams Price History

Fiscal Year	Model	Qty.	Price ($)	DoD Infl.	Price (1997 $)
1979	M1	110	1,475,731	0.4161	3,546,578
1980	M1	352	1,238,035	0.4641	2,667,604
1981	M1	569	926,423	0.5180	1,788,461
1982	M1	657	739,845	0.5921	1,249,527
1983	M1	686	749,676	0.6450	1,162,288
1983	IPM1	114	775,814	0.6450	1,202,812
1984	IPM1	780	753,740	0.6906	1,091,428
1984	M1A1	60	956,211	0.6906	1,384,609
1985	M1A1	840	933,966	0.7140	1,308,076
1986	M1A1	840	972,513	0.7340	1,324,950
1987	M1A1	720	1,016,036	0.7538	1,347,885
1988	M1A1	720	1,054,514	0.7756	1,359,611
1989	M1A1	720	1,080,202	0.8091	1,335,066
1990	M1A1	299	1,113,964	0.8423	1,322,526
1990	M1A1	393	1,155,121	0.8423	1,371,389
1991	M1A1	178	1,216,360	0.8785	1,384,587

impact of these engineering changes is not significant on the analysis presented here.

Engineering changes incorporated after award on one contract are included in the new unit prices for the follow-on year. For example, changes added to fiscal year 1981 would be included in the initial unit price for fiscal year 1982. The overall effect is to slightly reduce the slope of the learning curve. The effect on regression coefficients is probably negligible, as the engineering change content was consistently around 2 percent per year for this period of production. Inflation indexes are weighted in the same manner as for the M-60 and M-113.

LEARNING CURVE MODEL

To estimate the learning curve I modeled the data using the equation in Figure 3.

Model changes are often explicit in the original data. Where they are missing, model changes can be estimated by assigning the jump in prices at model change over to the model change. All model changes are treated as shifts in the learning curve that persist until a further change is encountered.

KNOWLEDGE DEPRECIATION MODEL

To estimate knowledge depreciation I used the equation in Figure 4 derived from Argot (1993).

The equation used for this article is:

$$\text{Ln } H_t = a_0 + \alpha \text{Ln } q_t + \gamma \text{Ln } K_{t-1} + M + u_t$$

where M is the model change matrix.

The key difference here is the lack of term for capital inputs. As explained in the text, there is no discernable impact from changes in capital and it was excluded from the analysis. The form of the equation is derived by algebraic substitution and translation of parameters.

The knowledge depreciation factor is an indicator of how quickly current knowledge becomes obsolete. A value of "one"

$\ln y = \ln A + b*\ln x + M_t + e$

where

      y is the unit price in constant dollars,

      x is the cumulative quantity produced,

      b is the learning coefficient b = log(curve slope)/log 2,

      M_t is the unique model change value at time t, and

      e is an error term.

Figure 3. Equation to Estimate Learning Curve

$$\text{Ln } \mathbf{q}_t = a_0 + \Sigma \mathbf{aD} + \alpha \text{Ln } \mathbf{H}_t + \beta \text{Ln } \mathbf{W}_t + \gamma \text{Ln } \mathbf{K}_{t-1} + \delta \mathbf{Z}_t' + \mathbf{u}_t$$

where

$\mathbf{q}_t$ is quantity of output,

$\mathbf{aD}$ are dummy variables with weights a, a_0 being initial,

$\mathbf{H}_t$ are total labor hours worked,

$\mathbf{W}_t$ is a surrogate for capital inputs,

$\mathbf{K}_t = \lambda \mathbf{K}_{t-1} + \mathbf{q}_t$, the knowledge equation,

λ is the knowledge depreciation parameter, defined on the interval [0,1],

$\mathbf{Z}_t$ is a vector of other influences (e.g., turnover),

$\mathbf{u}_t$ is the error vector, and

$\mathbf{t}$ is time period.

Figure 4. Argot's Equation (in vector notation)

indicates knowledge is fully retained and used. It is consistent with the learning curve hypothesis. A value of "zero" indicates no retention—that is, knowledge is fully renewed each period. An intermediate value indicates some knowledge becomes obsolete during each period and the factor provides an estimate of the percentage that becomes obsolete. Because the factor actually measures changes in output for a given labor input, it is an indirect measure of knowledge retention.