



# AN INDEX TO MEASURE A SYSTEM'S PERFORMANCE RISK

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Technical Performance Measures (TPMs) are traditionally defined and evaluated to assess how well a system is achieving its performance requirements. Typically, dozens of TPMs are defined for a system. Although they generate useful information and data about a system's performance, little is available in the program management community on how to integrate these measures into a meaningful measure of the system's overall performance risk. This paper presents how individual TPMs may be combined to measure and monitor the overall performance risk of a system. The approach consists of integrating individual technical performance measures in a way that produces an overall risk index. The computed index shows the degree of performance risk presently in the system. It identifies risk-driving TPMs, enables monitoring time-history trends, and reveals where management should target strategies to lessen or eliminate the performance risks of the system.

**A**s a system evolves through its acquisition and deployment phases, management defines and derives measures that indicate how well the system is achieving its performance requirements. These measures are known as Technical Performance Measures (TPMs) (Blanchard & Fabrycky, 1990; Department of Defense [DoD], 2002). Measures such as *Weight*, *Mean-Time-Between-Failure*, and *Detection Accuracy* are among the types of TPMs often defined on programs. Technical performance measurements can be taken from a variety of sources. This includes data from system testing, system simulations, and experimentation. Depending on the

source basis for these data, and the development phase of the system, performance data may be derived from a mix of actual or forecasted values.

The program management community has little in the way of methodology for quantifying performance risk as a function of a system's individual technical performance measures. The approach presented herein consists of computing a risk index derived from these individual performance measurements. The index shows the degree of performance risk presently in the system, supports identifying risk-driving TPMs, and can reveal where management should focus on improving technical performance and, thereby,

lessen risk. When the index is continuously updated, management can monitor the time-history trend of its value. This enables management to assess the effectiveness of risk reduction actions being targeted or achieved over time.

In general, TPMs are measures that, when evaluated over time, must either decrease to meet a system's performance requirements or increase to meet performance requirements. Thus, each TPM can be assigned to one of two categories. For this paper, define *Category A* as the collection of TPMs whose values must decrease to achieve a system's threshold performance requirements. Define *Category B* as the collection of TPMs whose values must increase to achieve a system's threshold performance requirements. This is illustrated in Figure 1.

In Figure 1, the horizontal axis represents measurement date. This is the date when the actual or forecasted value of the TPM was taken or derived. The vertical axis represents the value of the TPM at the corresponding measurement date.

In Figure 1,  $V_{thres}$  denotes the threshold performance value for the TPM. This is the minimum acceptable value for the TPM. It marks the boundary between the regions of acceptable versus unacceptable performance risk.

It is assumed that TPMs defined for a system are done judiciously; that is, only those TPMs truly needed to properly measure a system's overall technical performance are defined, measured, and monitored. Given this, *acceptable performance risk* can be defined as the condition when all TPMs reach, or extend beyond, their individual threshold performance values. Conversely, *unacceptable performance risk* can be defined as the condition when one or more TPMs have not reached their individual threshold performance values.

### A PERFORMANCE RISK INDEX MEASURE

The following presents an index designed to measure the performance risk of a system. The index provides a numerical indicator of how well a

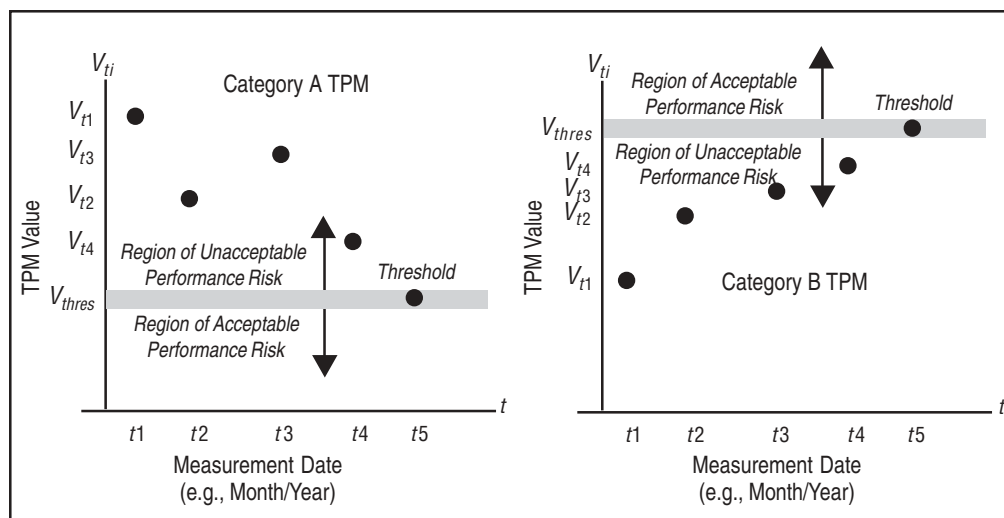
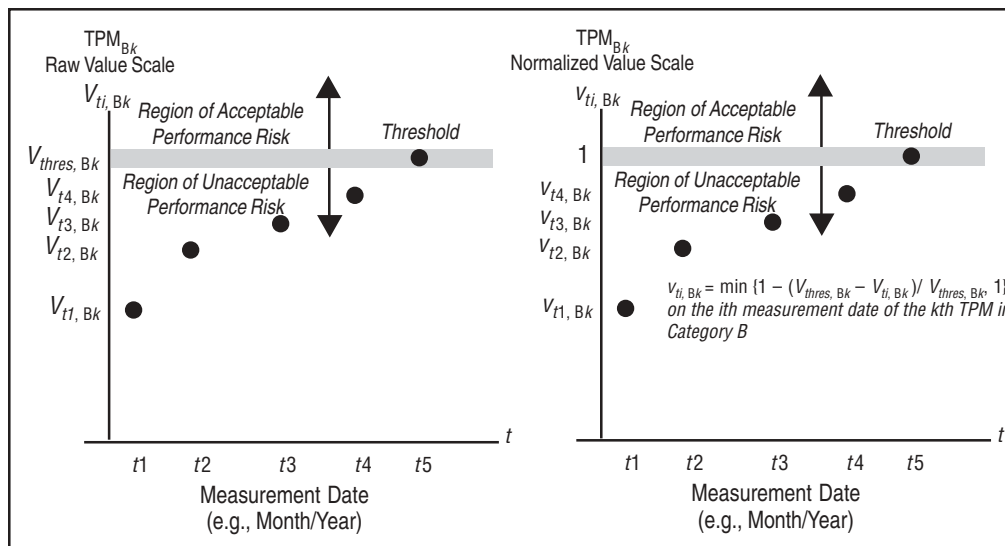
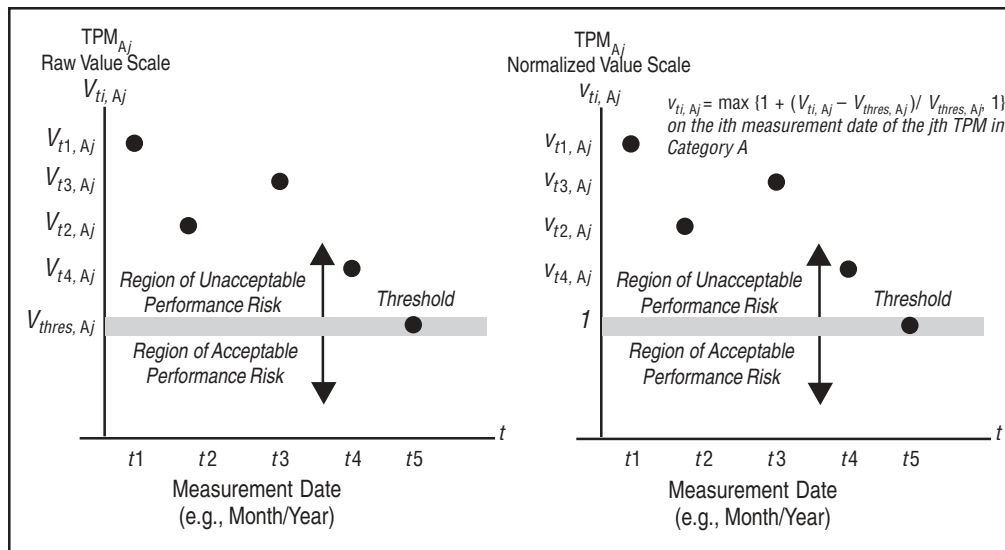


Figure 1. Category A and Category B Technical Performance Measures

**An Index to Measure a System's Performance Risk**

developing system is progressing toward its threshold performance requirements. It serves as a yardstick that enables management to measure the “distance” the system is from its minimum performance thresholds and to monitor trends over time.

To develop the risk index, it is necessary to normalize the TPM “raw” values into a common and dimensionless scale. Figures 2 and 3 show such scales for Category A and Category B TPMs. In these figures, the left-most vertical scales reflect TPM raw values (their native



units) taken from engineering measurements, tests, experiments, or prototypes. The right-most vertical scales reflect TPM normalized values. Here, threshold values are all normalized to one. This scale transformation is done for each TPM in each category. This allows management to compare the progress of each performance measure in a common and dimensionless scale. From these normalized scales, an overall measure of the extent to which the performance of the system meets its threshold requirements can then be determined. Next are formulas to derive this measure. This is followed by a computation example to illustrate the application context.

As mentioned previously, let Category A be the set of TPMs that need to be reduced to their threshold values. In Figure 2, let  $V_{ti, Aj}$  be the value at time  $ti$  for the  $j$ th TPM in Category A and  $V_{thres, Aj}$  be the threshold value to which the  $j$ th TPM is driven. Define  $v_{ti, Aj}$  to be a normalized TPM value against its threshold as follows (assuming both  $V_{ti, Aj}$  and  $V_{thres, Aj}$  are greater than 0):

$$\begin{aligned}
 v_{ti, Aj} &= \max\{V_{ti, Aj}, V_{thres, Aj}\} / V_{thres, Aj} \\
 &\text{(i.e., threshold met if } V_{ti, Aj} \leq V_{thres, Aj}\text{)} \\
 &= \max\{V_{ti, Aj} / V_{thres, Aj}, 1\} \\
 &= \max\{(V_{thres, Aj} - V_{thres, Aj} + V_{ti, Aj}) / \\
 &\quad V_{thres, Aj}, 1\} \\
 &= \max\{1 + (V_{ti, Aj} - V_{thres, Aj}) / \\
 &\quad V_{thres, Aj}, 1\} \quad (\geq 1) \quad \text{Eq 1}
 \end{aligned}$$

Equation 1 is the formula for  $v_{ti, Aj}$  in Figure 2, which brings out the overage above 1. Similarly, let Category B be the set of TPMs that need to be increased to their threshold values. In Figure 3, let  $V_{ti, Bk}$  be

the value at time  $ti$  for the  $k$ th TPM in Category B and  $V_{thres, Bk}$  be the threshold value to which the  $k$ th TPM is driven. Define  $v_{ti, Bk}$  to be a normalized TPM value against its threshold as follows (assuming both  $V_{ti, Bk}$  and  $V_{thres, Bk}$  are greater than 0):

$$\begin{aligned}
 v_{ti, Bk} &= \min\{V_{ti, Bk}, V_{thres, Bk}\} / V_{thres, Bk} \\
 &\text{(i.e., threshold met if } V_{ti, Bk} \geq V_{thres, Bk}\text{)} \\
 &= \min\{V_{ti, Bk} / V_{thres, Bk}, 1\} \\
 &= \min\{(V_{thres, Bk} - V_{thres, Bk} + V_{ti, Bk}) / \\
 &\quad V_{thres, Bk}, 1\} \\
 &= \min\{1 - (V_{thres, Bk} - V_{ti, Bk}) / \\
 &\quad V_{thres, Bk}, 1\} \quad (\leq 1) \quad \text{Eq 2}
 \end{aligned}$$

Equation 2 is the formula for  $v_{ti, Bk}$  in Figure 3, which brings out the underage below 1. From the normalized values, we now calculate their average difference from 1 for each category and use it as the category's *TPM Risk Index (TRI)*. Assume  $j = 1, 2, \dots, m$  for Category A ( $m$  elements) and  $k = 1, 2, \dots, n$  for Category B ( $n$  elements), then

$$\begin{aligned}
 TRI_{ti, A} &= [(v_{ti, A1} - 1) + (v_{ti, A2} - 1) + \dots \\
 &\quad + (v_{ti, Am} - 1)] / m \\
 &= [(v_{ti, A1} + v_{ti, A2} + \dots + v_{ti, Am}) / \\
 &\quad m] - 1 \quad \text{Eq 3}
 \end{aligned}$$

$$\begin{aligned}
 TRI_{ti, B} &= [(1 - v_{ti, B1}) + (1 - v_{ti, B2}) + \dots \\
 &\quad + (1 - v_{ti, Bn})] / n \\
 &= 1 - [(v_{ti, B1} + v_{ti, B2} + \dots + \\
 &\quad v_{ti, Bn}) / n] \quad \text{Eq 4}
 \end{aligned}$$

These two indices show the average overage or underage for TPMs in Category A or Category B when their

individual threshold values are re-scaled to 1. To combine all normalized values into an overall risk index, we first convert the TPMs in Category A into equivalent ones in Category B. This is because the normalized values for Category A can differ in orders of magnitude from those for Category B (e.g., 1000 vs. 0.5). An overall index, based on the normalized values as calculated, will be unduly influenced by large values. The result, though correct, can be difficult to interpret.

To make such a conversion, observe that for the  $j$ th TPM in Category A with value  $V_{ti, Aj}$  and threshold  $V_{thres, Aj}$ , an equivalent TPM in Category B can be constructed with value  $U_{ti, Aj} = 1/V_{ti, Aj}$  and threshold  $U_{thres, Aj} = 1/V_{thres, Aj}$ . Typically, the reciprocal of a TPM is just as practical. For example, a failure rate or a processing delay that is to be reduced can be taken in its reciprocal respectively as a mean time between failure or a completion rate that is to be increased.

The probability of a certain undesirable event (e.g., misclassification or an error exceeding the tolerance) or unavailability of a certain desirable state (e.g., system working or parts in hand) are more subtle. But their reciprocals can be viewed as the expected number of events that will contain one such undesirable event or the expected length of time that will contain one unit time of such a desirable state being unavailable. Although their complements (as opposed to reciprocals) can also be used as Category B TPMs, it is not recommended as the complements are usually close to 1 and their further improvements toward 1 do not show much difference when normalized.

The normalized value for a Category A TPM converted into a Category B TPM is, by definition

$$\begin{aligned}
 u_{ti, Aj} &= \min\{U_{ti, Aj}, U_{thres, Aj}\} / U_{thres, Aj} \\
 &= \min\{1/V_{ti, Aj}, 1/V_{thres, Aj}\} / (1/V_{thres, Aj}) \\
 &= [1 / \max\{V_{ti, Aj}, V_{thres, Aj}\}] / (1/V_{thres, Aj}) \\
 &= 1 / [\max\{V_{ti, Aj}, V_{thres, Aj}\} / V_{thres, Aj}] \\
 &= 1 / v_{ti, Aj} (\leq 1) \qquad \text{Eq 5}
 \end{aligned}$$

We can now treat all TPMs as being in Category B and then derive an overall risk index.

$$\text{Let } TRI_{ti, A}^* = 1 - [(u_{ti, A1} + u_{ti, A2} + \dots + u_{ti, Am}) / m] \qquad \text{Eq 6}$$

$$TRI_{ti, B} = 1 - [(v_{ti, B1} + v_{ti, B2} + \dots + v_{ti, Bn}) / n] \text{ as before} \qquad \text{Eq 7}$$

$$\begin{aligned}
 \text{then } TRI_{ti, All} &= 1 - [(u_{ti, A1} + u_{ti, A2} + \dots + u_{ti, Am} + v_{ti, B1} + v_{ti, B2} + \dots + v_{ti, Bn}) / (m + n)] \\
 &= 1 - [(m(1 - TRI_{ti, A}^*) + n(1 - TRI_{ti, B})) / (m + n)] \\
 &= [m(TRI_{ti, A}^*) + n(TRI_{ti, B})] / (m + n) \qquad \text{Eq 8}
 \end{aligned}$$

where  $TRI_{ti, All}$  is the overall *TPM Risk Index* for the system, computed across all of the system's TPMs. Finally, a non-negative weight  $w_{Aj}$  could be assigned

to  $(1 - u_{i, A_j})$  for the  $j$ th TPM in Category A and  $w_{Bk}$  to  $(1 - v_{i, Bk})$  for the  $k$ th TPM in Category B (as opposed to all having an equal weight, as assumed in the discussion above). In that case, it can also be shown that

$$TRI_{i, A}^* = 1 - [(w_{A1}u_{i, A1} + w_{A2}u_{i, A2} + \dots + w_{Am}u_{i, Am}) / W_A] \quad \text{Eq 9}$$

where  $W_A = w_{A1} + w_{A2} + \dots + w_{Am}$

$$TRI_{i, B} = 1 - [(w_{B1}v_{i, B1} + w_{B2}v_{i, B2} + \dots + w_{Bn}v_{i, Bn}) / W_B] \quad \text{Eq 10}$$

where  $W_B = w_{B1} + w_{B2} + \dots + w_{Bn}$

$$\text{and } TRI_{i, All} = [W_A TRI_{i, A}^* + W_B TRI_{i, B}] / W \quad \text{Eq 11}$$

where  $W = W_A + W_B$

Thus, equation 11 is the most general form of the system's overall *TPM Risk Index*.

From the above, note that  $TRI_{i, A}^*$ ,  $TRI_{i, B}$ , and  $TRI_{i, All}$ , equally or unequally weighted, are all bounded by 0 and 1. A value of 0 for the risk indices means *there are no unacceptable risks* in the included TPMs, each achieving (or extending beyond) its threshold value. The risk indices can be asymptotically near 1 and that implies that each TPM value in Category A is very large when compared to its threshold and/or that each TPM value in Category B is very small when compared to its threshold, i.e., all far away from their thresholds. When the TPMs are moving toward their thresholds, the risk indices are moving toward 0.

## COMPUTATION EXAMPLE AND TIME HISTORY GRAPH

Suppose Table 1 represents a system's set of Category A and Category B TPMs, along with their hypothetical threshold and raw values for six measurement dates. From these data, what is the system's overall technical performance risk index? How is it changing over time?

From the data in Table 1 and equations 9, 10, and 11, we can derive, for each measurement date, the TPM risk indices for the Category A and Category B TPMs, as well as for the system's overall TPM Risk Index. The results from these derivations are summarized in Table 2.

Note that TRI is a cardinal measure. This means its value is a measure of the "strength" or "distance" that the contributing TPMs are from their individual threshold performance values. A TRI equal to 0.5 is truly twice as "bad" as one equal to 0.25.

Figure 4 presents a time history trend of the TPM risk indices for the data in Tables 1 and 2. Here, the trend is good. All three TRIs are heading toward 0. This means all TPMs defined for the system are converging toward their individual threshold performance values. In practice, management should regularly produce a graphic summary such as this to monitor the extent that each risk index changes over time.

## SUMMARY

This paper provides an approach and formalism for developing an overall set of quantitative indices that measure a system's performance risk, as a function of its TPMs. Below are the general forms of the three principal risk indices.

**An Index to Measure a System's Performance Risk**

**Table 1. A Hypothetical Category A and Category B TPM Data Set**

CATEGORY A TPM					
	Vthres, A	Raw Value V(ti, A)	Eq 1 v(ti, A)	Eq 5 u(ti, A)	wt
<b>Measurement Date t1</b>					
Average Processing Delay (msecs)	1.000	3.000	3.000	0.333	1.000
Mean Time to Repair (mins)	10.000	50.000	5.000	0.200	1.000
Payload Weight (lbs)	950.000	2112.000	2.223	0.450	1.000
Time for Engagement Coordination (sec)	0.010	0.100	10.000	0.100	1.000
<b>TRI*(t1, A)</b>	<b>0.729</b>	<b>Eq 9</b>			
<b>Measurement Date t2</b>					
Average Processing Delay (msecs)	1.000	2.860	2.860	0.350	1.000
Mean Time to Repair (mins)	10.000	43.000	4.300	0.233	1.000
Payload Weight (lbs)	950.000	1764.000	1.857	0.539	1.000
Time for Engagement Coordination (sec)	0.010	0.040	4.000	0.250	1.000
<b>TRI*(t2, A)</b>	<b>0.657</b>	<b>Eq 9</b>			
<b>Measurement Date t3</b>					
Average Processing Delay (msecs)	1.000	1.180	1.180	0.847	1.000
Mean Time to Repair (mins)	10.000	43.000	4.300	0.233	1.000
Payload Weight (lbs)	950.000	1328.000	1.398	0.715	1.000
Time for Engagement Coordination (sec)	0.010	0.032	3.200	0.313	1.000
<b>TRI*(t3, A)</b>	<b>0.473</b>	<b>Eq 9</b>			
<b>Measurement Date t4</b>					
Average Processing Delay (msecs)	1.000	1.090	1.090	0.917	1.000
Mean Time to Repair (mins)	10.000	27.000	2.700	0.370	1.000
Payload Weight (lbs)	950.000	1189.000	1.252	0.799	1.000
Time for Engagement Coordination (sec)	0.010	0.020	2.000	0.500	1.000
<b>TRI*(t4, A)</b>	<b>0.353</b>	<b>Eq 9</b>			
<b>Measurement Date t5</b>					
Average Processing Delay (msecs)	1.000	1.030	1.030	0.971	1.000
Mean Time to Repair (mins)	10.000	12.000	1.200	0.833	1.000
Payload Weight (lbs)	950.000	1008.000	1.061	0.942	1.000
Time for Engagement Coordination (sec)	0.010	0.010	1.000	1.000	1.000
<b>TRI*(t5, A)</b>	<b>0.063</b>	<b>Eq 9</b>			
<b>Measurement Date t6</b>					
Average Processing Delay (msecs)	1.000	0.980	1.000	1.000	1.000
Mean Time to Repair (mins)	10.000	9.000	1.000	1.000	1.000
Payload Weight (lbs)	950.000	948.000	1.000	1.000	1.000
Time for Engagement Coordination (sec)	0.010	0.010	1.000	1.000	
<b>TRI*(t6, A)</b>	<b>0</b>	<b>Eq 9</b>			

*(continued on page 196)*

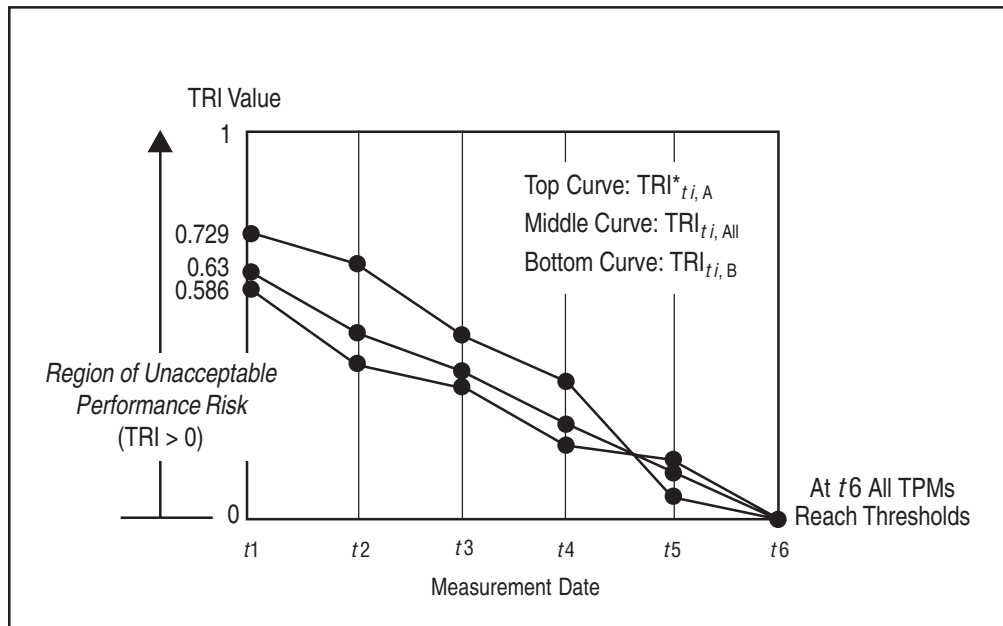


**Table 1. A Hypothetical Category A and Category B TPM Data Set (continued)**

CATEGORY B TPM				
	Vthres, B	Raw Value V(ti, B)	Eq 2 v(ti, B)	wt
<b>Measurement Date t1</b>				
Interceptors Available (no. of units)	150.000	67.000	0.447	1.000
Mean Time Between Failure (hours)	500.000	100.000	0.200	1.000
Single Shot Success Probability (%)	0.950	0.870	0.916	1.000
Damage Assessment Accuracy (%)	0.995	0.600	0.603	1.000
Software Coding (no. of modules coded)	763.000	578.000	0.758	1.000
<b>TRI(t1, B)</b>	<b>0.586</b>	<b>Eq 10</b>		
<b>Measurement Date t2</b>				
Interceptors Available (no. of units)	150.000	128.000	0.853	1.000
Mean Time Between Failure (hours)	500.000	189.000	0.378	5.000
Single Shot Success Probability (%)	0.950	0.890	0.937	1.000
Damage Assessment Accuracy (%)	0.995	0.878	0.882	1.000
Software Coding (no. of modules coded)	763.000	643.000	0.843	1.000
<b>TRI(t2, B)</b>	<b>0.399</b>	<b>Eq 10</b>		
<b>Measurement Date t3</b>				
Interceptors Available (no. of units)	150.000	134.000	0.893	1.000
Mean Time Between Failure (hours)	500.000	223.000	0.446	5.000
Single Shot Success Probability (%)	0.950	0.910	0.958	1.000
Damage Assessment Accuracy (%)	0.995	0.940	0.945	1.000
Software Coding (no. of modules coded)	763.000	687.000	0.900	1.000
<b>TRI(t3, B)</b>	<b>0.342</b>	<b>Eq 10</b>		
<b>Measurement Date t4</b>				
Interceptors Available (no. of units)	150.000	139.000	0.927	1.000
Mean Time Between Failure (hours)	500.000	348.000	0.696	5.000
Single Shot Success Probability (%)	0.950	0.934	0.983	1.000
Damage Assessment Accuracy (%)	0.995	0.945	0.950	1.000
Software Coding (no. of modules coded)	763.000	698.000	0.915	1.000
<b>TRI(t4, B)</b>	<b>0.194</b>	<b>Eq 10</b>		
<b>Measurement Date t5</b>				
Interceptors Available (no. of units)	150.000	142.000	0.947	1.000
Mean Time Between Failure (hours)	500.000	379.000	0.758	5.000
Single Shot Success Probability (%)	0.950	0.940	0.989	1.000
Damage Assessment Accuracy (%)	0.995	0.999	1.000	1.000
Software Coding (no. of modules coded)	763.000	723.000	0.948	1.000
<b>TRI(t5, B)</b>	<b>0.147</b>	<b>Eq 10</b>		
<b>Measurement Date t6</b>				
Interceptors Available (no. of units)	150.000	159.000	1.000	1.000
Mean Time Between Failure (hours)	500.000	521.000	1.000	5.000
Single Shot Success Probability (%)	0.950	0.990	1.000	1.000
Damage Assessment Accuracy (%)	0.995	1.000	1.000	1.000
Software Coding (no. of modules coded)	763.000	763.000	1.000	1.000
<b>TRI(t6, B)</b>	<b>0</b>	<b>Eq 10</b>		

**Table 2.**  
**Technical Performance Measure (TPM) Risk Index Summaries**

Measurement Date	TPM Risk Index for Category A TPMs $TRI_{t_i,A}^*$ Eq 9	TPM Risk Index for Category B TPMs $TRI_{t_i,B}$ Eq 10	Overall TPM Risk Index for the System $TRI_{t_i,All}$ Eq 11
t1	0.729	0.586	0.63
t2	0.657	0.399	0.478
t3	0.473	0.342	0.382
t4	0.353	0.194	0.243
t5	0.063	0.147	0.121
t6	0	0	0



**Figure 4. Illustrative Technical Performance Measure (TPM) Risk Index Time History Trend**

**Category A:**

$$TRI_{i,A}^* = 1 - [(w_{A1}u_{i,A1} + w_{A2}u_{i,A2} + \dots + w_{Am}u_{i,Am}) / W_A]$$

where  $W_A = w_{A1} + w_{A2} + \dots + w_{Am}$

**Category B:**

$$TRI_{i,B} = 1 - [(w_{B1}v_{i,B1} + w_{B2}v_{i,B2} + \dots + w_{Bn}v_{i,Bn}) / W_B]$$

where  $W_B = w_{B1} + w_{B2} + \dots + w_{Bn}$

**Overall Risk Index:**

$$TRI_{i,All} = [W_A TRI_{i,A}^* + W_B TRI_{i,B}] / W$$

where  $W = W_A + W_B$

To conclude, key features of the approach presented in the paper are summarized as follows:

- **Provides Integrated Measures of Technical Performance:** This approach provides management with a way to transform the typically dozen or more TPMs into common measurement scales. From this, all TPMs may then be integrated and combined in a way that provides management with meaningful and comparative

measures of the overall performance risk of the system, at any measurement time  $t$ .

- **Measures Technical Performance as a Function of the Physical Parameters of the TPMs:** This approach operates on actual or predicted values from engineering measurements, tests, experiments, or prototypes. As such, the physical parameters that characterize the TPMs provide the basis for deriving the TPM risk indices.
- **Measures the Degree of Risk and Monitors Change over Time:** The computed TPM risk indices show the degree of performance risk that presently exists in the system, supports the identification and ranking of risk-driving TPMs, and can reveal where management should focus on improving technical performance and, thereby, lessen risk. If the indices are continuously updated, then management can monitor the time-history trends of their values to assess the effectiveness of risk reduction actions being targeted or achieved over time.

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