United States
Department of
Agriculture

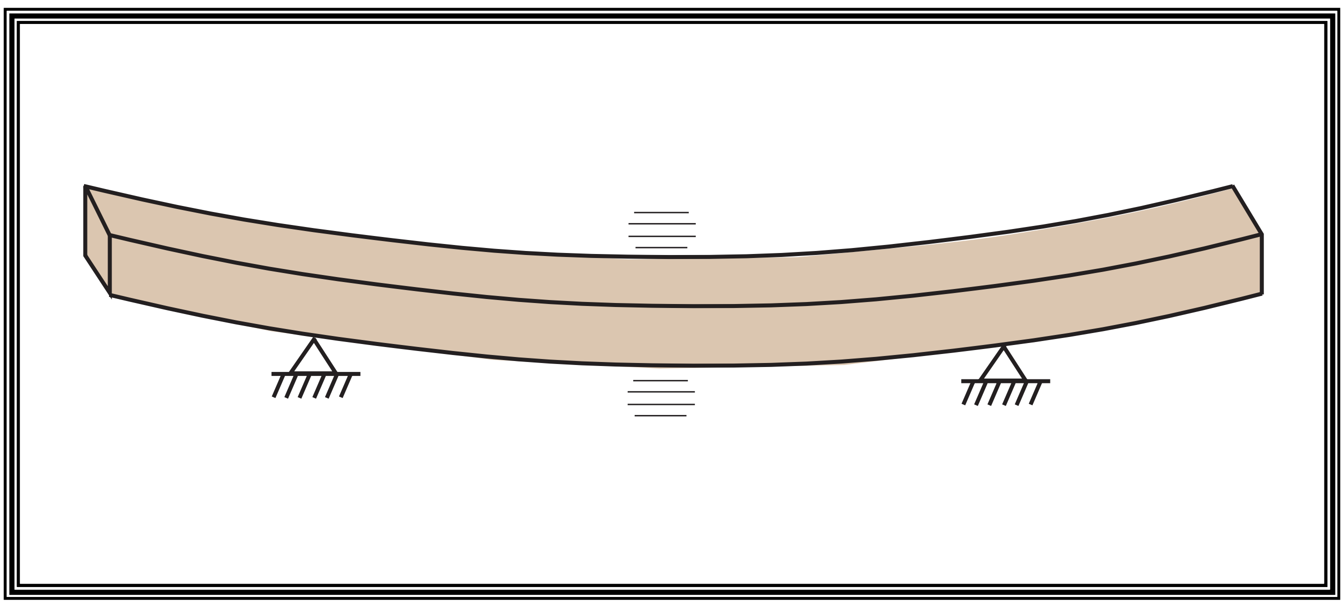
Forest Service

Forest
Products
Laboratory

Research
Note
FPL–RN–0276

Commentary on Factors Affecting Transverse Vibration Using an Idealized Theoretical Equation

Joseph F. Murphy



Abstract

An idealized theoretical equation to calculate flexural stiffness using transverse vibration of a simply end-supported beam is being considered by the American Society of Testing and Materials (ASTM) Wood Committee D07 to determine lumber modulus of elasticity. This commentary provides the user a quantitative view of six factors that affect the accuracy of using the idealized theoretical equation, idealized assumptions, and idealized boundary conditions. The six factors that affect the calculation of the flexural modulus of elasticity are ranked in order of importance, and recommendations are given. Not covered are the precision and accuracy of the physical measurements.

Keywords: flexural stiffness, transverse vibration, modulus of elasticity, accuracy

August 2000

Murphy, Joseph F. 2000. Commentary on factors affecting transverse vibration using an idealized theoretical equation. Res. Note FPL–RN–0276. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory. 4 p.

A limited number of free copies of this publication are available to the public from the Forest Products Laboratory, One Gifford Pinchot Drive, Madison, WI 53705–2398. Laboratory publications are sent to hundreds of libraries in the United States and elsewhere.

The Forest Products Laboratory is maintained in cooperation with the University of Wisconsin.

Commentary on Factors Affecting Transverse Vibration Using an Idealized Theoretical Equation

Joseph F. Murphy, Research Engineer
Forest Products Laboratory, Madison, Wisconsin

Introduction

This commentary on the calculation of flexural modulus of elasticity by the method of transverse vibration is aimed at providing the user a quantitative view of the factors that affect the accuracy of the method using an idealized theoretical equation, idealized assumptions, and idealized boundary conditions. The user, with little effort, can increase this accuracy. Recommendations are given for ways accuracy can be increased. Not covered are the precision and accuracy of the physical measurements.

Idealized Equation

The following idealized equation from Timoshenko and others (1974) is used to calculate flexural stiffness *EI* using transverse vibration of a simply end-supported beam:

$$EI = \frac{f^2 L^4}{(\pi/2)^2} \frac{W}{gL} \qquad (1)$$

where

EI is dynamic flexural stiffness (force × length^2),

f fundamental mode natural frequency (cycles/time),

L length of specimen (length),

W weight of specimen (force),

g acceleration of gravity (length/time^2) (that is, 980.665 cm/s^2, 386.089 in/s^2), and

W/gL mass per unit length of specimen.

For homogenous materials,

E is flexural modulus of elasticity (force/length^2) and

I moment of inertia (length^4).

Equation (1) was derived for an idealized homogenous material with constant cross section and rigid supports at the extreme ends of the member for boundary conditions. The American Society of Testing and Materials (ASTM) Wood Committee D07 proposes to limit the cross section to a rectangular shape (lumber), which, although not necessary (for example, the method is valid for constant cross section, I shape, T shape, glued-laminated beams, plywood, panel products), further defines the moment of inertia *I*:

$$I = \frac{bh^3}{12} \qquad (2)$$

where

b is base, horizontal dimension (length),

h height, vertical dimension (length), and

12 the constant for moment of inertia of member with rectangular cross section.

The idealized equation takes the final form

$$E = \frac{f^2 L^4}{Kbh^3}\frac{W}{L} = \frac{f^2 L^3 W}{Kbh^3} = \frac{f^2 W}{Kb}\left(\frac{L}{h}\right)^3 \qquad (3)$$

where the constant $K = (\pi/2)^2\, g/12$ (that is, 201.641 cm/s^2, 79.386 in/s^2).

This commentary discusses six factors that affect the calculated flexural modulus of elasticity *E* using Equation (3). The six factors are multiplicative:

$$E_{\text{calculated}} = E_{\text{true}}\, F_{\text{overhang}}\, F_{\text{shear}}\, F_{\text{support}}\, F_I\, F_W\, F_K \qquad (4)$$

where

$E_{\text{calculated}}$ is E calculated from Equation (3),

E_{true} best estimated E,

F_{overhang} factor to account for specimen overhang at supports,

F_{shear} factor to account for shearing deformations and rotary inertia,

F_{support} factor to account for nonrigid supports,

F_I factor to account for I using assumed cross section dimensions,

F_W factor to account for assumed beam weight distribution, and

F_K factor to account for a different K "constant."

The following sections discuss the six factors and offer recommendations.

Overhang

Murphy (1997) numerically investigated the effect of symmetric overhang on the transverse vibration of a beam (beam in the generic sense) with matching results for beams with rigid supports at the ends (span-to-length ratio $S/L = 1$, where S is span), rigid supports at midspan ($S/L = 0$), and a free–free condition ($S/L = 0.552$). He developed an analytic approximation to the vibration equation for small symmetric overhang ($1 \geq S/L \geq 0.8$). His equation accounting for overhang is

$$EI = \frac{f^2 S^4}{(\pi/2)^2} \frac{W}{gL} \tag{5}$$

Comparing Equations (5) and (1) results in the definition of the overhang factor

$$F_{\text{overhang}} = \left(\frac{L}{S}\right)^4 \tag{6}$$

Table 1 enumerates the unconservative error ($E_{\text{calculated}} > E_{\text{true}}$) associated with ignoring overhang.

Some transverse vibration systems that use software based on Equation (3) ask only for the input of L. For this software, error due to overhang could be minimized if span S were input in place of specimen length L. Then the error would be L/S rather than $(L/S)^4$ because the mass per unit length would be incorrect using span instead of length.

Considering overhang, it is recommended to

- use a corrected formula (Eq. (5)) to explicitly account for overhang (and apply to all wood-based products) or
- with systems that utilize Equation (3), use span S rather than specimen length L. (See section on Constant K.)

Table 1—Overhang (symmetric) factor[a]

Length (in.)	1-in. overhang		2-in. overhang		4-in. overhang	
	L/S	$(L/S)^4$	L/S	$(L/S)^4$	L/S	$(L/S)^4$
72	1.03	1.12	1.06	1.26	1.13	1.60
96	1.02	1.09	1.04	1.19	1.09	1.42
120	1.02	1.07	1.03	1.15	1.07	1.32
144	1.01	1.06	1.03	1.12	1.06	1.26

[a]See Table 2 for metric conversion factors.

Table 2—Metric conversion factors

From inch–pound unit	Multiply by conversion factor	To metric unit
inch	25.4	mm
lb/in	175.13	N/m

Shearing Deformations

Timoshenko and others (1974) account for shearing deformations on the flexural vibration of beams ("Effects of Rotary Inertia and Shearing Deformations"). Their correction term on the fundamental frequency f is

$$1 - \frac{1}{2}\frac{\pi^2}{S^2}\frac{I}{A}\left(1 + \frac{E}{k'G}\right) \tag{7}$$

where

A is cross sectional area (length^2),

G shear modulus (force/length^2), and

k' shear coefficient (cross section specific).

For a rectangular wood specimen ($k' = 5/6$ for rectangular members, and $E/G = 16$ for wood members), the factor accounting for shearing deformations and rotary inertia becomes approximately

$$F_{\text{shear}} \cong \left[1 - \left(\frac{26}{9}\frac{h}{S}\right)^2\right]^2 \tag{8}$$

Note that this factor is slightly conservative ($E_{\text{calculated}} < E_{\text{true}}$). For $h/S = 1.5/94 = 0.0166$, $F_{\text{shear}} = 0.996$; for $h/S = 1.5/70 = 0.021$, $F_{\text{shear}} = 0.992$. As h/S increases, the correction factor F_{shear} becomes more conservative.

Considering shearing deformations, it is recommended to

- specify a maximum height-to-span ratio (or conversely, minimum span-to-height ratio) and
- ignore the minor effect due to rotary inertia and shearing deformations.

Nonrigid Supports

Equation (3) assumes that the supports are rigid. Timoshenko and others (1974) address the effect of elastic supports (with equivalent vertical elastic springs k_s) ("Beams on Elastic Supports or Elastic Foundations"). The beam over the supports

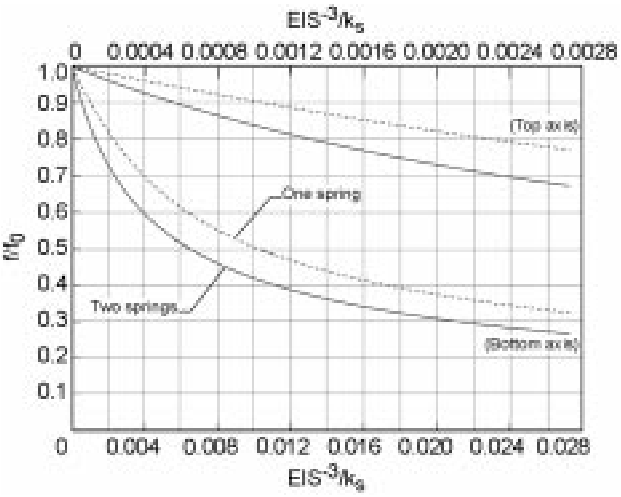


Figure 1. Reduced frequency of vibration as a function of specimen-to-support stiffness ratio.

is free to pivot. Using the method of Timoshenko and others, the solution to the equations of a vibrating beam on elastic support or supports depends on the ratio of beam–spring constant EI/S^3 to support–spring constant k_s. The frequency of oscillation of the *system* decreases as EIS^{-3}/k_s increases, that is, as the supports become less rigid (Fig. 1). In Figure 1, f_0 is the frequency with rigid supports. The solid curve is for two support springs, and the dashed curve is for only one support spring. For $EIS^{-3}/k_s < 0.0015$, the factor accounting for nonrigid support or supports is approximately

$$F_{\text{support}} \cong \left(\frac{f}{f_0}\right)^2 = \left(1 - \frac{7+7j}{0.15}\frac{EIS^{-3}}{k_s}\right)^2 \qquad (9)$$

where j is the number of elastic supports (1 or 2).

Note that the springiness of the support can be the spring constant of a load cell, flexing of a tripod, softness of earth or wood chip dirt mix, or even carpet pads—anything that adds springiness to the *system*.

Using an aluminum bar, a 6% difference was obtained just by changing a load cell. An aluminum calibration bar having $(EIS^{-3})_{cb} = 2.66$ lb/in and a load cell having a spring constant of 5,000 lb/in yields a ratio of 0.00053, $f/f_0 = 0.95$ from Figure 1, and $F_{\text{support}} = 0.90$ from Equation (9). (See Table 2 for metric conversion factors.) If the E_{cb} is forced to be that of aluminum and a wood member is tested with $EIS^{-3} = 1.48$, $f/f_0 = 0.97$, $(f/f_0)^2 = 0.94$, then the calculated flexural modulus of elasticity $E_{\text{calculated}}$ is 4.4% too high. Conversely, if the tested wood member has an $EIS^{-3} = 3.59$ (shorter span S), $f/f_0 = 0.93$, $(f/f_0)^2 = 0.865$, then the $E_{\text{calculated}}$ is 4% too low.

Note that this factor, in the absence of using a calibration bar, is conservative ($E_{\text{calculated}} < E_{\text{true}}$).

Considering nonrigid supports, it is recommended to

- assume the supports to be rigid and
- use the measured frequency of oscillation to calculate a conservative stiffness/flexural modulus of elasticity.

If a calibration bar is used to account for elasticity of the supports and the calibration bar's beam–spring constant is $(EIS^{-3})_{cb}$, then the support factor becomes

$$F_{\text{support}} = \left[\frac{0.15k_s(7+7j)(EIS^{-3})}{0.15k_s(7+7j)(EIS^{-3})_{cb}}\right]^2 \qquad (10)$$

All test specimens with beam–spring constants greater than $(EIS^{-3})_{cb}$ will result in conservative calculations of beam stiffness EI, and more importantly, all test specimens with beam–spring constants less than $(EIS^{-3})_{cb}$ will result in *unconservative* calculations of beam stiffness EI, the support–spring constant k_s remaining the same.

Note that calibration to a bar will produce accurate results only for tested members with EIS^{-3} equal to that of the calibration bar.

Considering nonrigid supports when using a calibration bar, it is recommended to

- use a calibration bar only as a verification check and not to calibrate a system,
- assume the supports are rigid, and
- measure a conservative frequency and calculate a conservative beam stiffness EI.

Assumed Moment of Inertia

The moment of inertia factor is the result of using assumed, sometimes nominal, cross-section dimensions rather than measured dimensions in calculating moment of inertia I. The factor is

$$F_I = \frac{bh^3}{b_{\text{assumed}}h_{\text{assumed}}^3} = \left(\frac{b}{b_{\text{assumed}}}\right)\left(\frac{h}{h_{\text{assumed}}}\right)^3 \qquad (11)$$

The moment of inertia factor is conservative when the beam dimensions are less than those assumed and *unconservative* when the beam dimensions are greater than the assumed. For example, if the beam height is assumed to be 1.5 in. and it is really 1/8 in. oversize, then $E_{\text{calculated}} = 1.27\ E_{\text{true}}$, and if the beam height is 1/8 in. undersize, then $E_{\text{calculated}} = 0.77\ E_{\text{true}}$.

From some in-grade data on nominal 2- by 4-in. dimensions, the dimension 5th percentile has $h = 1.48$, $b = 3.44$, $F_I = 0.944$, and the dimension 95th percentile has $h = 1.58$, $b = 3.57$, $F_I = 1.192$.

Transverse vibration is used to first calculate stiffness *EI* and then *E*, using a measured or assumed *I*. It is extremely important to be consistent in reporting which *I* is used.
If a regression is developed (for example, strength versus *E* or moisture adjustment of *E*) using one *I* (for example, measured), then it should be noted so that the other *I* (for example, assumed nominal) is not accidentally used in it subsequently. Also, an apparent change in the quality of the forest resource, as measured by $E_{\text{calculated}}$, might actually be attributed to a change in dimension if nominal dimensions are used. Buckling is dependent on *E*; therefore, the engineer using *E* measured by transverse vibration has to be made aware of exactly how *E* was calculated.

Considering moment of inertia, it is recommended to

- measure and use actual dimensions, either as input to or to correct the output of computer programs and
- if assumed or nominal dimensions are used, then use the notation $(EI)_{\text{tv}}/I_{\text{n}}$ (instead of transverse vibration E_{tv}) in all reports, papers, equations, figures, tables, graphs, plots, regressions, and correspondence that use—either based on or reduced from—these data.

Nonhomogenous Weight

In some measuring systems, weight is assumed to be twice that measured by a load cell at one support. This assumes that the beam has a uniform density and the supports are symmetrically placed with respect to center span. The factor to account for nonhomogenous weight distribution is

$$F_W = \frac{2W_{\text{half}}}{W} \qquad (12)$$

where W_{half} is the weight measured at one support and *W* is the actual weight of the specimen.

In an end-weight system, if twice the weight measured is less than the actual, then this factor is conservative; if greater, then $E_{\text{calculated}} > E_{\text{true}}$.

Considering nonhomogenous weight distribution, it is recommended to

- accurately calibrate the weight-measuring device for static loads close to the expected weight to be measured, using calibrated dead weights, and
- if one end of a beam is suspected to vary in weight by more than 5% from the other end, use total beam weight and do not double the weight of either end.

Constant *K*

The constant *K* in Equation (3) theoretically should be $[(\pi/2)^2\,(g/12)]$. If a different *K* is used, then the factor accounting for this is

$$F_K = \frac{(\pi/2)^2\,(g/12)}{K} \qquad (13)$$

As shown, if the *K* used is greater than the theoretical value, then $E_{\text{calculated}} < E_{\text{true}}$, and if less, $E_{\text{calculated}} > E_{\text{true}}$.

Considering the constant *K*, it is recommended to

- set *K* to its theoretical value or
- set *K* to *L/S* times its theoretical value only when *S* is substituted for *L* in Equation (3). (See section on Overhang.)

Concluding Remarks

Six factors affect the accuracy of the calculated flexural modulus of elasticity *E* using the idealized theoretical Equation (3). The factors with recommendations to increase accuracy, ranked in order of importance, are as follows:

1.	F_{overhang}	Use correct Equation (5) or use span *S* instead of *L* in Equation (3).
2.	F_I	Use measured dimensions as input or measured dimensions to correct output or measure stiffness *EI*, not modulus *E*. (This latter recommendation also works with other wood-based products.)
3.	F_K	Use theoretical value.
4.	F_W	With long beams, flip beam end-for-end and average the two results.
5.	F_{support}	Assume rigid supports; use stiff load cell and tripods and put on solid base.
6.	F_{shear}	Exceed minimum span-to-height ratio.

After accounting for each of the six factors, the only steps needed are to calibrate the load cell with dead weights and check that frequency data are acquired correctly.

References

Murphy, J.F. 1997. Transverse vibration of a simply supported beam with symmetric overhang of arbitrary length. Journal of Testing and Evaluation. 25(5): 522–524.

Timoshenko, S.; Young, D.H.; Weaver, W. Jr. 1974. Vibration problems in engineering. 4th ed. New York, NY: John Wiley & Sons.