

**The problem:** A certain ground motion has an  $x$  percent probability of being exceeded in  $Y$  years. What is the probability,  $w$ , that that same ground motion is exceeded in  $Z$  years?

**The solution:** In the algorithm used to create the maps, one specifies a probability of exceedance for a given number of years. The computer finds the ground motion that has an annual rate that satisfies the following equation:

$$1 - r(a) = F(a) = e^{-T\varphi(a)}$$

where  $r(a)$  is the exceedance probability of the ground motion  $a$ ,  $F(a)$  is the corresponding probability of non-exceedance,  $e$  is the base of the natural logarithm scale,  $T$  is the number of years for which we want to know the corresponding probability, and  $\varphi(a)$  is the annual rate of exceedance of ground motion  $a$ . This equation is nothing more than the Poisson probability that given an expected number  $n$  of events,  $e^{-n}$  is the probability of getting none.

Given a ground motion,  $a$ , the annual rate at a site  $\varphi(a)$  is constant. Taking logs and solving for  $\varphi(a)$ , we can make the equation,

$$\frac{\ln F(a)}{T} = -\varphi(a)$$

and equate for the two cases, number of years =  $Y$  and  $Z$

$$\frac{\ln(1 - x)}{Y} = \frac{\ln(1 - w)}{Z}$$

Solving for  $1 - w$ ,

$$\ln(1 - w) = \frac{Z}{Y} \ln(1 - x)$$

$$(1 - w) = e^{\frac{Z}{Y} \ln(1 - x)} = (1 - x)^{\frac{Z}{Y}}$$

So, for instance, for  $r = 0.10$ ,  $Y = 50$ , and  $Z = 500$ ;  $F = 0.9$  and

$$(1 - w) = (0.90)^{10} = 0.35, \text{ hence } w = 0.65$$

In a similar way, for  $r=0.05$ ,  $w = 0.40$ , and for  $r = 0.02$ ,  $w = 0.18$

If a ground motion has this probability of being exceeded in 50 years,	the same ground motion has the following probability of being exceeded in 500 years.	Prob of exceedance in 500 ————— Prob of exceedance in 50
0.02	0.18	9.0
0.05	0.40	8.0
0.10	0.65	6.5