Several Nondestructive Evaluation Techniques for Assessing Stiffness and MOE of Small-Diameter Logs

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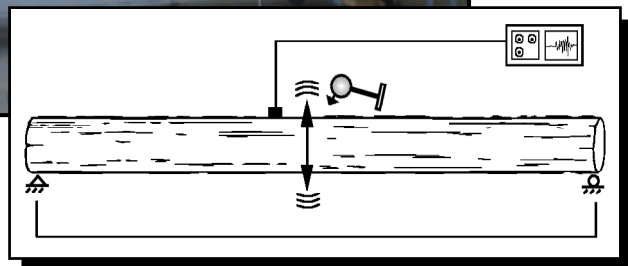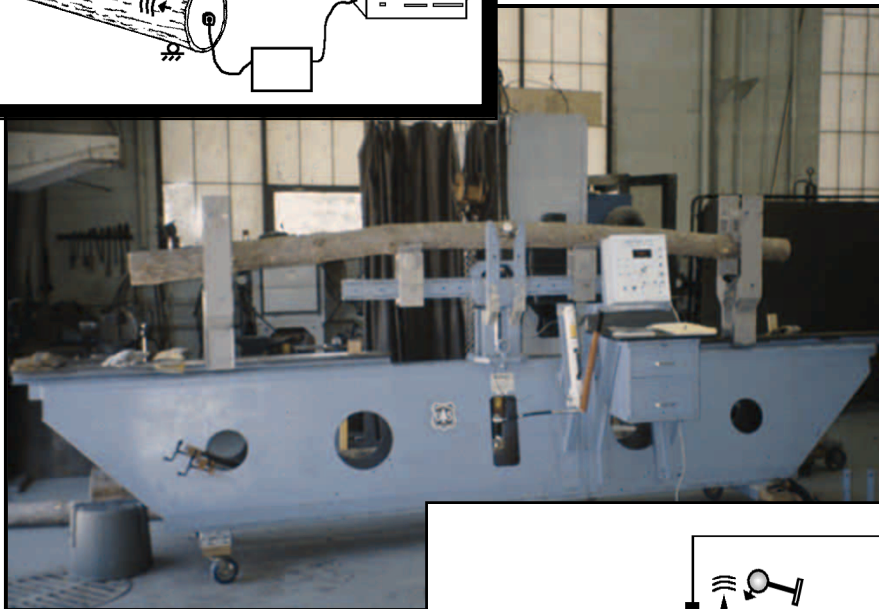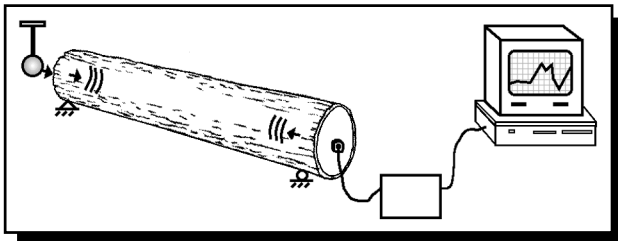
Abstract

Many forests of the United States have large areas that contain trees of small diameter, mixed species, and undefined quality. Because these areas are at risk from attack by insects, disease, and uncontrollable wild fires, it is essential to find ways to increase the incentive to harvest this small-diameter material. One way to do this is to develop cost-effective products for the fiber from these trees. Nondestructive technology needs to be developed to evaluate the potential quality of stems and logs obtained from trees in such ecosystems. Static bending, transverse vibration, and longitudinal stress wave techniques are frequently used to assess the modulus of elasticity (MOE) of lumber. Excellent correlations between MOE values obtained from these techniques have been reported. The objective of this study was to investigate the use of these techniques to evaluate the flexural stiffness and MOE of small-diameter logs. A total of 159 red pine and jack pine logs were obtained from northern Michigan and were assessed nondestructively using these techniques. Statistical comparisons between stiffness and MOE values obtained from each technique were then examined. Results of this study demonstrated that strong relationships exist between the log properties determined by the three techniques, longitudinal stress wave, transverse vibration, and static bending. Developed models allow for the prediction of static bending properties of logs at levels of accuracy previously considered unattainable. This indicates that any of these techniques can be used to sort small-diameter logs with reasonable accuracy.

Keywords: small-diameter log, nondestructive evaluation, stress wave, transverse vibration, static bending, modulus of elasticity (MOE), flexural stiffness

Contents

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Introduction

Many decades of inappropriate management practices, or lack of management altogether, have resulted in large amounts of U.S. forests being overstocked with small-diameter trees of mixed species. These stands are typically low in value, and the value of the harvestable material will not cover the costs of needed management treatments. A specific example is the interior west region of the United States, where 39 million acres of ponderosa-pine-type forest have lost ecological integrity due to major changes in vegetative structure and composition. These changes have been caused by control of fire in an ecosystem where historically there were frequent, low-intensity stand maintenance fires. Exclusion of fire led to the current condition, and these stands are now at high risk of attack by insects, disease, and stand-destroying wildfires. Restoration, either mechanical or through prescribed fires, can cost $150 to $500 per acre. It is essential to find cost-effective products that can be produced from the materials available in these stands so that needed management operations such as thinning can be implemented to improve the condition of these stands. Economical and value-added uses for this material can help offset forest management costs, provide economic opportunities for many small, forest-based communities, and avoid future loss caused by catastrophic wildfires. The variability and lack of predictability of the strength and stiffness of this material cause problems in engineering applications. Cost-effective technology needs to be developed for evaluating the potential quality of stems and logs obtained from trees in such ecosystems.

In the forest products industry, a variety of nondestructive evaluation (NDE) techniques are now being used to assess the engineering properties of structural lumber. Static bending, transverse vibration, and longitudinal stress wave techniques are frequently used to assess the modulus of elasticity (MOE) of lumber. Commercially available equipment that applies these techniques is readily available. The objective of this study was to investigate the use of these three techniques to evaluate the stiffness and MOE of small-diameter logs.

Static Bending

Measuring the MOE of a member by static bending is the foundation of a commonly used process to grade structural lumber (Ross and Pellerin 1994). As currently employed for grading lumber, this relatively simple measurement involves utilizing the load–deflection relationship of a simply supported beam with different loading patterns (Fig. 1). The MOE is computed directly by using equations derived from the fundamental mechanics of materials.
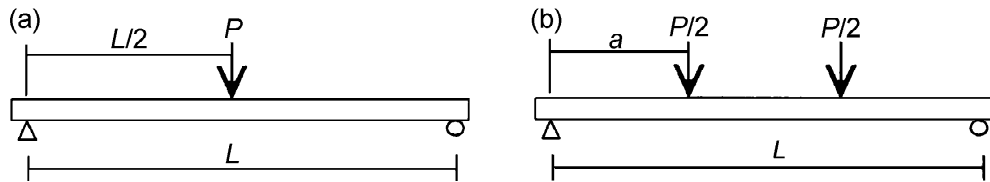
Figure 1—Static bending configuration; (a) center-point bending, and (b) general bending (*L*, beam span; *P*, load within proportional limit; *a*, distance from end support to nearest load point).
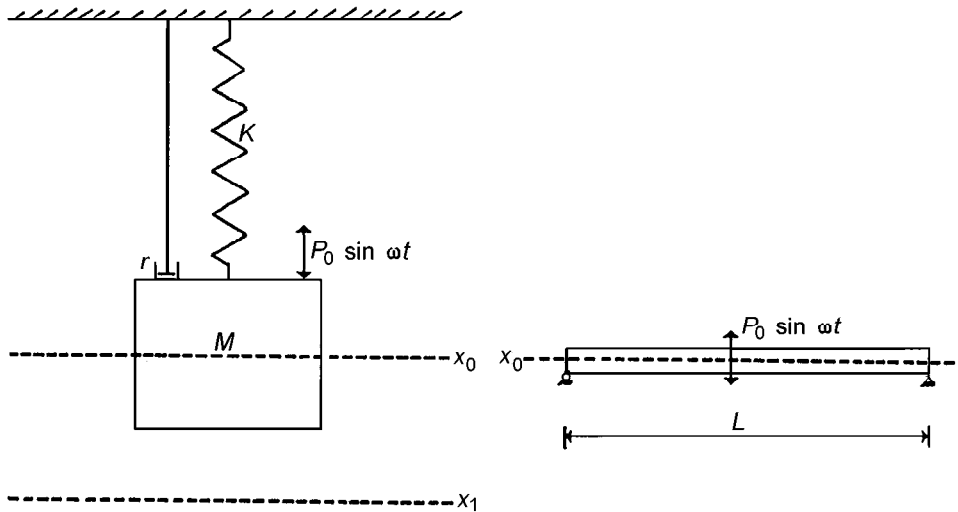

Figure 2—Mass-spring dashpot vibration model (left) and transverse vibrating beam (right) (P_0 sin •*t*, a sinusoidal force; *M*, mass; *K*, elastic constant of the spring; *r*, viscous damping coefficient of the dashpot; *L*, beam span).

Figure 1a shows a standard bending configuration. The load is applied at the midspan of the beam. An ordinary dial gauge or electronic recording equipment is normally used to obtain load–deflection data. Bending MOE is then calculated from data taken from the linear elastic region of the load–deflection curve:

$$\text{MOE} = \frac{PL^3}{48\delta I} \qquad (1)$$

where MOE is static modulus of elasticity (lb/in^2 (Pa)), *P* is load within the proportional limit (lb (N)), *L* is beam span (in. (m)), • is deflection at midspan within the proportional limit (in. (m)), and *I* is beam moment of inertia (in^4 (m^4)).

Figure 1b shows a general bending configuration used for testing structural lumber. The loads are equally applied at two points of the beam. Bending MOE is calculated by the following equation:

$$\text{MOE} = \frac{Pa(3L^2 - 4a^2)}{48\delta I} \qquad (2)$$

where *P* is load (lb (N)), *a* is distance from the end support to nearest load point (in. (m)), *L* is beam span (in. (m)), • is midspan deflection (in. (m)), and *I* is beam moment of inertia (in^4 (m^4)).

Transverse Vibration

Transverse vibration techniques have received considerable attention for NDE applications (Jayne 1959, Kaiserlik 1977, Pellerin 1965, Ross and others 1991, Ross and Pellerin 1994). To illustrate these methods, an analogy can be drawn between the behavior of a vibrating beam and the vibration of a mass that is attached to a weightless spring and internal damping force (Fig. 2). In Figure 2, mass *M* is supported from a rigid body by a weightless spring whose elastic constant is denoted by *K*. Internal friction or damping is given by *r*, which represents the viscous damping coefficient of the dashpot. A forcing function equaling P_0 sin •*t* or zero is applied for forced and free vibration, respectively. When *M* is set into vibration, its equation of motion can be expressed by the following:

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$$M\left(\frac{d^2x}{dt^2}\right) + r\left(\frac{dx}{dt}\right) + Kx = P_0 \sin \omega t \qquad (3)$$

Equation (3) can be solved for either K or r.

A solution for K will lead to an expression for MOE where

$$\text{MOE} = \frac{f_r^2 W L^3}{12.65 I g} \qquad (4)$$

for a beam freely supported at two nodal points and

$$\text{MOE} = \frac{f_r^2 W L^3}{2.46 I g} \qquad (5)$$

for a beam simply supported at its ends.

In Equations (4) and (5), MOE is dynamic modulus of elasticity (lb/in² or Pa), f_r is resonant frequency (Hz), W is beam weight (lb or kg), L is beam span (in. or m), I is beam moment of inertia (in⁴ or m⁴), and g is acceleration due to gravity (386 in/s² or 9.8 m/s²).

Stress Wave

Several techniques that utilize stress wave propagation have been researched for use as NDE tools. Speed-of-sound transmission and attenuation of induced stress waves in a material are frequently used as NDE parameters (Ross and others 1994, 1996, 1999; Schad and others 1995; Wang 1999; Wang and others 2000).

To illustrate these techniques, consider application of one-dimensional wave theory to the homogeneous viscoelastic bar (Fig. 3). After an impact hits the end of the bar, a compressive stress wave is generated in the bar that travels from left to right at a speed C. As the wave reaches the free end, it is reflected as a tension wave and begins traveling back down the bar. Energy is dissipated as the wave travels through the bar; therefore, although the speed of the wave remains constant, movement of particles diminishes with each successive passing of the wave. Eventually all particles of the bar come to rest.

Monitoring the movement of a cross section near the end of such a bar in response to a propagating stress wave results in waveforms that consist of a series of equally spaced pulses whose magnitude decreases exponentially with time (Fig. 4). The propagation speed C of such a wave can be determined by coupling measurements of the time between pulses •t and the length of the bar L by

$$C = \frac{2L}{\Delta t} \qquad (6)$$

The MOE can be computed using C and the mass density of the bar ρ:
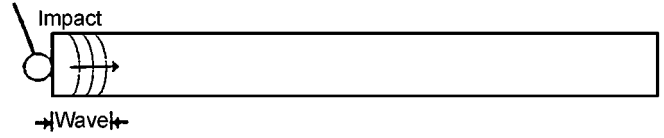
$$\text{MOE} = C^2 \rho \qquad (7)$$

Figure 3—Viscoelastic bar of length *L* subjected to an impact.
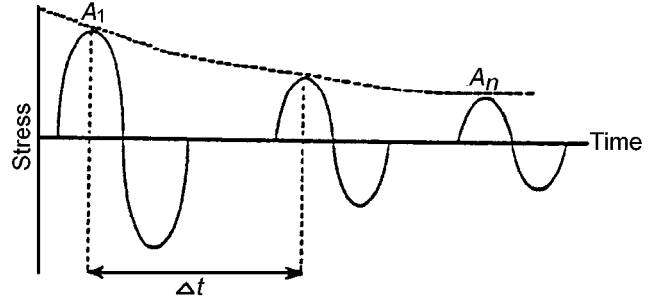

Figure 4—Theoretical response of the end of a viscoelastic bar (cross section) in response to a propagating stress wave.

Although this equation was derived for an idealized, one-dimensional case, it has been shown to exist for actual three-dimensional members as long as the length of the wave is large relative to the member's lateral dimensions.

Materials and Methods

A flow chart that outlines the experimental procedures is shown in Figure 5. First, a sample of small-diameter trees was selected from stands and harvested to obtain logs. Physical properties (diameter, moisture content, and green density) of logs were then measured. This was followed by a sequence of nondestructive tests using longitudinal stress wave, transverse vibration, and static bending techniques to obtain the MOE and flexural stiffness of these logs. Statistical analyses were then used to examine the relationships between log properties determined by the different techniques.

In this study, 159 small-diameter logs (109 jack pine (*Pinus banksiana* Lamb.) and 50 red pine (*Pinus resinosa* Ait.)) were nondestructively evaluated. These logs came from trees that were grown on the Ottawa National Forest and the Lake Superior State Forest, both in northern Michigan.

The jack pine logs used in this study were obtained from an overage stand of jack pine, which was beginning to show signs of deterioration. Ranger district personnel in the Ottawa National Forest were able to visually identify four categories of trees in this stand: live healthy trees (merchantable live), live trees that are showing signs of being under stress (suspect), trees that are dead but still containing
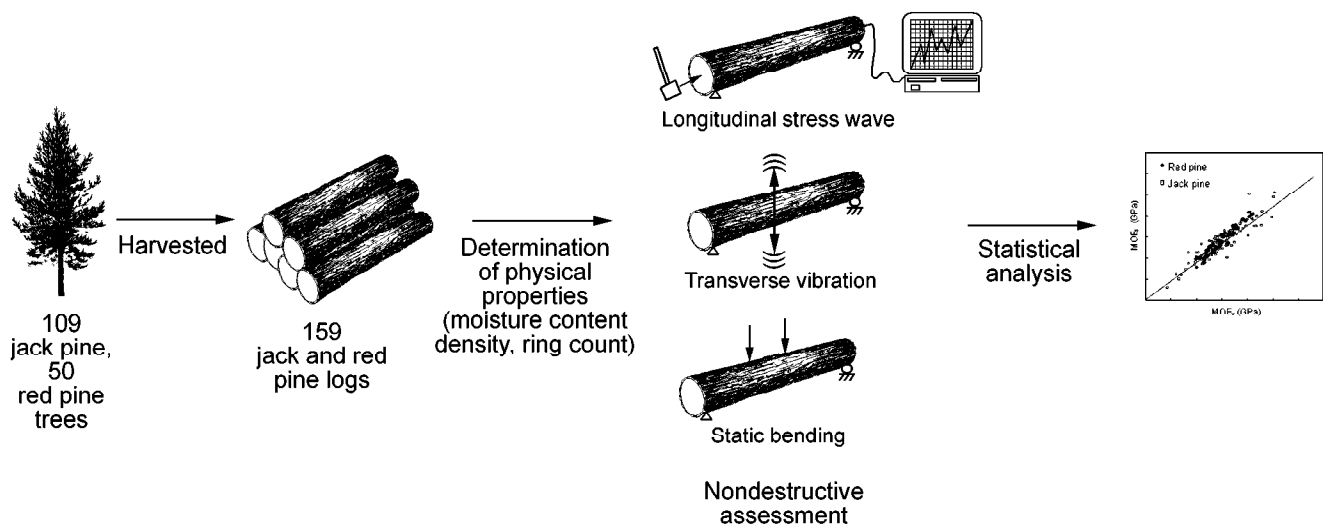
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Figure 5—Flowchart of study.

merchantable material (merchantable dead), and dead trees that have deteriorated to the point of having no merchantable material (unmerchantable dead). To manage these jack pine stands, forest managers have been holding commercial salvage sales on considerable acreages. To properly estimate the value of these stands, better information on the value of each of the four categories of trees is needed. Trees in each of these categories were selected for this study. The estimated ages of these jack pine trees ranged from 50 to 70 years old. The diameter at breast height (DBH) of sampled trees ranged from 5.0 to 12.2 in. (127 to 310 mm).

Red pine logs were obtained from 38-year-old research plots in the Lake Superior State Forest. The objective of the original research on these plots was to examine the growth of red pine with time at various stocking levels and correlate volume yield with financial yield at different initial stocking levels. Plots at five levels of stocking were available: 220, 320, 420, 620, and 820 trees per acre. Ten trees were harvested from each of the stocking level plots. The DBH of sampled trees ranged from 4.7 to 11.5 in. (11.9 to 29.2 cm).

After these sampled trees were harvested, a 16-ft- (4.9-m-) long butt log was bucked from each tree on site and then transported to the Forestry Sciences Laboratory, USDA Forest Service, North Central Forest Experiment Station, Houghton, Michigan, for various nondestructive testing.

Upon arrival at the Forestry Sciences Laboratory, a 2-ft- (61-cm-) long section from each end of the butt log was cut off and sent to the USDA Forest Service, Forest Products Laboratory, in Madison, Wisconsin, for pulping studies. The remaining 12-ft- (3.7-m-) long logs were then used for this study. To determine moisture content (MC) of sampled trees, we cut three disks from each tree, one from the butt, the middle, and the top. Green weight and ovendry weight of these disks were then obtained and used to determine tree MC. For each 12-ft- (3.7-m-) long log, the green weight and the diameters of both ends were measured to obtain the green density and the moment of inertia of the log.

Each log was first evaluated using a longitudinal stress wave technique to obtain an estimate of dynamic MOE (stress wave MOE (MOE_{sw})) of the log. Figure 6 shows the experimental setup for stress wave measurements on the logs. An accelerometer was attached to one end of the log. A stress wave was introduced to the log through a hammer impact on the opposite end, and the resulting stress wave was recorded with a personal computer. A detailed description of the instrumentation and analysis procedures can be found in a previous article (Ross and others 1994), and a discussion of its application to large specimens is included in Schad and others (1995). From stress wave measurement, the stress wave speed (C) in a log was determined by Equation (6). Dynamic MOE (MOE_{sw}) was then calculated using Equation (7).

After stress wave tests, the logs were vibrated using a transverse vibration technique. The experimental setup is shown in Figure 7. A digital oscilloscope and an accelerometer were used in this test. The log under test was supported at one end by a knife-edge support and at the opposite end by a point support. The accelerometer was located in the middle of the log and glued on the upside surface, where the bark was removed or polished to improve the contact between the accelerometer and the log. The log was then set into vibration by impacting the middle part of the log with a rubber hammer. The free vibration response of the log was observed in the oscilloscope. The signal observed was a series of pulses with a gradually decreasing (decaying) amplitude. The vibrational parameter measured was fundamental natural frequency. The dynamic MOE of logs determined by transverse vibration technique (vibration MOE (MOE_v)) was calculated from Equation (5).
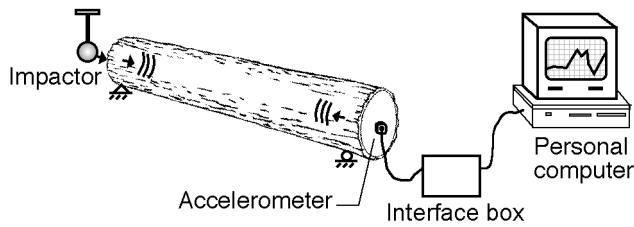
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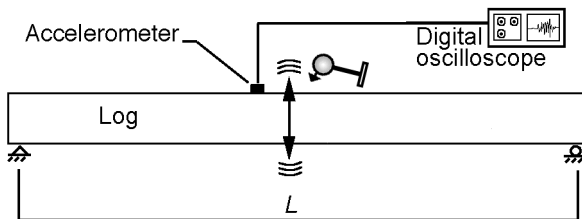**Figure 6—Experimental setup for stress wave test on logs.**

Figure 7—Experimental setup for transverse vibration test on logs.

Static bending tests were then performed on the logs to obtain the static MOE (MOE_s) and flexural stiffness (*EI*) of these logs. Measuring MOE of a member by static bending techniques has been widely considered the foundation of lumber grading and NDE of wood and wood-based materials. However, this technique is rarely used to evaluate the MOE of logs as a standard method. Consequently, no standard testing procedure exists for testing small-diameter logs. However, we assumed the MOE_s of logs to be the true MOE for logs, and we used it to evaluate the dynamic MOE of logs determined by stress wave and transverse vibration techniques. A Metriguard (Pullman, Washington) Model 312 bending proof tester (Fig. 8a) was used to conduct static bending tests on all logs. Figure 8b shows the bending configuration (general bending) we used. The testing machine was originally designed for proof-loading dimensional lumber. To test logs, we modified the two end supports to fit the geometrical shape of small-diameter logs (Fig. 9). The modified supports allowed testing of logs with a maximum diameter of 12 in. (30.5 cm). The span between two supports was set at 115.5 in. (2.93 m). The distance from loading point to the nearest support was 38.5 in. (0.98 m), which was one-third of the span. A load was applied to the log through two bearing blocks. Deflection was measured in the central region, a zone of pure bending without shear deformation. The log under test was first preloaded to 100 lb (445 N), and the deflection was set to zero. This procedure was mainly used to improve the contact between the log, supporters, and bearing blocks and thus to eliminate the effect of bark on the deflection measurement. The log was then loaded to 0.2-in. (5.08-mm) deflection. The load value corresponding to this deflection was then recorded. Static MOE (MOE_s) of the log was calculated using Equation (2).
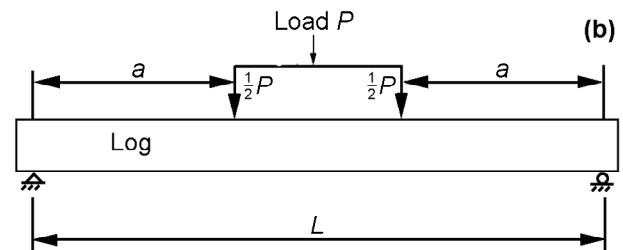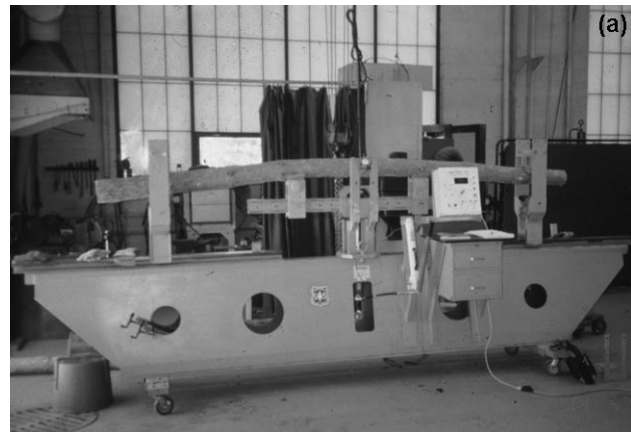

Figure 8—(a) Static bending test on logs, and (b) bending configuration (*a*, distance from end support to nearest load point; *P*, load within proportional limit; *L*, beam span).

Results and Discussion

Physical Characteristics of Logs

Table 1 summarizes the physical characteristics of the red pine and jack pine logs. The average diameters of the butt logs obtained from the trees ranged from 4.4 to 10.2 in. (11.2 to 25.9 cm) for red pine and from 4.7 to 11.0 in. (11.9 to 27.9 cm) for jack pine. This is a typical diameter range for small-diameter timber (Wolfe 2000). For both species, the average MC exceeded the fiber saturation point (about 30%). However, the red pine logs apparently had much higher MC level than the jack pine logs. The individual values ranged from 88% to 145% for red pine logs and from 31% to 65% for jack pine logs. The low MC level for jack pine logs was caused by the fact that they came from different categories that included live, suspect, and dead trees. The suspect and dead trees had already lost a lot of moisture by the time they were harvested. Therefore, the MC of some logs obtained from dead trees was close to or even lower than the fiber saturation point.

Also, red pine logs have higher density than jack pine logs. The green density for the red pine logs ranged from 48.0 to 56.5 lb/ft³ (770 to 900 kg/m³), and that for jack pine logs ranged from 28.7 to 53.7 lb/ft³ (460 to 860 kg/m³). The lower value and large range of density for jack pine logs was also due to the different tree categories.
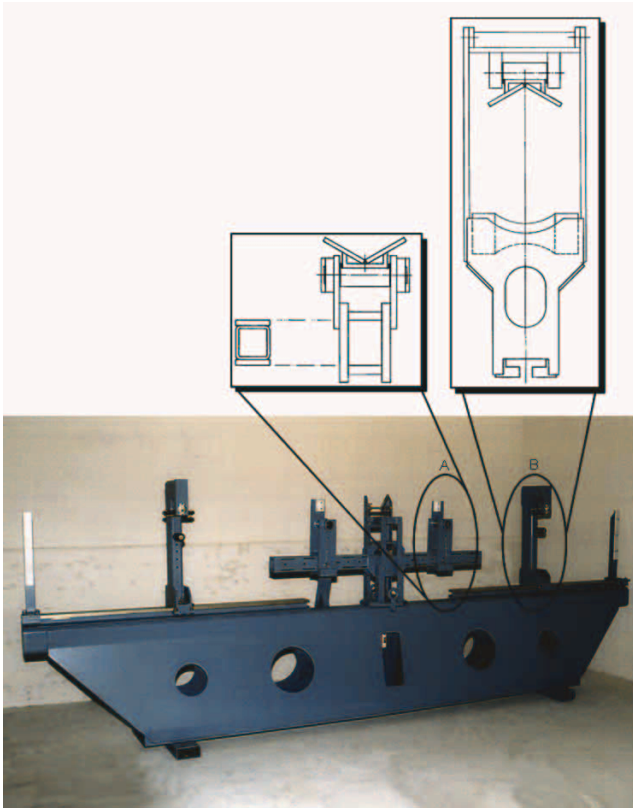
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Figure 9—Modified fixture of supports used for static bending test.

Jack pine logs differ from red pine logs in cross section of stem shape and in straightness. Red pine logs are mostly round and very straight. Jack pine logs tend to have a more irregular shape in cross section and a more curved stem, which could introduce errors in the determination of density and moment of inertia of these logs.

Modulus of Elasticity of Logs

Results obtained from various NDE measurements of both red pine and jack pine logs are summarized in Table 2. The basic statistics of dynamic MOE (determined by stress wave and transverse vibration techniques) and static MOE (determined by static bending technique within elastic region) for both species are given in Table 2.

The static MOE (MOE_s) of logs ranged from 0.45 to 1.21 $\times10^6$ lb/in^2 (3.10 to 8.34 GPa) with a mean value of 0.80 $\times10^6$ lb/in^2 (5.52 GPa) for red pine and 0.17 to 1.48 $\times10^6$ lb/in^2 (1.17 to 10.20 GPa) with a mean value of 0.81 $\times10^6$ lb/in^2 (5.58 GPa) for jack pine. The stress wave technique produced a higher estimate of MOE for both species. For red pine logs, the mean MOE_{sw} was 11.8% and 18.8% greater than its vibrational and static counterpart,

respectively. For jack pine logs, the mean MOE_{sw} was 21.6% and 24.7% greater than its vibrational and static counterparts, respectively. We believe that the higher value of MOE_{sw} could be related to the wave propagation mechanism, the dimension of the logs, or the moisture state of the wood in the logs.

In previous studies (Wang 1999, Wang and others 2000), we found that the stress wave behaved differently in logs than in small, clear wood and lumber because of the relative large size of the logs. As the wave travels through a log in the longitudinal direction, the outer portion of the wood (mature wood) may have a dominating effect on the propagation of waves. This led to a higher stress wave speed for a log compared with small, clear specimens cut from the log, which increased the value of MOE_{sw} and in turn overestimated the MOE of the log. It was also found that the diameter-to-length ratio could be a critical factor that may affect stress wave behavior in logs. Quantitative analyses of the overestimation in MOE_{sw} of logs have not been reported.

Compared with MOE_{sw} of logs, the dynamic MOE of logs determined from transverse vibration technique (MOE_v) is much closer to static MOE of logs. The MOE_v of red pine logs ranged from 0.58 to 1.22 $\times10^6$ lb/in^2 (4.00 to 8.40 GPa), and the range for jack pine logs was 0.25 to 1.47 $\times10^6$ lb/in^2 (1.72 to 10.14 GPa). For both species, the mean value of MOE_v was about 7% greater than the mean MOE_s.

Modulus of Elasticity Relationships

Statistical analysis procedures were used to examine the relationships between the various MOE of red pine and jack pine logs. The results obtained from regression analyses are presented in Tables 3 and 4.

Univariable Regression Models

The correlations among various MOE could be represented by linear regression models ($y = a + bx$). The results of the comparison of the three different techniques are reported in terms of correlation coefficients. These results show how reliable the method might be for prediction purposes. The square of the correlation coefficient expresses the percentage of the total variability explained by the regression line.

In general, the dynamic MOE (MOE_{sw} and MOE_v) of logs was very closely correlated with the static MOE (MOE_s) for both red pine and jack pine logs. The correlation coefficients were 0.87 (MOE_{sw} versus MOE_s) and 0.97 (MOE_v versus MOE_s) for red pine logs. Those for jack pine logs were 0.77 (MOE_{sw} versus MOE_s) and 0.92 (MOE_v versus MOE_s). The linear regression analyses indicated that the developed regression models were statistically significant at the 0.01 confidence level.

Table 1—Physical characteristics of red pine and jack pine logs[a]

Species and log group	Number of logs	DBH[b] of trees (in.)		Diameter of butt logs (in.)		Density (lb/ft^3)		Moisture content (%)	
		Average	Minimum–maximum	Average	Minimum–maximum	Average	Minimum–maximum	Average	Minimum–maximum
Red pine									
220[c]	10	9.82	8.50–10.35	9.16	7.90–10.13	53.3	49.7–56.0	113.6	114.1–144.6
320	10	9.87	8.12–11.10	9.16	7.63–10.16	53.8	52.3–56.7	121.1	114.5–140.4
420	10	8.67	7.88–9.02	7.99	7.37–8.31	54.2	51.9–56.5	115.3	88.2–132.6
620	10	7.23	5.48–8.70	6.79	5.30–8.16	50.9	48.0–52.8	109	121.7–134.5
820	10	6.72	4.70–7.86	6.24	4.40–7.42	52.7	49.4–54.4	116.7	116.7–145.2
Jack pine									
Merchantable live	30	9.4	7.4–11.0	8.59	6.83–9.90	47.04	42.18–53.73	65	50.9–107.2
Suspect	29	9.4	6.4–12.0	8.47	6.14–10.99	42.26	29.01–51.38	54.1	40.2– 74.3
Merchantable dead	32	7.9	5.0–12.2	7.11	4.67–10.56	34.91	28.66–43.34	36.8	21.0– 55.7
Nonmerchantable dead	13	7.6	6.3–9.7	6.65	5.58–8.50	34.58	29.46–42.16	31.2	20.6– 38.4

[a]1 in. = 25.4 mm, 1 lb/ft^3 = 16.01 kg/m^3.
[b]DBH, diameter at breast height.
[c]Number of trees per acre.

Table 2—Modulus of elasticity (MOE) of red pine and jack pine logs[a]

Species	Number of logs	Dynamic MOE (MOE_{sw})			Dynamic MOE (MOE_v)			Static MOE (MOE_s)		
		Mean	SD	Minimum–maximum	Mean	SD	Minimum–maximum	Mean	SD	Minimum–maximum
Red pine	50	0.95	0.110	0.76–1.22	0.85	0.164	0.58–1.22	0.80	0.187	0.45–1.21
Jack pine	109	1.11	0.230	0.47–1.84	0.87	0.246	0.25–1.47	0.81	0.235	0.17–1.48

[a]MOE_{sw}, dynamic MOE of a log determined by stress wave technique; MOE_v, dynamic MOE of a log determined by transverse vibration technique; MOE_s, static MOE of a log determined by general static bending; SD, standard deviation.

Table 3—Results of linear regression analyses of various moduli of elasticity (MOE) of red pine and jack pine logs[a]

Species	Linear regression model $y = a + bx$				r^2	r	S_{yx}	F-test[b]
	y	x	a	b				
Red pine								
	MOE_v	MOE_{sw}	−0.3927	1.3101	0.77	0.88	0.079	162.1***
	MOE_s	MOE_{sw}	−0.5986	1.4740	0.75	0.87	0.095	143.1***
	MOE_s	MOE_v	−0.1443	1.1105	0.95	0.97	0.044	835***
Jack pine								
	MOE_v	MOE_{sw}	−0.0318	0.8167	0.58	0.76	0.160	142.4***
	MOE_s	MOE_{sw}	−0.0644	0.7883	0.60	0.77	0.150	150.9***
	MOE_s	MOE_v	0.0425	0.8782	0.85	0.92	0.092	567.1***
Combined								
	MOE_v	MOE_{sw}	0.0478	0.7748	0.55	0.74	0.150	182.9***
	MOE_s	MOE_{sw}	0.0089	0.7555	0.53	0.73	0.151	172.4***
	MOE_s	MOE_v	0.0820	0.9175	0.86	0.93	0.082	946.9***

[a]MOE_{sw}, MOE of a log determined by stress wave method; MOE_v, MOE of a log determined by transverse vibration method; MOE_s, MOE of a log determined by general static bending; r^2, coefficient of determination; r, correlation coefficient; S_{yx}, standard error of estimate.
[b] ***Highly significant (0.01 confidence level).

Table 4—Results of regression analyses relating static bending modulus of elasticity (MOE$_s$) to stress wave MOE (MOE$_{sw}$) and ratio of log diameter to log length (D/L)[a]

Species		Regression model	$y = ax_1^b x_2^c$						
	y	x_1	x_2	a	b	c	r^2	r	S_{yx}
Red pine	MOE$_s$	MOE$_{sw}$	D/L	0.0968	0.9078	−0.7326	0.91	0.953939	0.055
Jack pine	MOE$_s$	MOE$_{sw}$	D/L	1.7826	1.1957	−0.5060	0.74	0.860233	0.120

[a]MOE$_{sw}$, MOE of a log determined by stress wave method; MOE$_s$, MOE of a log determined by general static bending; r^2, coefficient of determination; r, correlation coefficient; S_{yx}, standard error of estimate.

Figures 10 to 13 show the relationships of dynamic MOE predicted by stress wave technique to dynamic MOE predicted by transverse vibration technique and static bending MOE for both species. The red pine logs produced a better correlation ($r = 0.87$ to 0.88) than the jack pine logs ($r = 0.76$ to 0.77). This could be attributed to the geometrical differences between the two species. It was evident that the irregular shape (irregular cross section and curved stem) of some jack pine logs could introduce errors in diameter measurements, thus causing errors in density and MOE determination, especially in MOE$_{sw}$ determination.

Also, the plotted data points were more heavily concentrated below the 45° line than above, thus indicating that the stress wave technique yields higher MOE values than its vibrational and static counterparts. As was discussed earlier, the higher value of MOE$_{sw}$ could have been caused by several factors such as wave propagation mechanism, log size, and moisture state of wood. Of these factors, log size (diameter D and length L) seems more important because it could affect stress wave behavior in logs. Wang (1999) reported that high diameter-to-length ratio D/L could cause significant changes in wave propagation path in the longitudinal direction in stress wave measurements of logs. Therefore, it seems that the effect of log size should not be neglected in the MOE regression models.

The relationships between MOE$_v$ and MOE$_s$ of red pine and jack pine logs are shown in Figures 14 and 15. The MOE correlations from transverse vibration tests were significantly improved compared with those from stress wave tests. In the relationship between MOE$_v$ and MOE$_s$, it was found that the two species could be combined and represented as a single population. The correlation coefficient relating the MOE$_v$ to MOE$_s$ was 0.93 for the two species combined. This value clearly indicates that the variation caused by these two species does not affect the relationship.

Multivariable Regression Models

For MOE$_{sw}$, the univariable linear regression models resulted in a correlation coefficient of 0.77 to 0.87 with static bending MOE. These values, although significant, indicate a
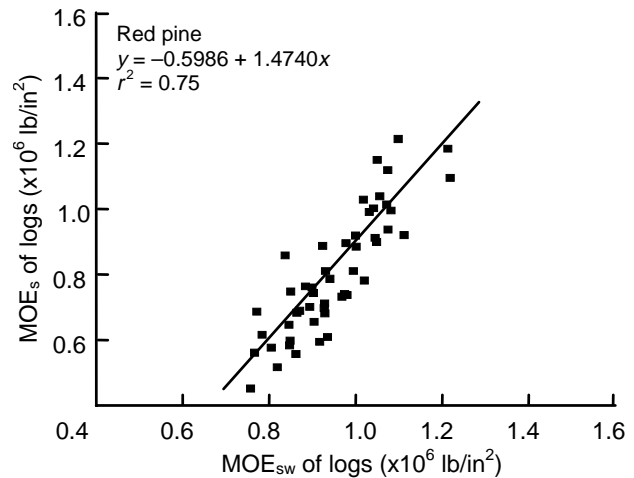

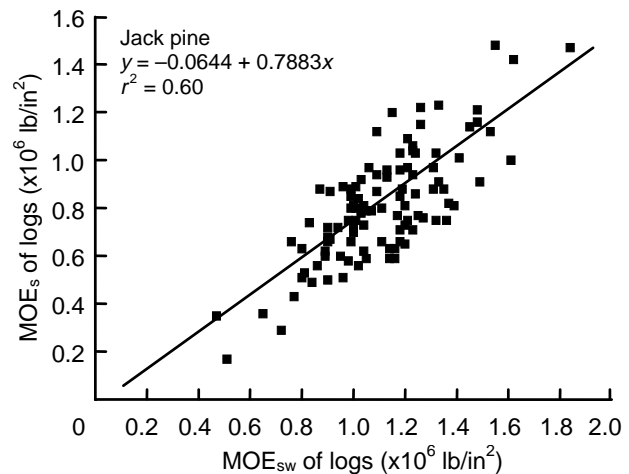**Figure 10—Relationship of stress wave MOE (MOE$_{sw}$) to static MOE (MOE$_s$) for red pine logs (1 lb/in^2 = 6.9 kPa).**

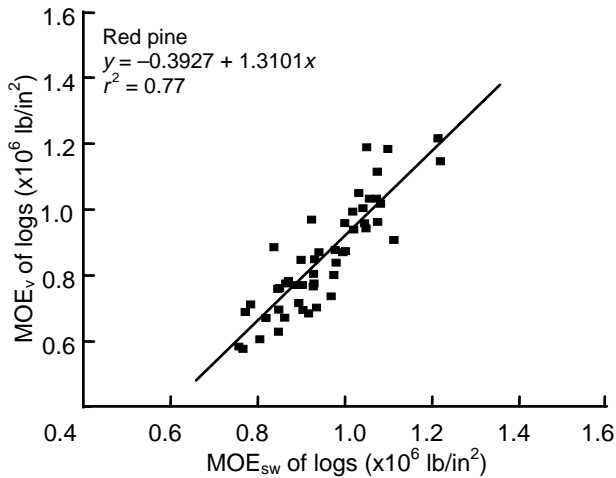**Figure 11—Relationship of stress wave MOE (MOE$_{sw}$) to static MOE (MOE$_s$) for jack pine logs (1 lb/in^2 = 6.9 kPa).**

8

Figure 12—Relationship of stress wave MOE (MOE$_{sw}$) to static MOE (MOE$_s$) for red pine logs (1 lb/in^2 = 6.9 kPa).
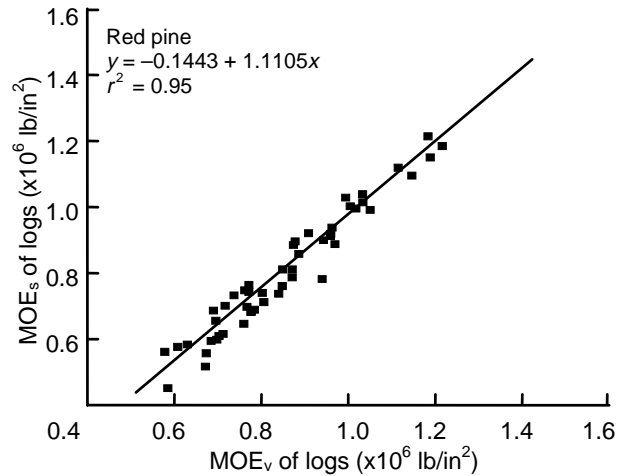

Figure 14—Relationship of vibration MOE (MOE$_v$) to static MOE (MOE$_s$) for red pine logs (1 lb/in^2 = 6.9 kPa).
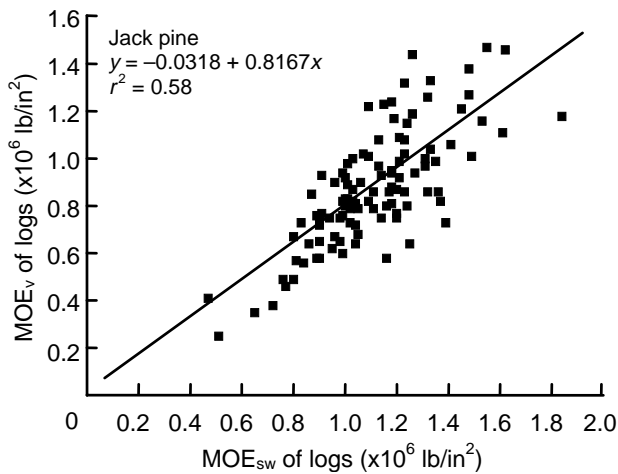

Figure 13—Relationship of stress wave MOE (MOE$_{sw}$) to vibration MOE (MOE$_v$) for jack pine logs (1 lb/in^2 = 6.9 kPa).
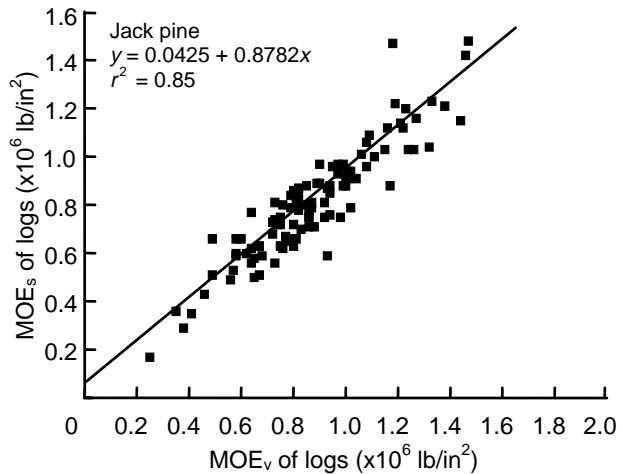

Figure 15—Relationship of vibration MOE (MOE$_v$) to static MOE (MOE$_s$) for jack pine logs (1 lb/in^2 = 6.9 kPa).

greater scatter of points about the regression line than had occurred in MOE$_v$. In an effort to obtain a better prediction model for MOE of logs, a multivariable regression model relating MOE$_s$ to MOE$_{sw}$ and diameter-to-length ratio was developed. The mathematical regression models used in this analysis were assumed to be of the following form:

$$y = ax_1^b x_2^c \qquad (8)$$

where y is modulus of elasticity being estimated; a, b, c are empirical constants; x_1 is nondestructive parameter MOE$_{sw}$; and x_2 is ratio of log diameter to log length.

The MOE of logs predicted by Equation (8) was then compared with the static bending MOE of logs. Results of the

regression analyses and values for the constants in the equations are summarized in Table 4.

The relationship between stress-wave-predicted MOE using the developed multivariable model and the static MOE of logs is shown in Figures 16 and 17. Figures 16 and 17 indicate that a strong relationship existed between stress-wave-predicted MOE and static MOE. Compared with the univariable linear regression model, the multivariable models showed significantly better correlation. The correlation coefficient r increased from 0.87 (red pine) and 0.77 (jack pine) for the univariable model to 0.95 (red pine) and 0.86 (jack pine) for the multivariable model. This showed that the diameter-to-length ratio had an interactive effect that contributed significantly when used in conjunction with MOE$_{sw}$.
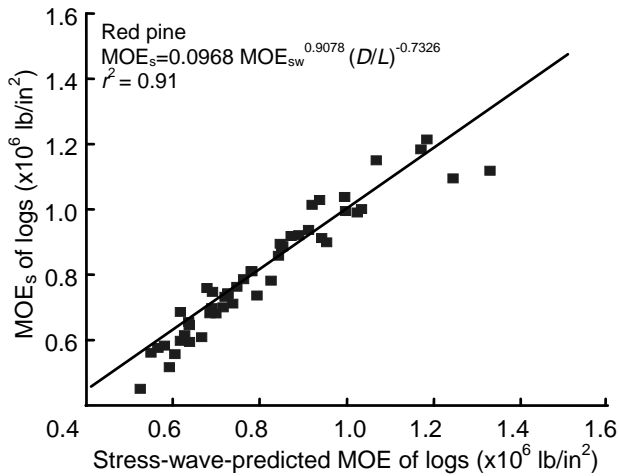
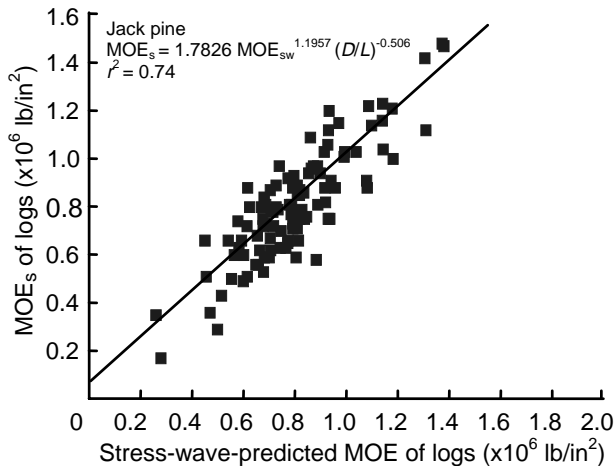**Figure 16—Relationship of stress-wave-predicted MOE to static MOE (MOE_s) for red pine logs (1 lb/in^2 = 6.9 kPa).**

Figure 17—Relationship of stress-wave-predicted MOE to static MOE (MOE_s) for jack pine logs (1 lb/in^2 = 6.9 kPa).

Flexural Stiffness Relationships

Of the parameters that can be measured nondestructively (density, appearance, MOE, and stiffness), stiffness is used most frequently to predict the strength of wood materials. Therefore, it is important to know the relationships between the stiffness determined by the three techniques covered in this report.

Flexural stiffness is expressed as the product of the moment of inertia (*I*) and MOE in bending. For logs, the moment of inertia is given by

$$I = \frac{\pi D^4}{64} \tag{9}$$

where *D* is the average diameter of a log (in. (m)).

When the MOE of logs determined by these techniques is known, the flexural stiffness of logs can be easily calculated.

The relationships between various log stiffness (stress wave *EI*, vibration *EI*, and static *EI*) are shown in Table 5 and Figures 18 to 20. Red pine and jack pine logs showed no distinction in the stiffness relationships. Therefore, we combined these two species and treated them as a single population.

The results revealed that the correlations between these nondestructively determined stiffness values were extraordinarily strong. In Figures 18 and 19, the static *EI* and vibration *EI* were plotted as a function of stress wave *EI*. Compared with the MOE relationships, the stress wave technique and the transverse vibration and static bending techniques were significantly better correlated in terms of flexural stiffness. Regression analyses indicated that a second-order polynomial regression model ($y = a + bx + cx^2$) could best

Table 5—Results of regression analyses of flexural stiffness (*EI*) of logs of both species determined by different techniques[a]

Regression model		$y = a + bx + cx^2$ or $y = a + bx$						
y	*x*	*a*	*b*	*c*	r^2	*r*	S_{yx}	*F*-test[b]
EI_s	EI_{sw}	11.6810	0.7530	−0.0003	0.94	0.97	23.37	2,246***
EI_v	EI_{sw}	5.1330	0.9035	−0.0005	0.97	0.98	18.03	1,226***
EI_s	EI_v	−1.8262	0.9434	—	0.94	0.97	22.83	2,575***

[a]EI_{sw}, stiffness of log determined by longitudinal stress wave technique; EI_v, stiffness of log determined by transverse vibration technique; EI_s, stiffness of log determined by general static bending; r^2, coefficient of determination; *r*, correlation coefficient, S_{yx}, standard error of estimate.
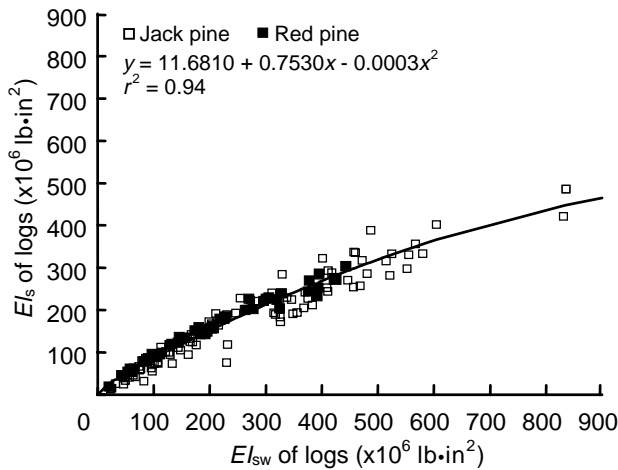[b]***Highly significant (0.01 confidence level).

Figure 18—Stress wave flexural stiffness (EI) versus static EI (1 lb·in^2 = 0.00287 N·m).
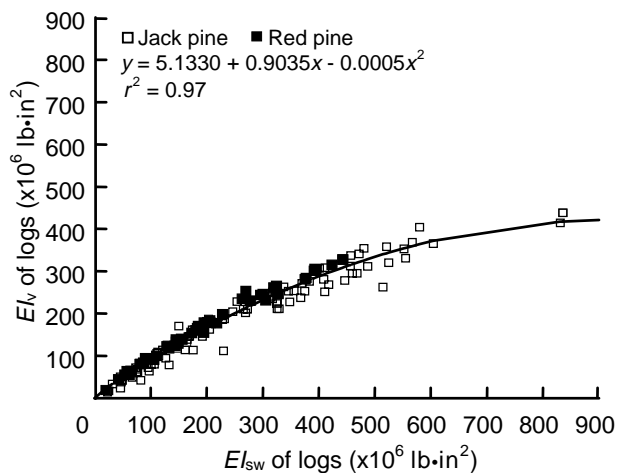
Figure 19—Stress wave flexural stiffness (EI) versus vibration EI (1 lb·in^2 = 0.00287 N·m).
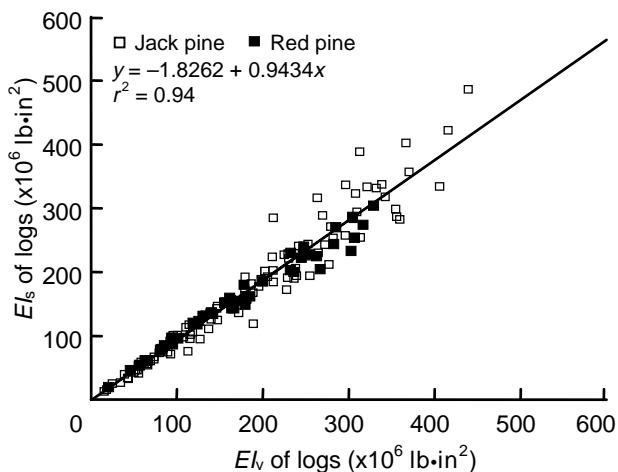
Figure 20—Vibration flexural stiffness (EI) versus static EI (1 lb·in^2 = 0.00287 N·m).

fit the experimental data. The correlation coefficients were 0.97 (stress wave EI versus static EI) and 0.98 (stress wave EI versus vibration EI), respectively. In other words, the developed regression models accounted for 97% and 94% of observed behavior.

Figure 20 shows the relationship between flexural stiffness of logs measured by transverse vibration and static bending techniques. Just as in the case of the MOE relationship, a linear regression model was found to be the best fitting function to the experimental data. The correlation coefficient was 0.97, indicating that 94% of observed behavior has been accounted for.

Conclusions

The results of these experiments show that small-diameter red pine and jack pine logs can be successfully evaluated by longitudinal stress wave, transverse vibration, or static bending techniques. The dynamic MOE (MOE$_{sw}$ and MOE$_v$) of logs was found to be well correlated with the static MOE for both species.

The experimental results demonstrated that the stress wave technique was sensitive to the geometrical imperfections of logs. Round and straight logs produced better correlation between MOE$_{sw}$ and MOE$_s$ than did logs that were not round or straight. More importantly, the diameter-to-length ratio D/L had an interactive effect that contributed significantly when used in conjunction with MOE$_{sw}$. The developed multivariable model relating MOE$_s$ to MOE$_{sw}$ and diameter-to-length ratio was found to be a better predictor for static MOE of logs. This could allow for the prediction of static bending properties of logs using stress wave techniques at levels of accuracy previously considered unattainable.

The results from transverse vibration tests demonstrated that a significant improvement in predicting MOE of logs was achieved compared with the results from stress wave tests. In regard to the relationship of MOE$_v$ and MOE$_s$, transverse vibrational parameters were found to be less sensitive to the geometrical imperfections of logs than were the stress wave parameters. Red pine and jack pine logs therefore could be combined and represented as a single population in the prediction model.

Strong relationships were found between the various nondestructively determined log stiffnesses. Compared with the MOE relationships, the stress wave technique and the transverse vibration and static bending techniques were significantly better correlated in terms of flexural stiffness.

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