Probabilistic Forecast Distribution Verification Primer

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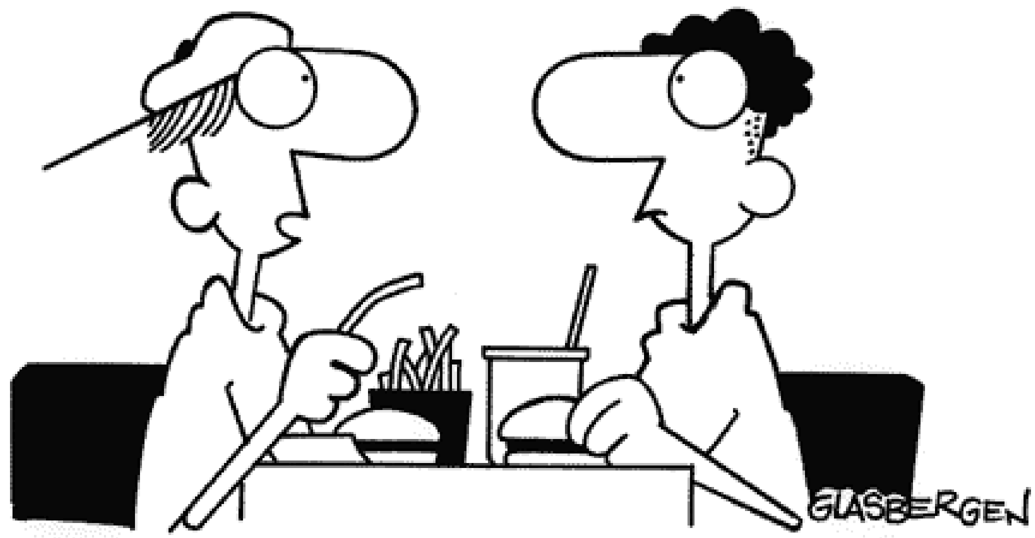


"I forgot to make a back-up copy of my brain, so everything I learned last semester was lost."

What is a Probabilistic Forecast?

A probabilistic forecast forecasts a probability distribution over a range of values. Simply put, rather than forecasting a specific value, forecasted probabilities are assigned to each particular value or range of values.

Example 1: A river flow forecast might be:

0-150 cfs	10%
150-300 cfs	40%
300-450 cfs	25%
450-600 cfs	15%
600-750 cfs	10%

What does a probabilistic forecast look like?

Two common methods for displaying a probability forecast are (1) Probability Density Function (PDF) and (2) Cumulative Distribution Function (CDF).

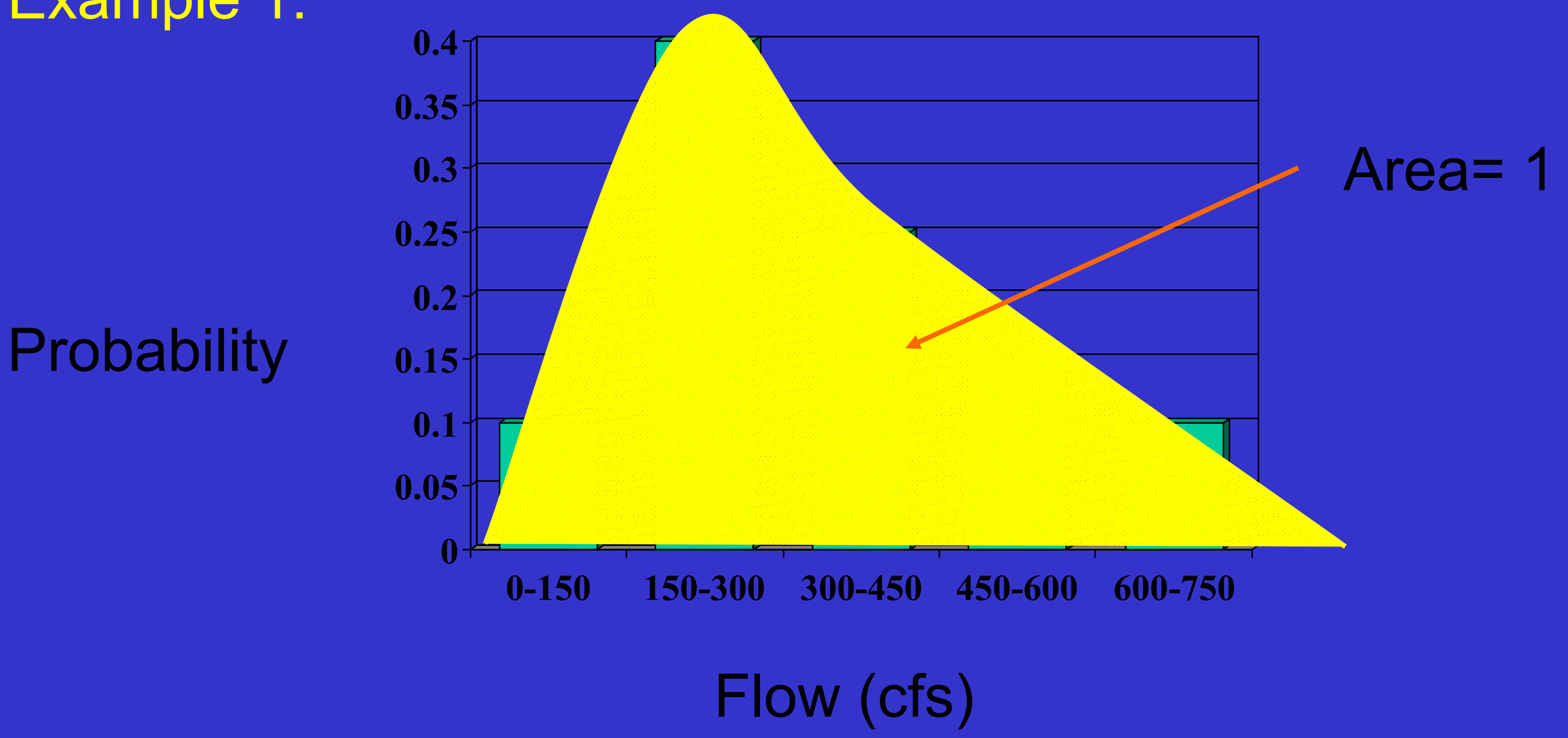
A single probabilistic forecast may be represented by either of these methods. The observation corresponding to one particular forecast will be a single value – not a probability distribution. Still a distribution of observed data may be created by using the history of observations as data points.

A large number of probabilistic forecasts may be represented by an average probability distribution and displayed as either a PDF or CDF. Similarly, multiple observations may be taken as a probability distribution and displayed as either a PDF or CDF.

What is a Probability Density Function (PDF)?

A PDF is basically a histogram, possibly smoothed, normalized such that the area under the curve is unity. Forecast values are on the x-axis, probability on the y-axis.

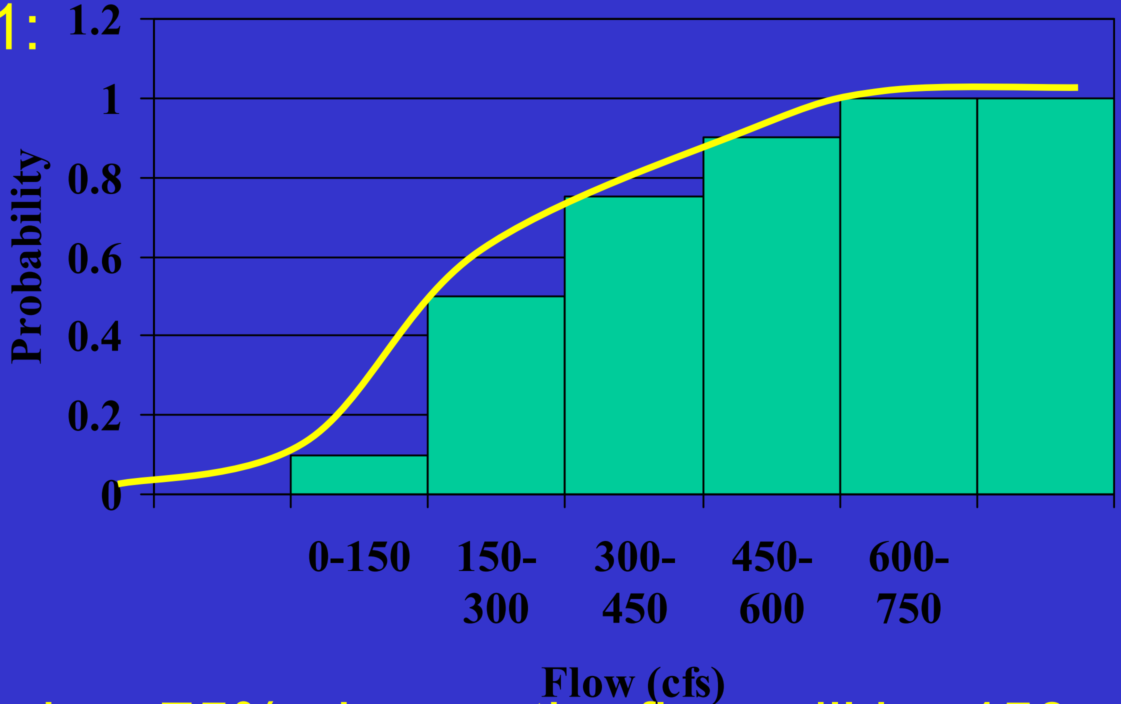
Example: Using the sample probabilistic forecast from Example 1:



What is a Cumulative Distribution Function (CDF)?

A CDF is related to the PDF. The CDF is the probability that the observation will be *less than* a value for every value on the x-axis. Probability is on the y-axis. The CDF is the inegral of the PDF.

Example 2: Using the sample probabilistic forecast from Example 1:



Here there is a 75% chance the flow will be 450 cfs or less.

How are probabilistic forecasts constructed?

Probabilistic forecasts are usually constructed with ensembles. That is, two or more forecasts for the same quantity may be combined into a single probabilistic forecast. This is accomplished by fitting a continuous distribution (i.e. normal , log-normal or empirical or…) to the discreet data points.

How are probabilistic forecasts constructed? (con't)

For example, the hypothetical forecast given in the previous slides may have been created from the following ensembles:

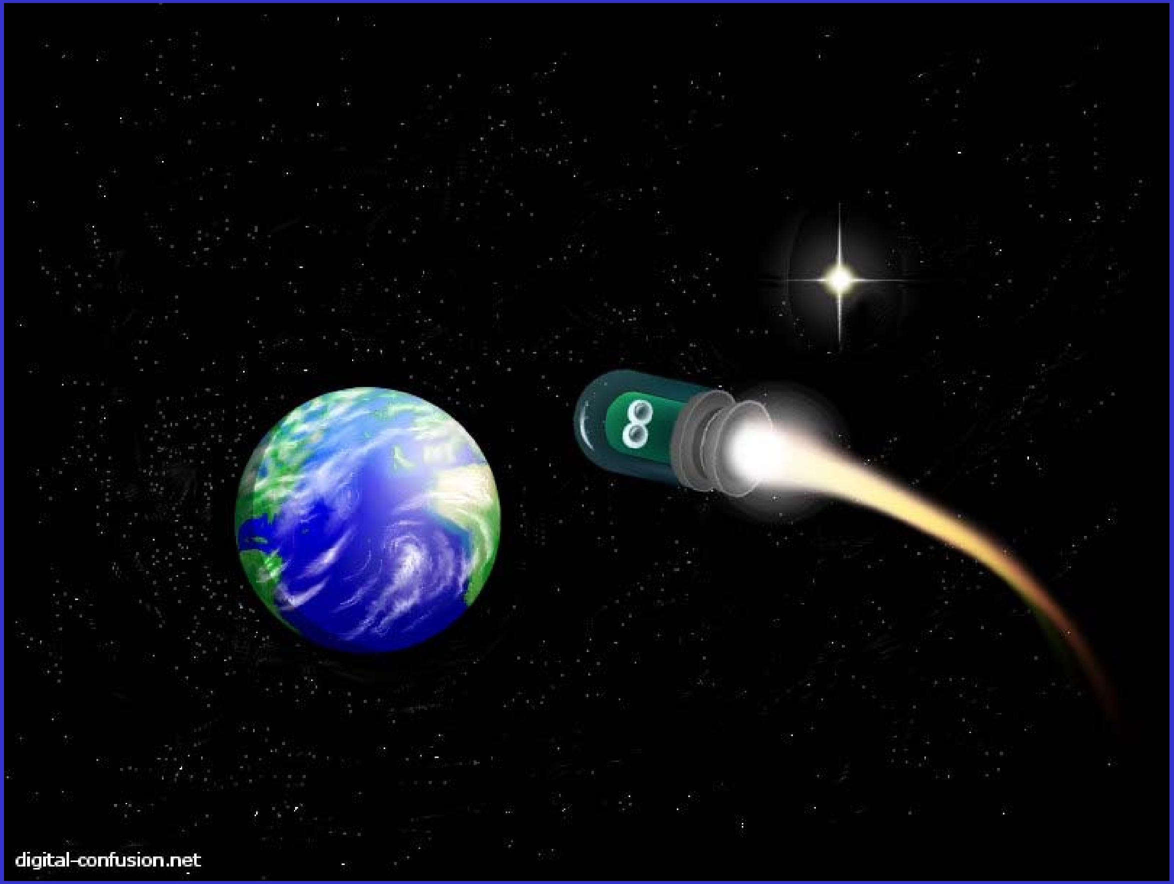
Category	Forecast(s)	Number in cat	% of tot
0-150 cfs	123, 144 cfs	2	10
150-300	166, 178, 202, 248,		
	249, 270, 279, 290	8	40
300-450	302, 350, 378, 400, 433	5	25
450-600	490, 505, 545	3	15
600-750	603, 625	2	10

How do I choose a continuous distribution function?

Choosing a particular distribution to fit a data set takes both art and science. There is a large body of knowledge on this subject which is outside the scope of this primer. Whatever distribution is chosen should fit the data well and represent whatever else may be known about the entire distribution. For the purposes of this primer, we will use only empirical distributions.

An EMPIRICAL DISRIBUTION simply gives equal weight to each ensemble member or data point and connects them to form the distribution.

OK, I made a probabilistic forecast… How can I tell if its any good?



"Probabilistic forecasting means never having to say I'm sorry." – Craig Peterson - CBRFC

Probabilistic Forecast Verification 101

Caveats:

(1) A large (> ~20) number of *independent* observations are required.

(1) No “one size fits all” measure of success.

(1) Concepts are similar to deterministic forecast evaluation; However the application of the concepts is different.

Talagrand Diagram

A Talagrand Diagram is an excellent tool to detect systematic flaws of an ensemble prediction system. It indicates how well the probability distribution has been sampled. It does not indicate that the ensemble will be of practical use.

It allows a check where the verifying analysis usually falls with respect to the ensemble data (arranged in increasing order at each point).

Sample Verification Data Set

For illustrative purposes, a small sample data set of ensemble forecasts was created. Each of the verification techniques will be applied to this dataset.

Four forecasts of peak flow (cfs) were made on May 14 for a two week window ending May 29 for a number of years. The four Sample Ensemble Members (E1 – E4) were ranked lowest to highest and correlated with the corresponding observation.

YEAR	E1	E2	E3	E4	OBS
1981	42	74	82	90	112
1982	65	143	223	227	206
1983	82	192	295	300	301
1984	211	397	514	544	516
1985	142	291	349	356	348
1986	114	277	351	356	98
1987	98	170	204	205	156
1988	69	169	229	236	245
1989	94	219	267	270	233
1990	59	175	244	250	248
1991	108	189	227	228	227
1992	94	135	156	158	167

ALRIGHT RUTH, I ABOUT GOT THIS ONE RENORMALIZED.

Talagrand Diagram Description

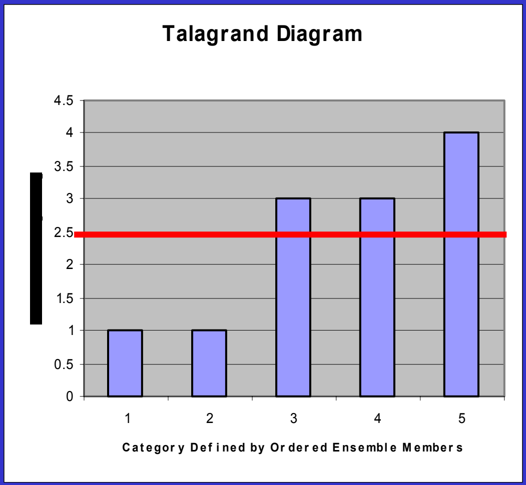
Four Sample Ensemble Members (E1 – E4) Ranked Lowest to Highest For Daily Flow

Produced From Reforecasts Using Carryover In Each Year

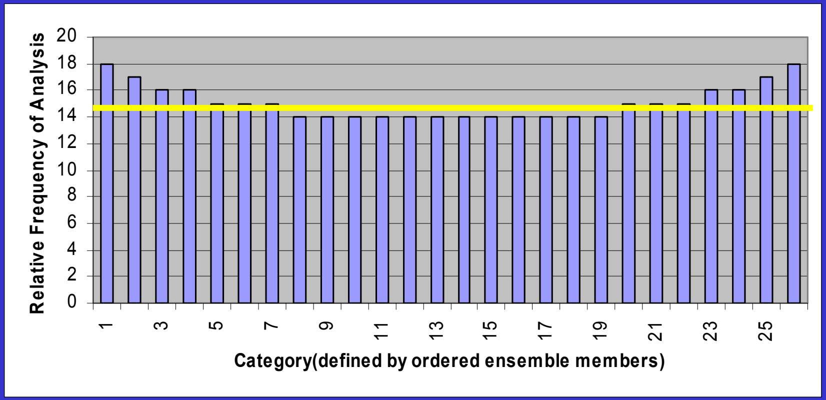
Bin1 Bin2 Bin3 Bin4 Bin5

YEAR	E1	E2	E3	E4	OBS	Bin #
1981	42	74	82	90	112	5
1982	65	143	223	227	206	3
1983	82	192	295	300	301	5
1984	211	397	514	544	516	4
1985	142	291	349	356	348	3
1986	114	277	351	356	98	1
1987	98	170	204	205	156	2
1988	69	169	229	236	245	5
1989	94	219	267	270	233	3
1990	59	175	244	250	248	4
1991	108	189	227	228	227	4
1992	94	135	156	158	167	5

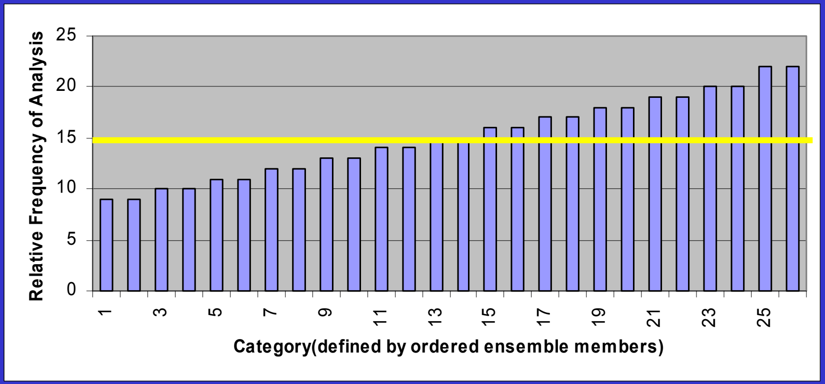
Bin #	Tally
1	1
2	1
3	3
4	3
5	4



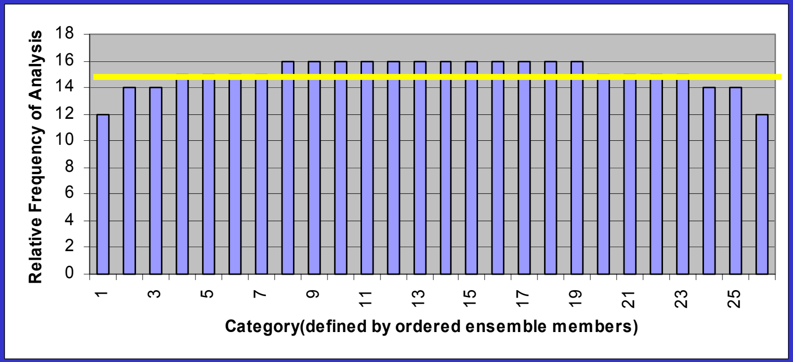
Talagrand Diagram Examples



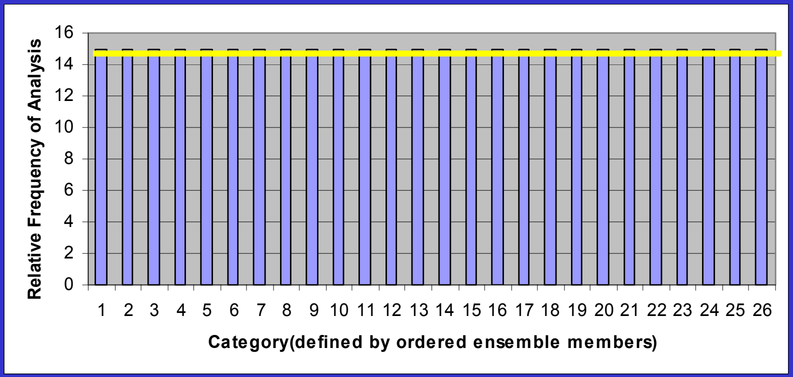
Talagrand Diagram (Rank Histogram)
Example: “U-Shaped”
Indicates Ensemble Spread Too Small



Talagrand Diagram (Rank Histogram)
Example: “L-Shaped”
Indicates Over or Under Forecasting Bias



Talagrand Diagram (Rank Histogram)
Example: “N-Shaped” (domed shaped)
Indicates Ensemble Spread is Too Big



Talagrand Diagram (Rank Histogram)
Example: “Flat-Shaped”
Indicates Ensemble Distribution Has Been Sampled Well

Brier Score

"A number of scalar accuracy measures for verification of probabilistic forecasts of dichotomous events exist, but the most common is the Brier Score (BS). The BS is essentially the mean-squared error of the probability forecasts, considering that the observation is o=1 if the event occurs and o=0 if the event does not occur. The score averages the squared differences between pairs of forecast probabilities and the subsequent observations." (Wilkes, 1995)

$$BS = \frac{1}{n}\sum_{k=1}^{n}(y_k - o_k)^2$$

BS is bounded by 0 and 1. A forecast with BS=0 is perfect.

Brier Score

A key feature of the Brier Score is its application to dichotomous events; either the event happened or it didn't happen. Therefore it is necessary to define the event for verification. This could be a major weakness if the probabilistic forecast is being made for purposes beyond a simple percent "yes" and percent "no" for the particular event being verified.

As an example, we present probability above and below flood stage. We will assume the flood stage for the sample data set is 300 cfs.

Brier Score Example

Step 1: Compute probability of flood / no flood based on flood flow of 300 cfs.

YEAR	E1	E2	E3	E4	OBS	P(flood)	P(no flood)
1981	42	74	82	90	112	0	1
1982	65	143	223	227	206	0	1
1983	82	192	295	300	301	0.25	0.75
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	516	0.75	0.25
1985	142	291	349	356	348	0.5	0.5
1986	114	277	351	356	98	0.5	0.5
1987	98	170	204	205	156	0	1
1988	69	169	229	236	245	0	1
1989	94	219	267	270	233	0	1
1990	59	175	244	250	248	0	1
1991	108	189	227	228	227	0	1
1992	94	135	156	158	167	0	1

In 1984, 3 of 4 forecasts were for above flood flow. Therefore the probability of flooding is ¾ or 0.75 whereas the probability of no flooding is 1-0.75 = 0.25

Brier Score Example

Step 2: Determine whether event (flooding) occurred (1) or not (0).

YEAR	OBS	P(flood)	P(no flood)	Flooding?
1981	112	0	1	0
1982	206	0	1	0
1983	301	0.25	0.75	1
1984	516	0.75	0.25	1
1985	348	0.5	0.5	1
1986	98	0.5	0.5	0
1987	156	0	1	0
1988	245	0	1	0
1989	233	0	1	0
1990	248	0	1	0
1991	227	0	1	0
1992	167	0	1	0

Brier Score Example

Step 3: Calculate $(y - o)^2$

YEAR	P(flood)	Flooding?	$(y - o)^2$
1981	0	0	0
1982	0	0	0
1983	0.25	1	0.56
1984	0.75	1	0.06
1985	0.5	1	0.25
1986	0.5	0	0.25
1987	0	0	0
1988	0	0	0
1989	0	0	0
1990	0	0	0
1991	0	0	0
1992	0	0	0

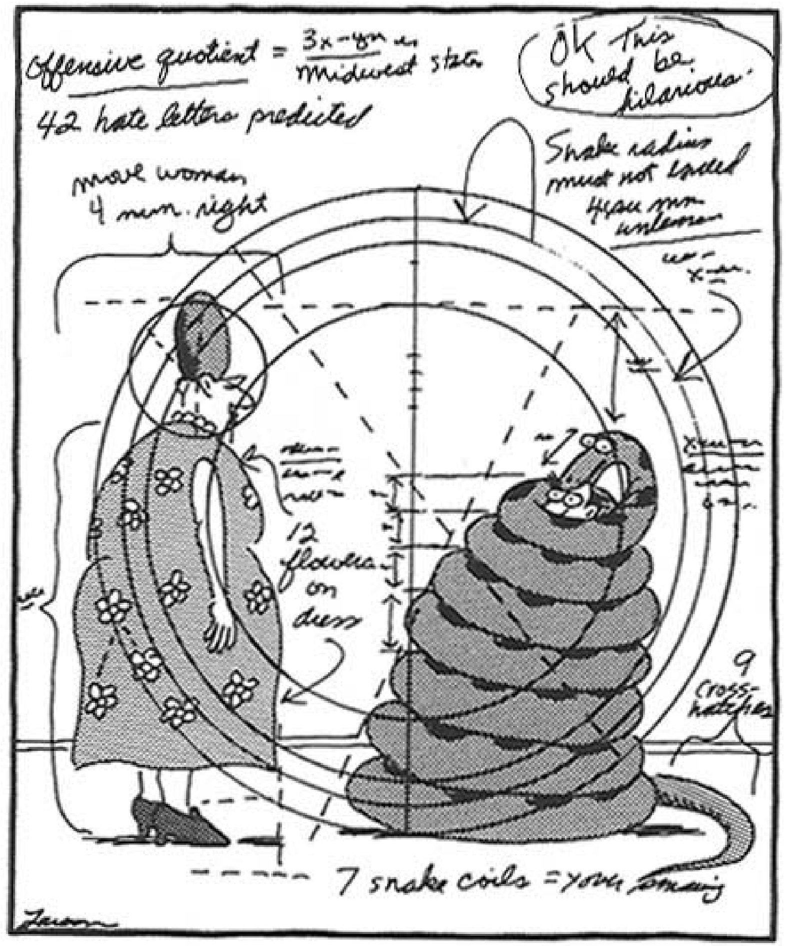
Brier Score Example

Step 4: Calculate BS = 1/n SUM[$(y - o)^2$]

YEAR	P(flood)	Flooding?	$(y - o)^2$
1981	0	0	0
1982	0	0	0
1983	0.25	1	0.56
1984	0.75	1	0.06
1985	0.5	1	0.25
1986	0.5	0	0.25
1987	0	0	0
1988	0	0	0
1989	0	0	0
1990	0	0	0
1991	0	0	0
1992	0	0	0

BS = 1/12 * (0.56+0.06+0.25+0.25+…)

BS = 0.093



Revealing some of the mathematical computations every cartoonist must know.

Ranked Probability Score (RPS)

The Ranked Probability Score (RPS) is used to assess the overall forecast performance of the probabilistic forecasts.

Similar to Brier Score but includes more than two categories.

A perfect forecast would result in a RPS of zero.

Gives credit for forecasts close to observation…
Penalizes forecasts further from the observation.

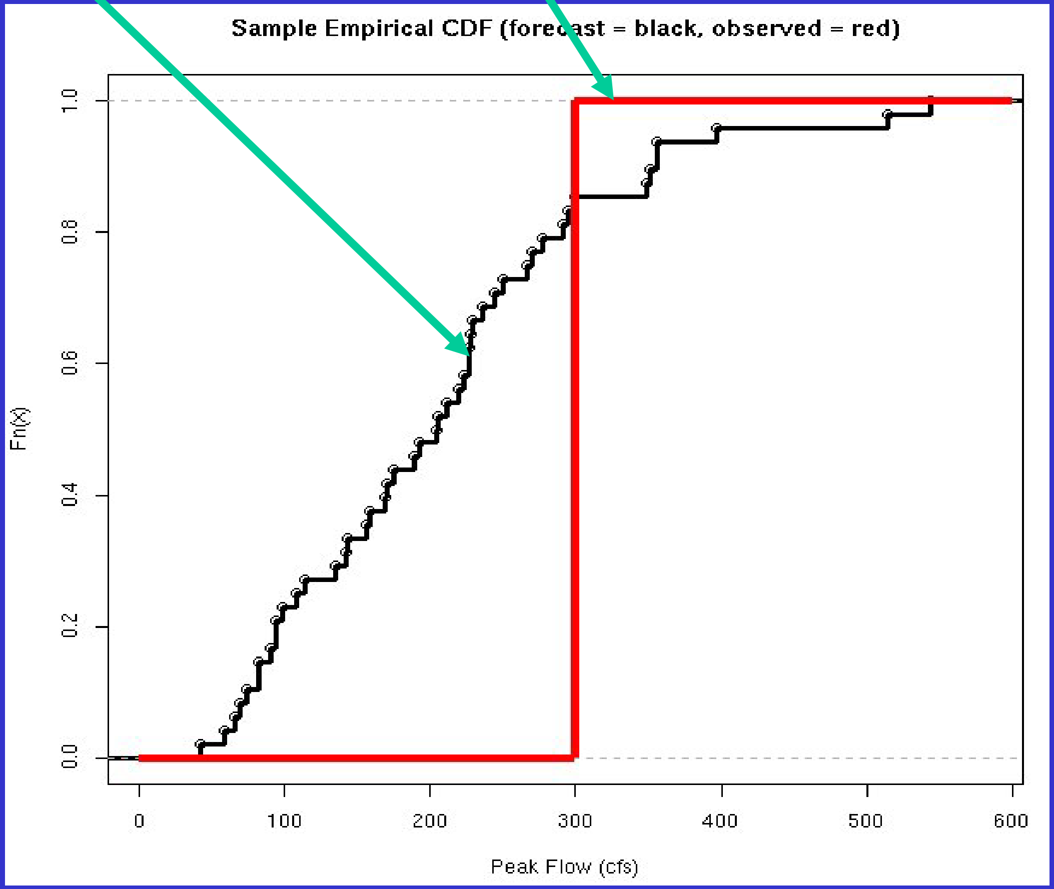
Looks at the entire distribution (all traces).

RPS Formulation

Goal: Compare forecast CDF to observed CDF

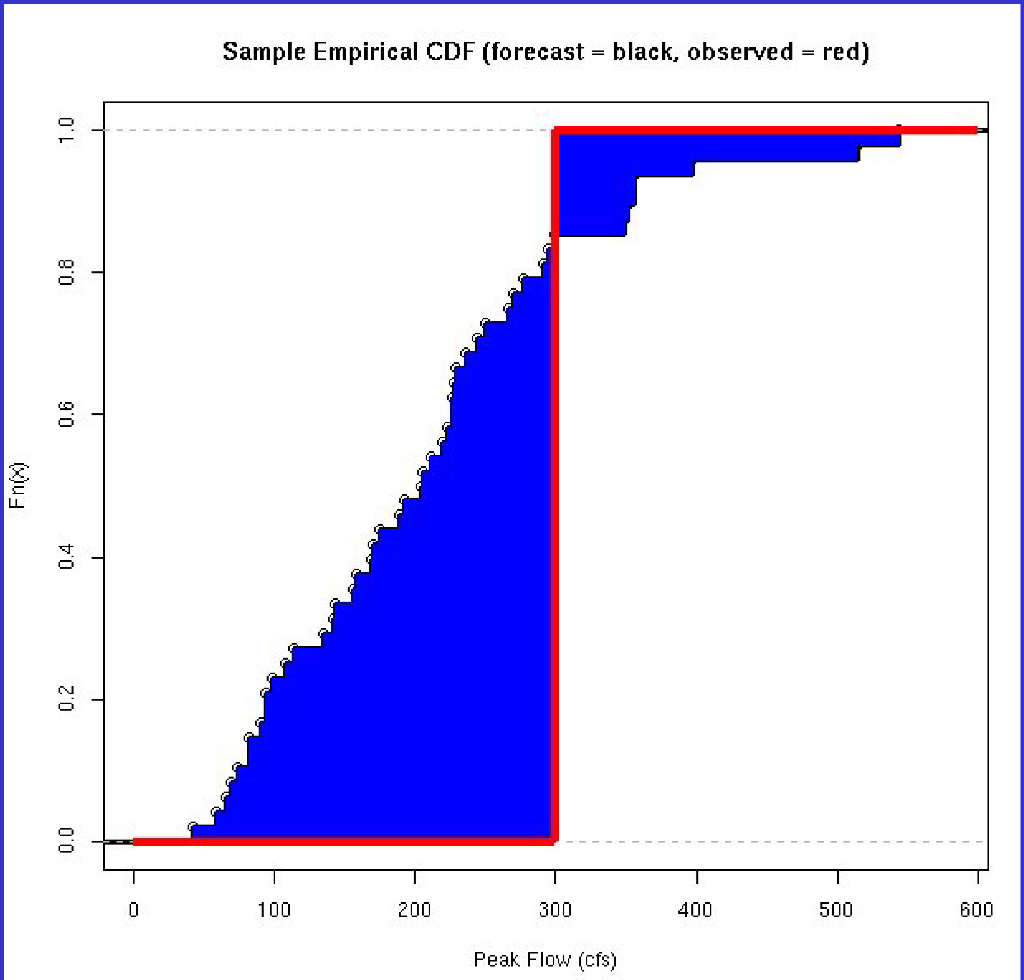
Notes:

1. Here an empirical distribution is assumed (not necessary).
2. Observation is one value, in this case 3.0.



RPS Formulation

Graphically, the RPS is this area:



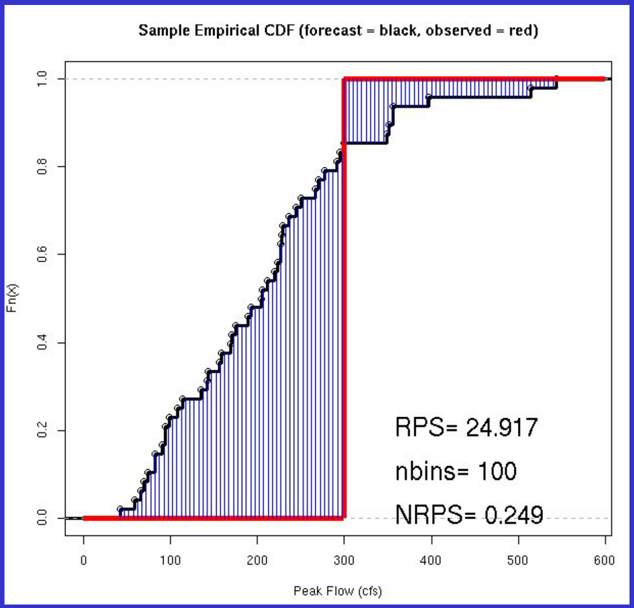
RPS Formulation

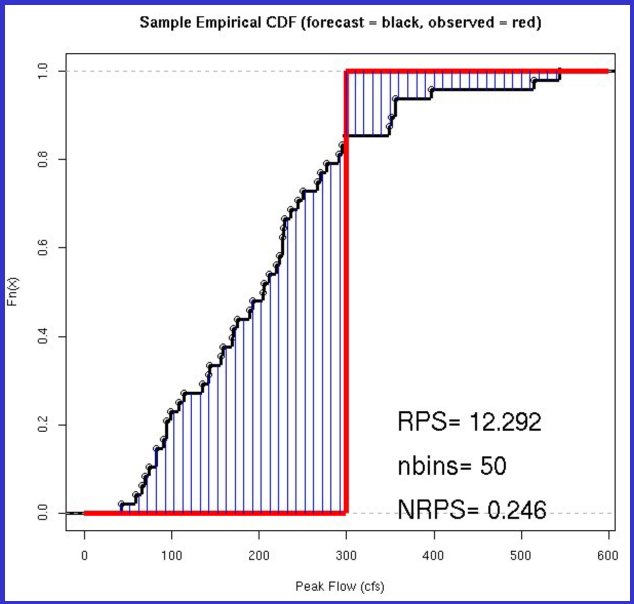
Mathematically, RPS is given by:

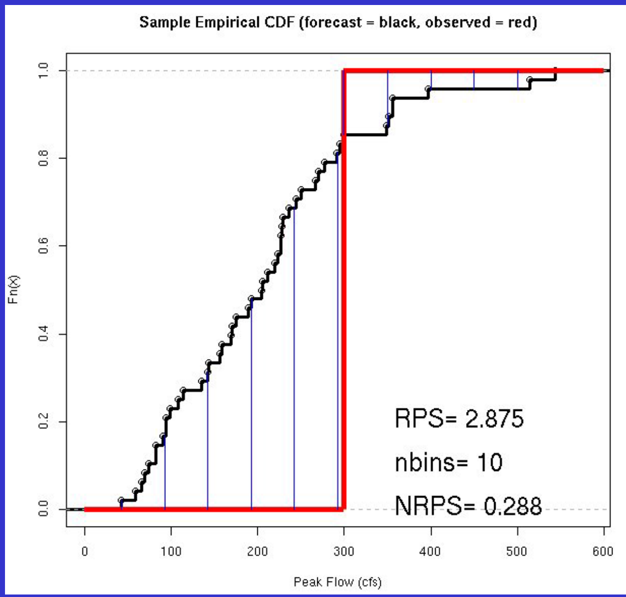
$$RPS = \sum_{i=bin\#1}^{bin\#n} [P(forecast < i) - P(observed < i)]^2$$

Where the summation indices are over n bins whose number and spacing are determined by the user. In order to best approximate the area between the forecast and observed CDFs, a large number of bins should be chosen. The larger the number of bins the more computationally intense the calculation becomes.

RPS Formulation







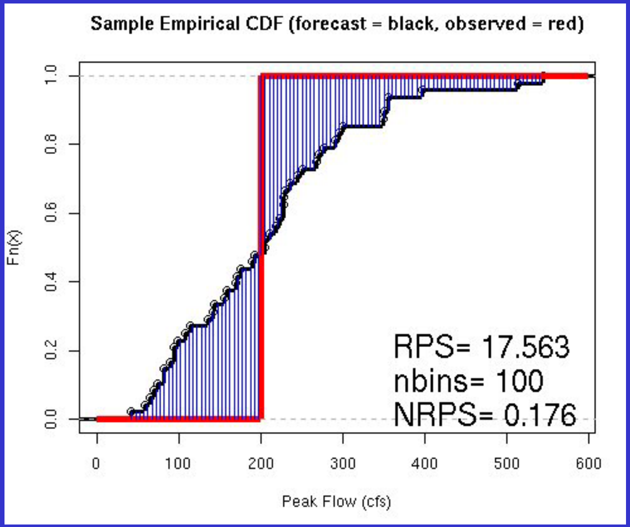
RPS is hugely dependent on bin choice. Here three different bin spacing are shown along with the calculated RPS.

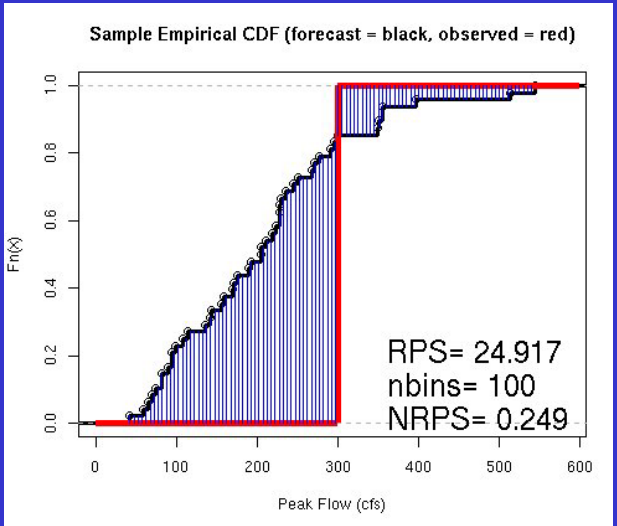
A “normalized RPS” may be defined as

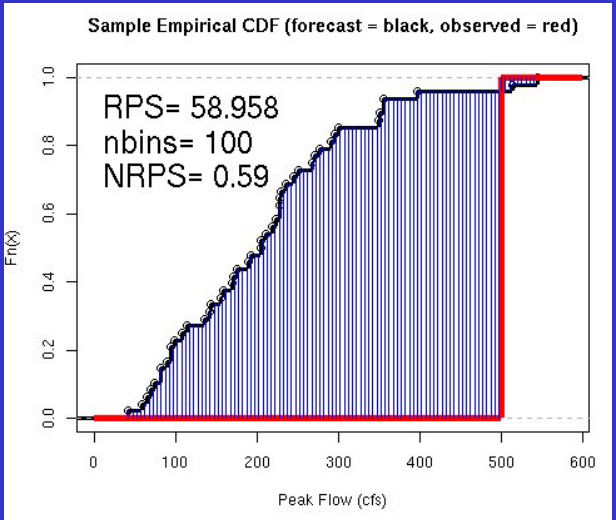
$$NRPS = RPS / (\#\ of\ bins)$$

NRPS allows comparison between RPSs calculated with different numbers of bins and is bounded by 0 and 1. Again a score of zero indicates a perfect forecast.

RPS Formulation



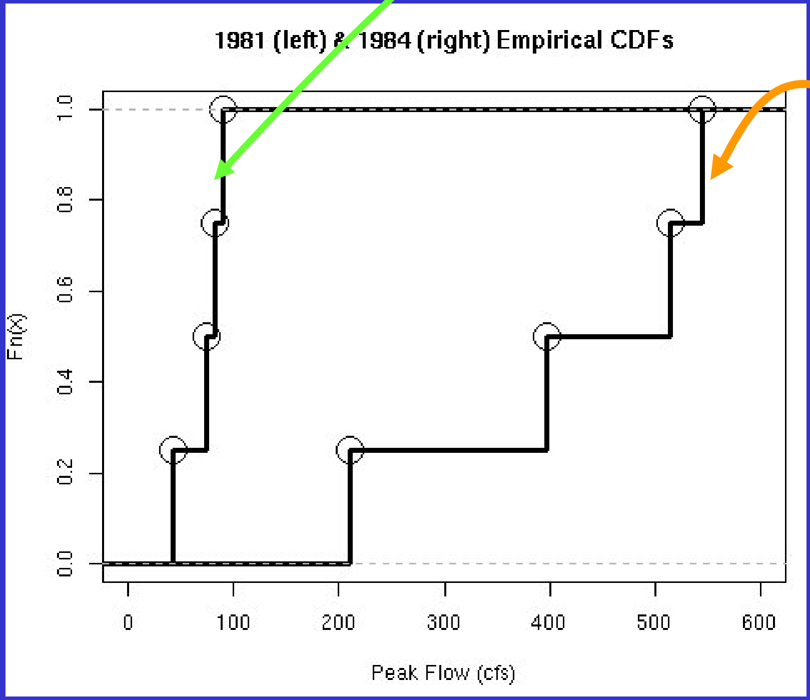




RPS is *sensitive to distance*. Here RPS is calculated with the same forecast CDF against 3 different observations. The smaller the blue area, the “better” the forecast is and the smaller the RPS is.

RPS with sample data set

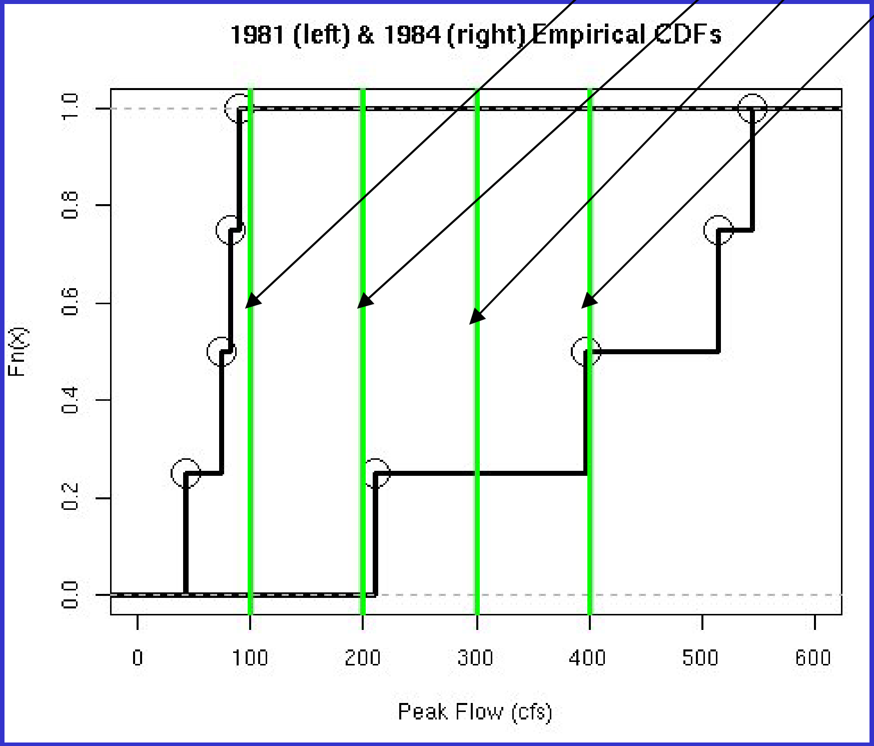
1. Assume an empirical CDF; For example, 1981 and 1984 are shown below. Note the Y values are simply 0/4,1/4,2/4,3/4, and 4/4.



YEAR	E1	E2	E3	E4	OBS
1981					112
1982	65	143	223	227	206
1983	82	192	295	300	301
1984					516
1985	142	291	349	356	348
1986	114	277	351	356	98
1987	98	170	204	205	156
1988	69	169	229	236	245
1989	94	219	267	270	233
1990	59	175	244	250	248
1991	108	189	227	228	227
1992	94	135	156	158	167

RPS with sample data set

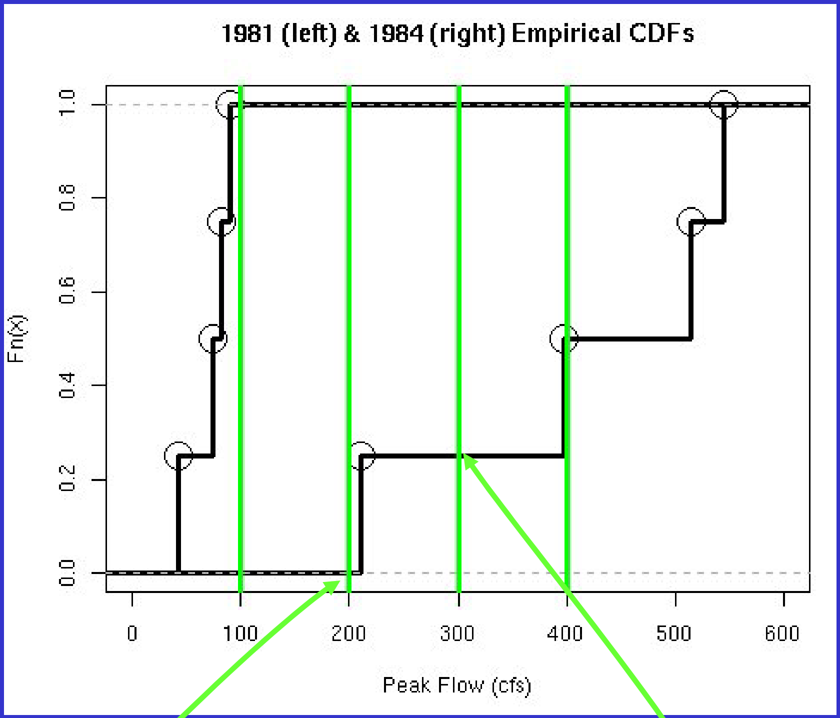
2. Choose number of bins and bin spacing. For Simplicity, let's choose four bins and set them to be: (100,200,300,400)



YEAR	E1	E2	E3	E4	OBS
1981	42	74	82	90	1.12
1982	65	143	223	227	2.06
1983	82	192	295	300	3.01
1984	211	397	514	544	5.16
1985	142	291	349	356	3.48
1986	114	277	351	356	0.98
1987	98	170	204	205	1.56
1988	69	169	229	236	2.45
1989	94	219	267	270	2.33
1990	59	175	244	250	2.48
1991	108	189	227	228	2.27
1992	94	135	156	158	1.67

RPS with sample data set

3. Pick off the probability that the volume will be less than the break point at each bin (i.e. non-exceedence probability). For example, 1984 would be:



Peak	Probability
100	0
200	0
300	0.25
400	0.5

YEAR	E1	E2	E3	E4	OBS
1981	42	74	82	90	112
1982	65	143	223	227	206
1983	82	192	295	300	301
1984	211	397	514	544	516
1985	142	291	349	356	348
1986	114	277	351	356	98
1987	98	170	204	205	156
1988	69	169	229	236	245
1989	94	219	267	270	233
1990	59	175	244	250	248
1991	108	189	227	228	227
1992	94	135	156	158	167

RPS with sample data set

3 (con't). Pick off probabilities at each bin. All years:

						Probability(forecast peak < …)			
YEAR	E1	E2	E3	E4	OBS	100 cfs	200 cfs	300 cfs	400 cfs
1981	42	74	82	90	112	1.0	1.0	1.0	1.0
1982	65	143	223	227	206	0.25	0.5	1.0	1.0
1983	82	192	295	300	301	0.25	0.5	1.0	1.0
1984	211	397	514	544	516	0.0	0.0	0.25	0.5
1985	142	291	349	356	348	0.0	0.25	0.5	1.0
1986	114	277	351	356	98	0.0	0.25	0.5	1.0
1987	98	170	204	205	156	0.25	0.5	1.0	1.0
1988	69	169	229	236	245	0.25	0.5	1.0	1.0
1989	94	219	267	270	233	0.25	0.25	1.0	1.0
1990	59	175	244	250	248	0.25	0.5	1.0	1.0
1991	108	189	227	228	227	0.0	0.5	1.0	1.0
1992	94	135	156	158	167	0.25	1.0	1.0	1.0

RPS with sample data set

4. Compute Probabilities for observations. Since there is only one observation the probability will be either 0 or 1.

		P(forecast peak < …)				P(observed peak < …)			
YEAR	OBS	100 cfs	200	300	400	100 cfs	200	300	400
1981	112	1.0	1.0	1.0	1.0	0.0	1.0	1.0	1.0
1982	206	0.25	0.5	1.0	1.0	0.0	0.0	1.0	1.0
1983	301	0.25	0.5	1.0	1.0	0.0	0.0	0.0	1.0
1984	516	0.0	0.0	0.25	0.5	0.0	0.0	0.0	0.0
1985	348	0.0	0.25	0.5	1.0	0.0	0.0	0.0	1.0
1986	98	0.0	0.25	0.5	1.0	1.0	1.0	1.0	1.0
1987	156	0.25	0.5	1.0	1.0	0.0	1.0	1.0	1.0
1988	245	0.25	0.5	1.0	1.0	0.0	0.0	1.0	1.0
1989	233	0.25	0.25	1.0	1.0	0.0	0.0	1.0	1.0
1990	248	0.25	0.5	1.0	1.0	0.0	0.0	1.0	1.0
1991	227	0.0	0.5	1.0	1.0	0.0	0.0	1.0	1.0
1992	167	0.25	1.0	1.0	1.0	0.0	1.0	1.0	1.0

RPS with sample data set

5. Compute the RPS. 1983 is done as an example…

$$RPS = \sum_{i=1}^{n} [P(forecast < i) - P(observed < i)]^2$$

	P(forecast peak < …)				P(observed peak < …)			
YEAR	100	200	300	400	100	200	300	400
1983	0.25	0.5	1.0	1.0	0.0	0.0	0.0	1.0

$$RPS = [0.25 - 0.0]^2 + [0.5 - 0.0]^2 + [1.0 - 0.0]^2 + [1.0 - 1.0]^2$$

$$= 1.31$$

RPS with sample data set

5 (con't). RPS computed for all years…

	P(forecast peak < …)				P(observed peak < …)				
YEAR	100	200	300	400	100	200	300	400	RPS
1981	1.0	1.0	1.0	1.0	0.0	1.0	1.0	1.0	1.0
1982	0.25	0.5	1.0	1.0	0.0	0.0	1.0	1.0	0.31
1983	0.25	0.5	1.0	1.0	0.0	0.0	0.0	1.0	1.31
1984	0.0	0.0	0.25	0.5	0.0	0.0	0.0	0.0	0.31
1985	0.0	0.25	0.5	1.0	0.0	0.0	0.0	1.0	0.31
1986	0.0	0.25	0.5	1.0	1.0	1.0	1.0	1.0	1.81
1987	0.25	0.5	1.0	1.0	0.0	1.0	1.0	1.0	0.31
1988	0.25	0.5	1.0	1.0	0.0	0.0	1.0	1.0	0.31
1989	0.25	0.25	1.0	1.0	0.0	0.0	1.0	1.0	0.13
1990	0.25	0.5	1.0	1.0	0.0	0.0	1.0	1.0	0.31
1991	0.0	0.5	1.0	1.0	0.0	0.0	1.0	1.0	0.25
1992	0.25	1.0	1.0	1.0	0.0	1.0	1.0	1.0	0.06

RPS with sample data set

6. Now you can make statements such as…

YEAR	RPS
1981	1.0
1982	0.31
1983	1.31
1984	0.31
1985	0.31
1986	1.81
1987	0.31
1988	0.31
1989	0.13
1990	0.31
1991	0.25
1992	0.06

1986 was worst forecast (RPS was highest)

1992 was best forecast (RPS lowest)

Etc.

Because the actual RPS value is difficult to evaluate independently, the use of the RPS in the absence of reference forecasts is limited to forecast comparison among different forecast locations. (Franz: Nov 2002)

Can be used to analyze regional consistency, i.e., possible need for recalibration.)

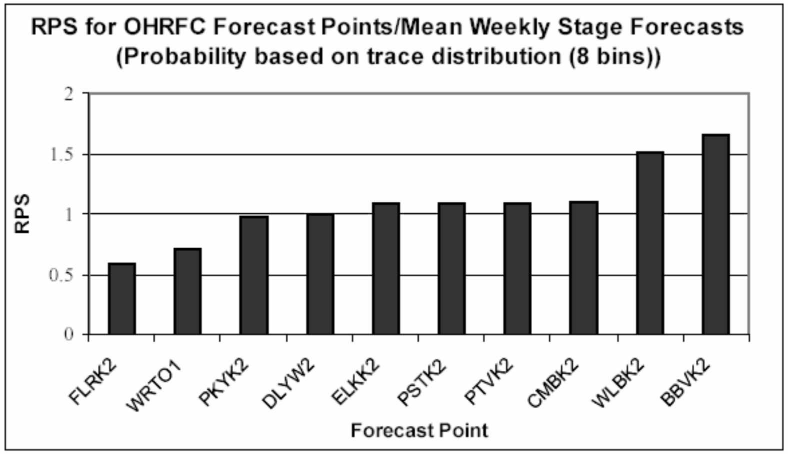


Figure 3: RPS analysis results for mean weekly stage forecasts.

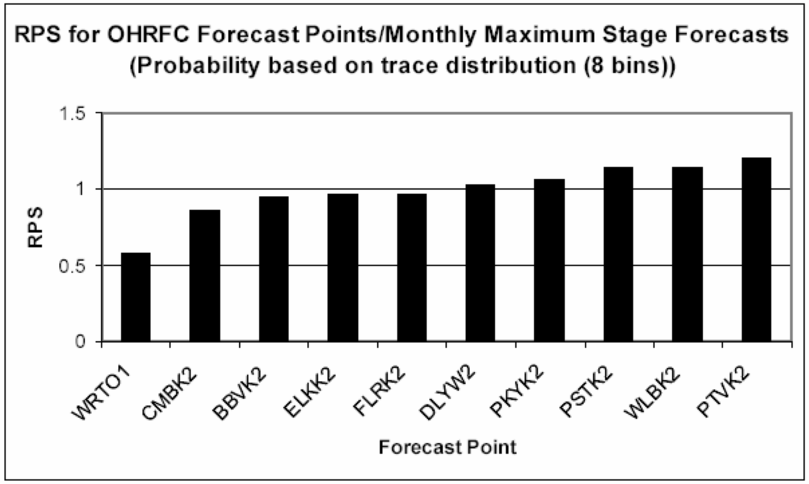
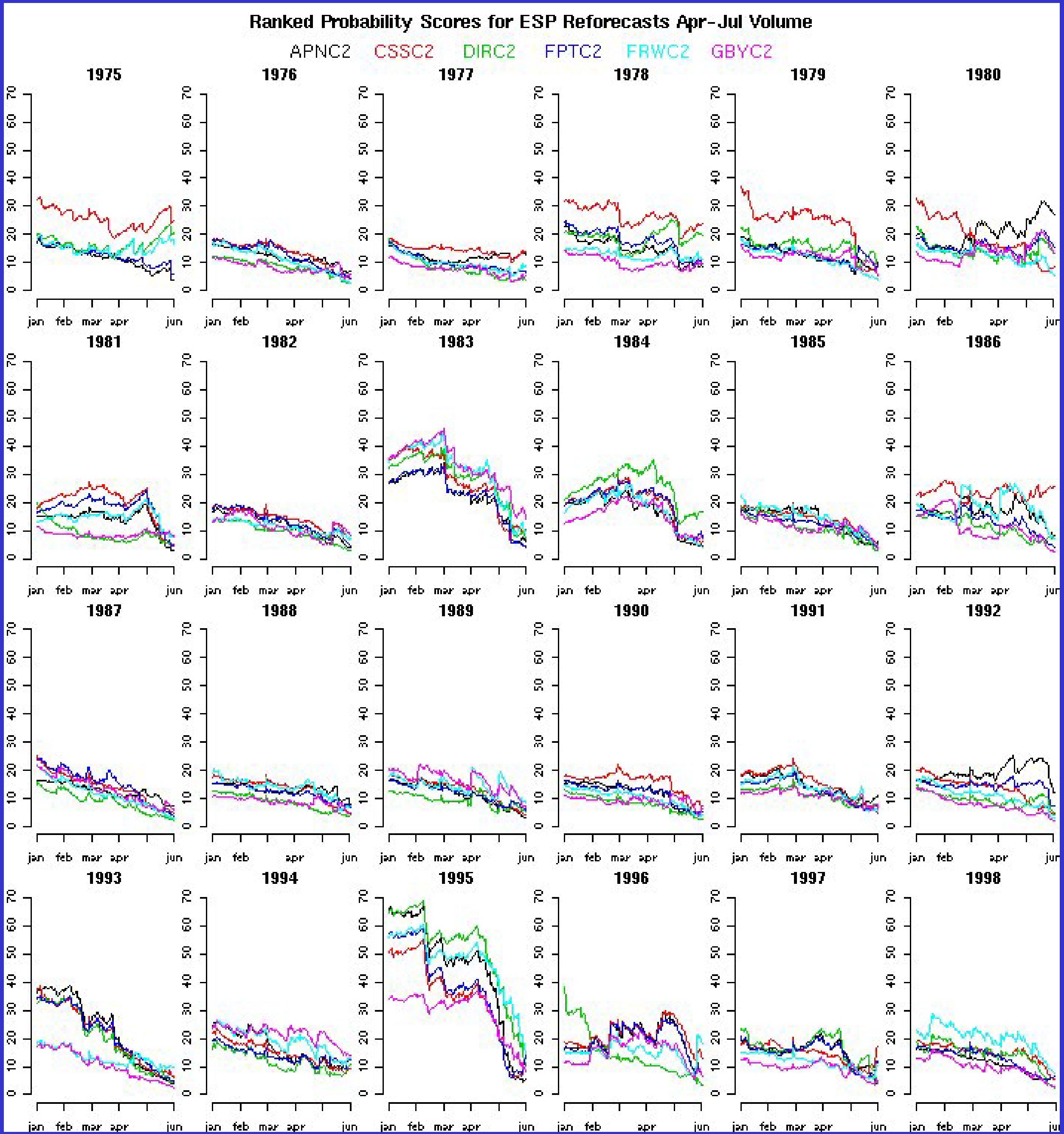
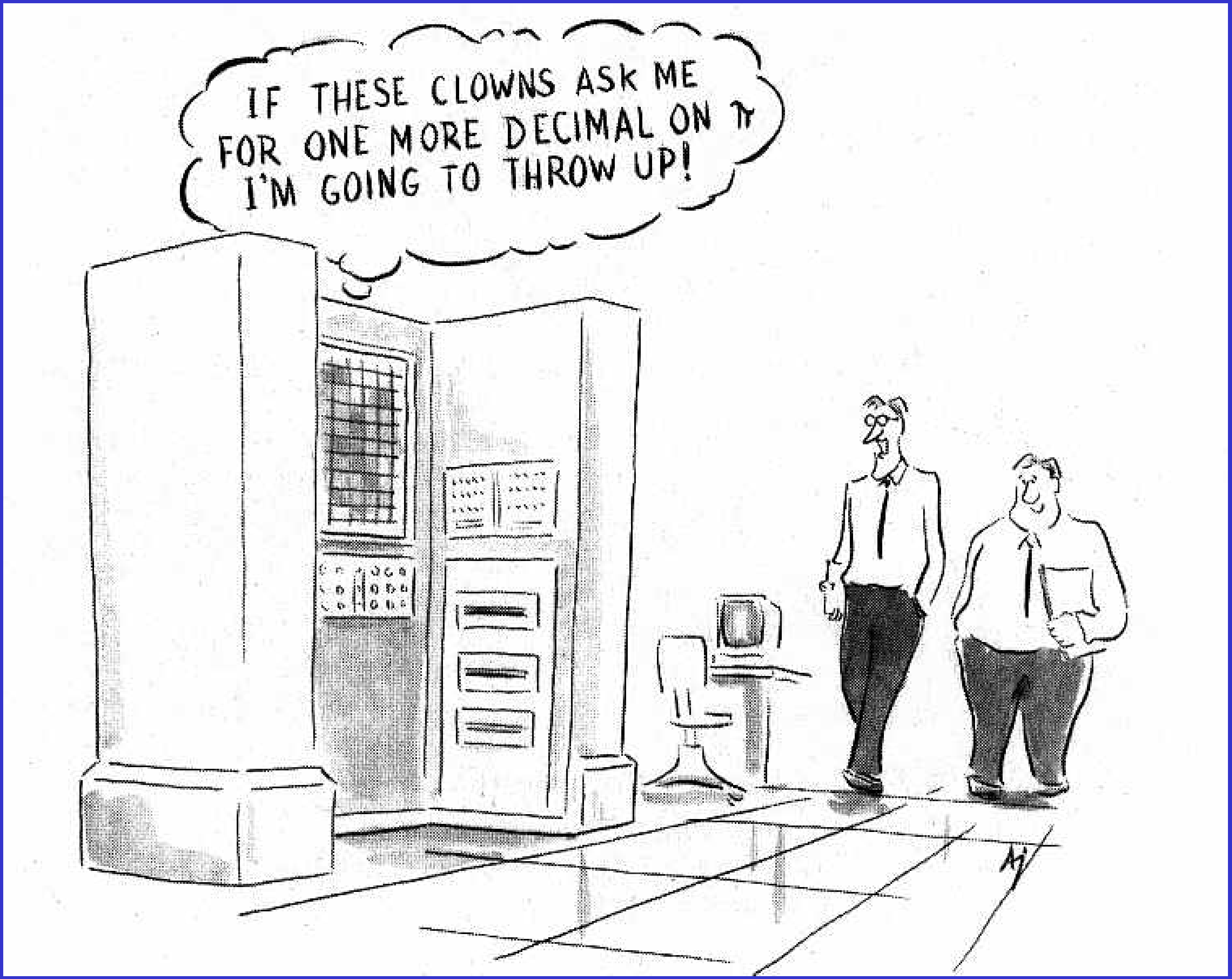


Figure 4: RPS analysis results for maximum monthly stage forecasts.

Credit: “Verification on NWS Probabilistic Hydrologic Forecasts” – Franz/Sorooshian – U. of AZ

RPS used to compare various basins. (Note RPS here was computed with 100 bins.)




IF THESE CLOWNS ASK ME FOR ONE MORE DECIMAL ON π I'M GOING TO THROW UP!

Ranked Probability Skill Score RPSS

Useful to compare the forecast of interest to a reference forecast, e.g., climatology.

It is expressed as a percent improvement, e.g., over climatology (or reference forecast).

Perfect score is 100%.

Negative score indicates forecasts performed worse than reference forecast.

Credit: Presentation-"Evaluation of NWS Ensemble Sreamflow Prediction" – Kristie Franz – U. of AZ

Ranked Probability Skill Score RPSS

$$RPSS = \frac{RPS_{f} - RPS_{cl}}{0 - RPS_{cl}} \times 100\%$$

RPS_{f} = Rank Probability Score (forecasts)

RPS_{cl} = Rank Probability Score (climatology)

RPS_f and RPS_{cl} must be calculated with the same bins!

RPSS with sample data set

1. Take the RPS vector calculated in the RPS sample section and call it Forecast RPS or RPS_{for}.

YEAR	RPS_{for}
1981	1.0
1982	0.31
1983	1.31
1984	0.31
1985	0.31
1986	1.81
1987	0.31
1988	0.31
1989	0.13
1990	0.31
1991	0.25
1992	0.06

RPSS with sample data set

2. Calculate a reference RPS vector. This may be a climatology RPS that used the climatological values as forecasts. Call it RPS_{clim}.

YEAR	RPS_{for}	RPS_{clim}
1981	1.0	1.83
1982	0.31	1.33
1983	1.31	1.50
1984	0.31	3.00
1985	0.31	1.33
1986	1.81	2.00
1987	0.31	1.25
1988	0.31	1.00
1989	0.13	1.08
1990	0.31	0.83
1991	0.25	1.67
1992	0.06	1.67

RPSS with sample data set

3. Apply the formula, RPSS = 1 – mean(RPS_{for})/mean(RPS_{clim}). Note this formula is equalivent to the one a few slides back.

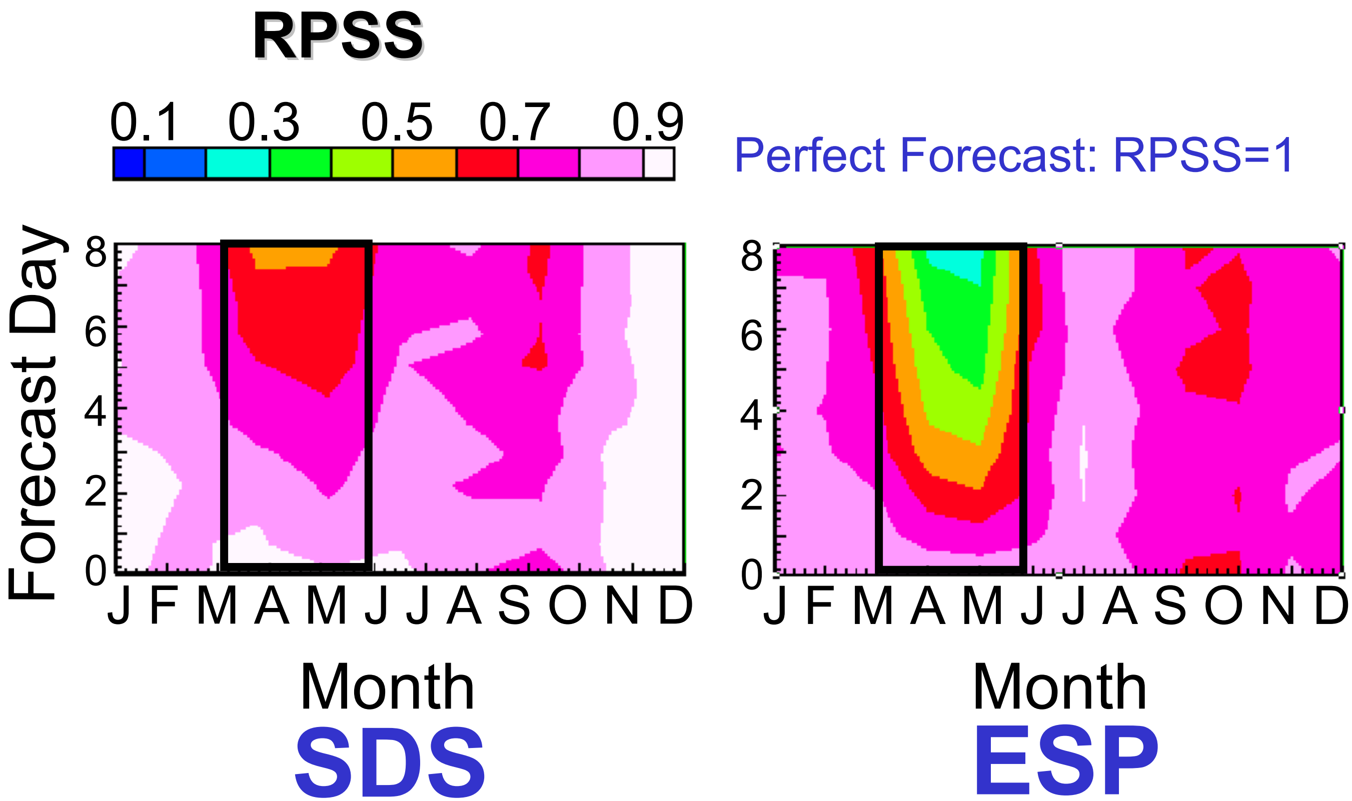
YEAR	RPS_{for}	RPS_{clim}
1981	1.0	1.83
1982	0.31	1.33
1983	1.31	1.50
1984	0.31	3.00
1985	0.31	1.33
1986	1.81	2.00
1987	0.31	1.25
1988	0.31	1.00
1989	0.13	1.08
1990	0.31	0.83
1991	0.25	1.67
1992	0.06	1.67

$$RPSS = 1 – 0.54/1.46$$

$$= +0.63$$

In words, this means the forecasts are 63% better than climatology!

Ranked Probability Skill Score (RPSS) for each forecast day and month using measured runoff and simulated runoff produced using: (1) SDS output and (2) ESP technique




EXPLAINING THE ODDS
We will now announce the winner of the lottery, followed by the names of the 4,347,608 losers.
S. Harris

DISCRIMINATION

"Measures of discrimination summarize the conditional distributions of the forecasts given the observations $p(y_i \mid o_j)$… The discrimination attribute reflects the ability of the forecasting system to produce different forecasts for those occasions having different realized outcomes of the predictand. If a forecasting system forecasts f = snow with equal frequency when o = snow and o = sleet, the two conditional probabilities of a forecast of snow are equal, and the forecasts are not able to discriminate between snow and sleet events." Wilkes (1995)

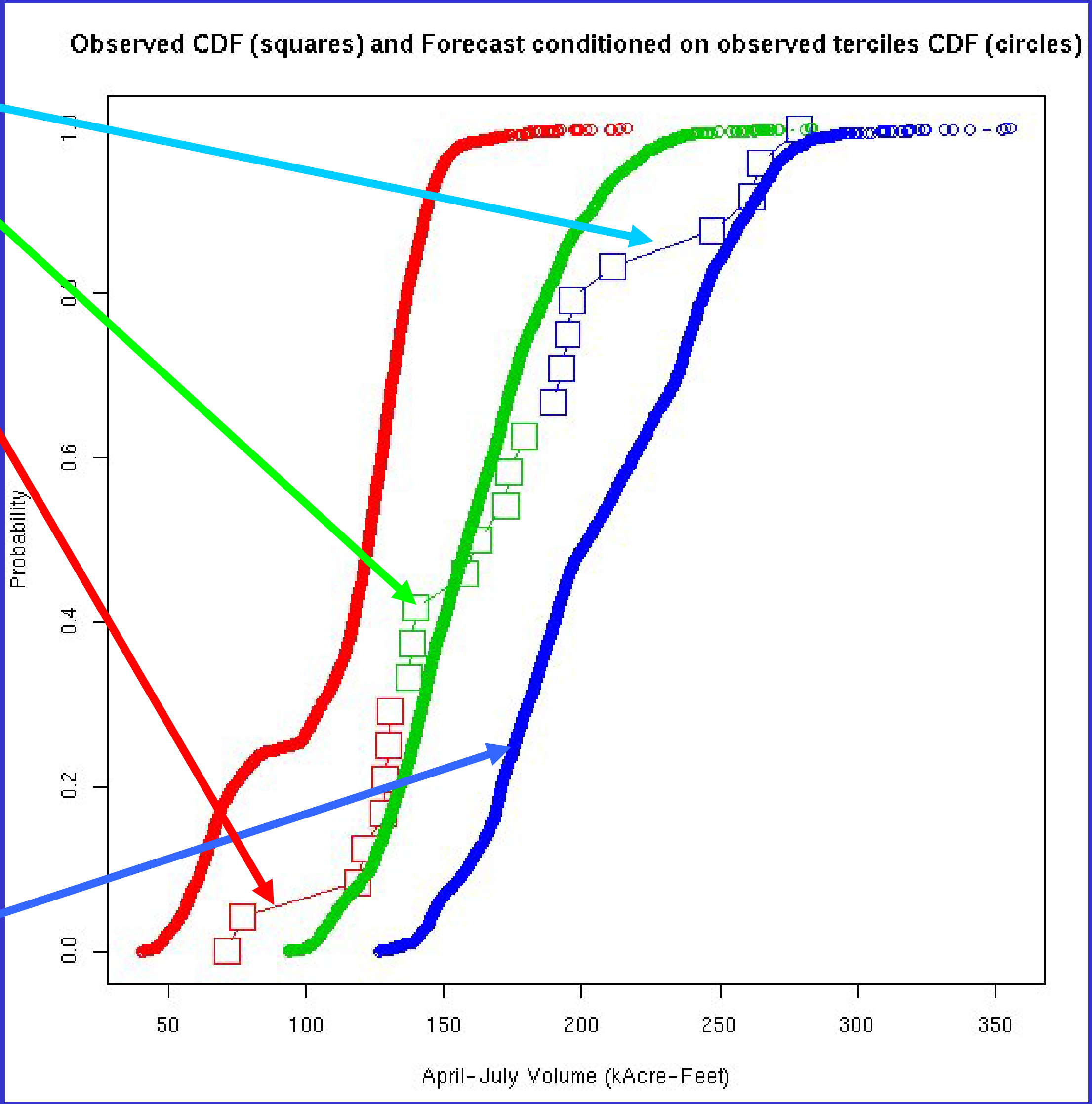
We will approach discrimination through examination of conditional probability densities both in PDFs and CDFs.

DISCRIMINATION Example

All observation CDF is plotted and color coded by tercile.

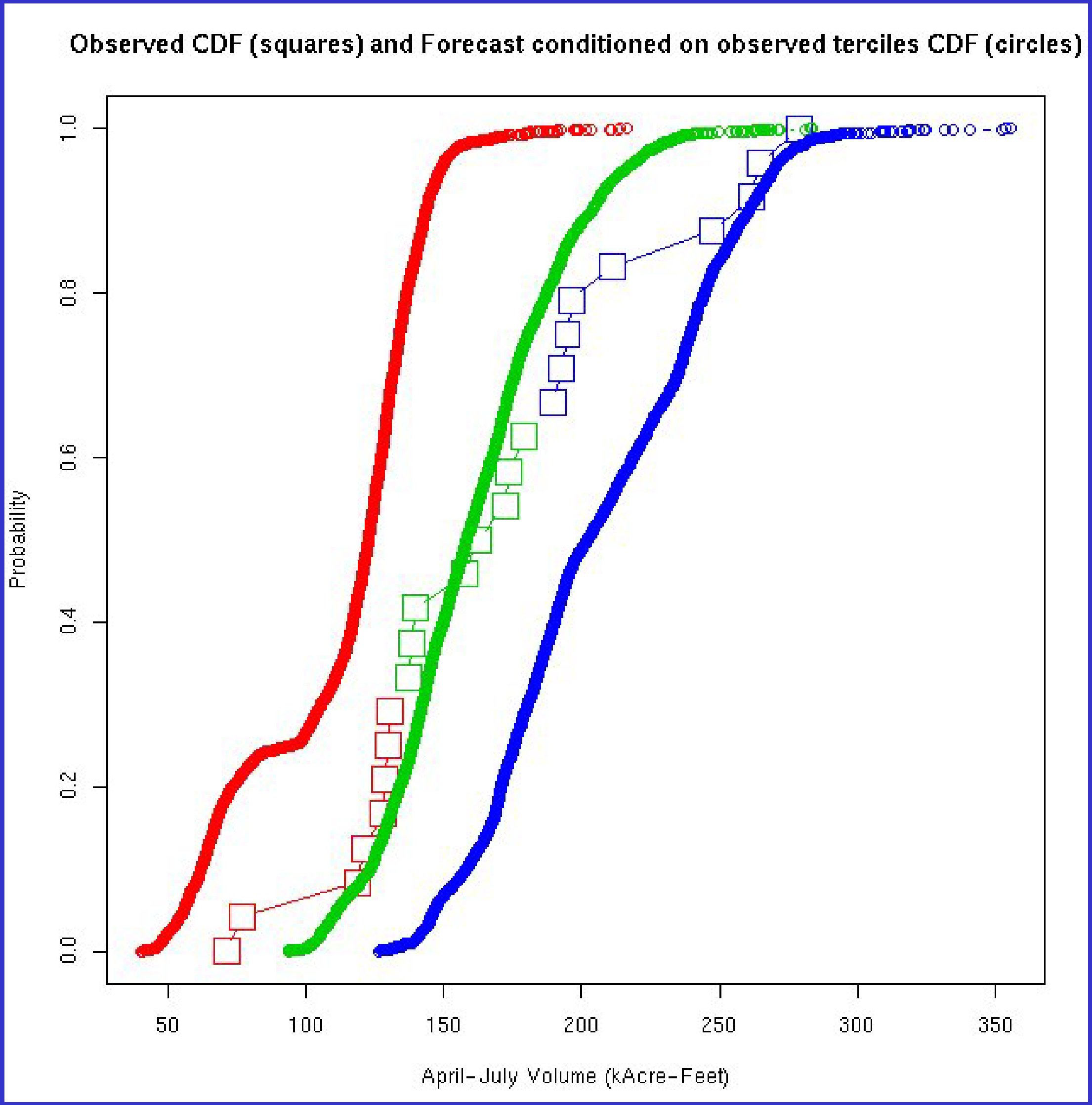
Forecast ensemble members are sorted into 3 groups according to which tercile its associated observation falls into.

The CDF for each group is plotted in the appropriate color. i.e. high is blue.



DISCRIMINATION Example

In this case, there is relatively good discrimination since the three conditional forecast CDFs separate themselves.



Discrimination with sample data set

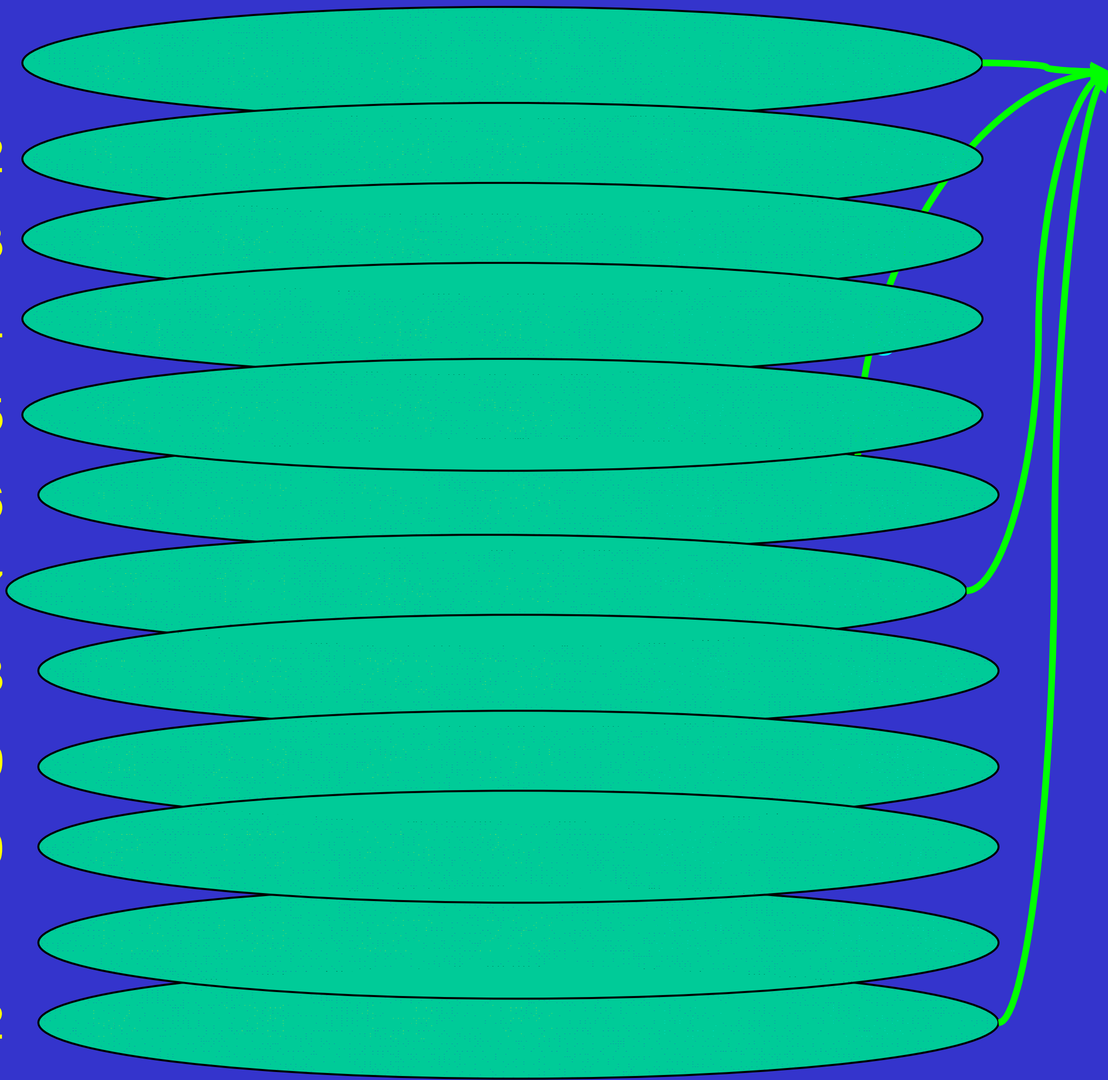
1. Order observations and divide ordered list into categories. Here we will use terciles.

YEAR	E1	E2	E3	E4	OBS	OBS Tercile
1981	42	74	82	90	112	Low
1982	65	143	223	227	206	Middle
1983	82	192	295	300	301	High
1984	211	397	514	544	516	High
1985	142	291	349	356	348	High
1986	114	277	351	356	98	Low
1987	98	170	204	205	156	Low
1988	69	169	229	236	245	Middle
1989	94	219	267	270	233	Middle
1990	59	175	244	250	248	High
1991	108	189	227	228	227	Middle
1992	94	135	156	158	167	Low

Discrimination with sample data set

2. Group forecast ensemble members according to OBS tercile.

YEAR	E1	E2	E3	E4	OBS	OBS Tercile
1981						
1982						
1983						
1984						
1985						
1986						
1987						
1988						
1989						
1990						
1991						
1992						



Low OBS Forecasts:
42, 74, 82, 90,
114, 277, 351, 356,
98, 170, 204, 205, 94,
135, 156, 158

Mid OBS Forecasts:
65, 143, 223, 227,
69, 169, 229, 236,
94, 219, 267, 270,
108, 189, 227, 228

Hi OBS Forecasts:
82, 192, 295, 300,
211, 397, 514, 544,
142, 291, 349, 356,
59, 175, 244, 250

Discrimination with sample data set

3. Plot all-observation CDF color coded by tercile

OBS

112

206

301

516

348

98

156

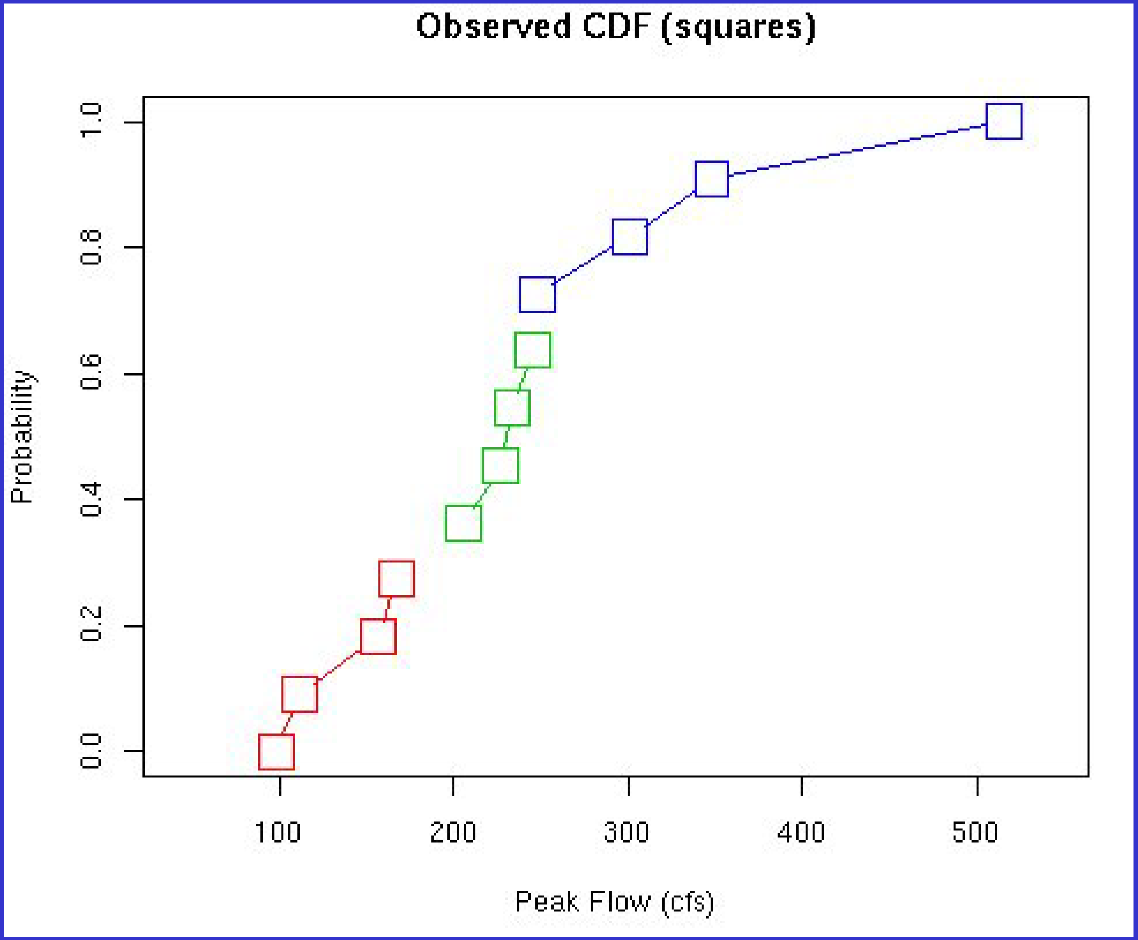
245

233

248

227

167



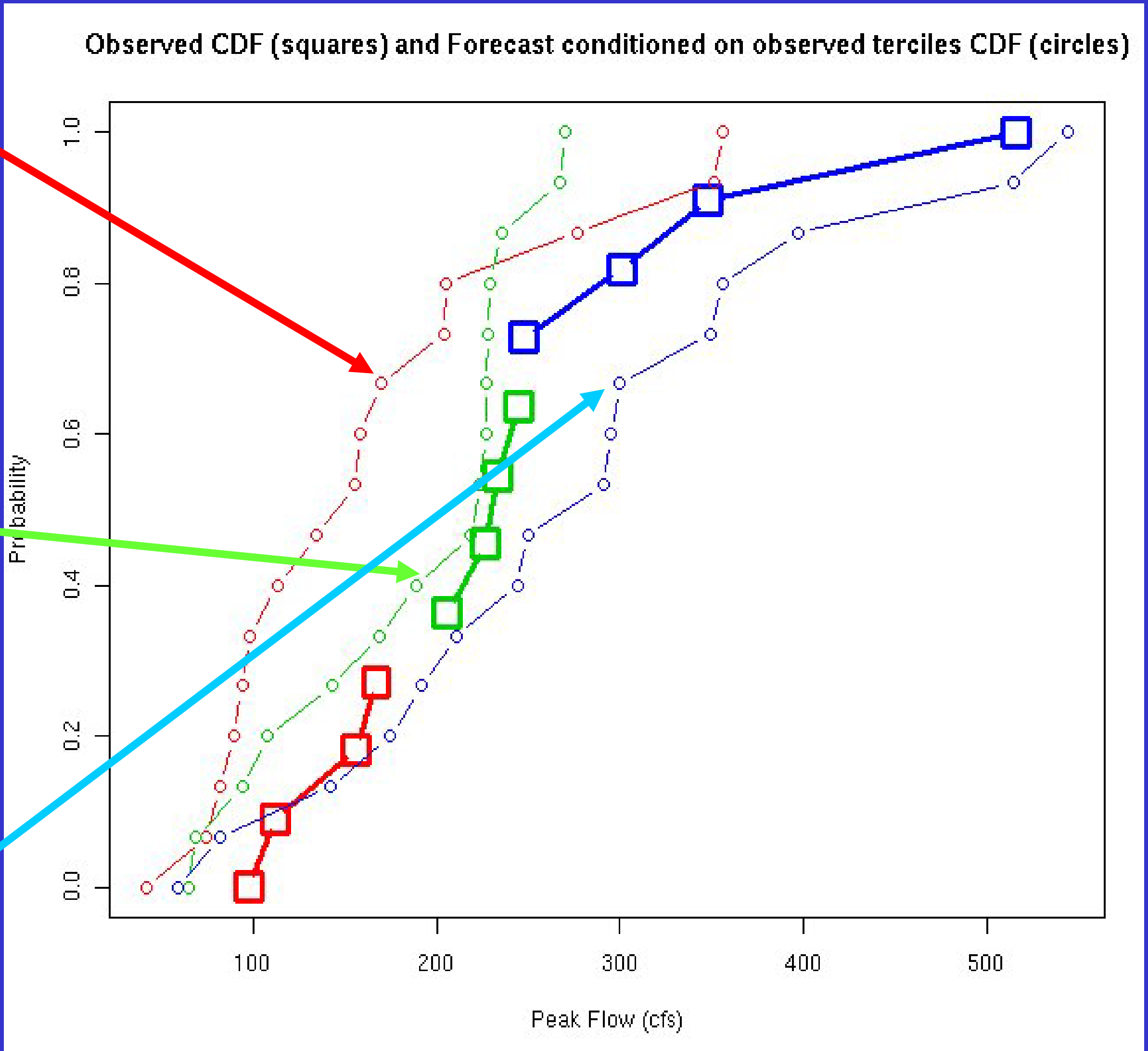
Discrimination with sample data set

4. Add forecasts conditioned on observed terciles CDFs to plot

Low OBS Forecasts:
42, 74, 82, 90,
114, 277, 351, 356,
98, 170, 204, 205,
94, 135, 156, 158

Mid OBS Forecasts:
65, 143, 223, 227,
69, 169, 229, 236,
94, 219, 267, 270,
108, 189, 227, 228

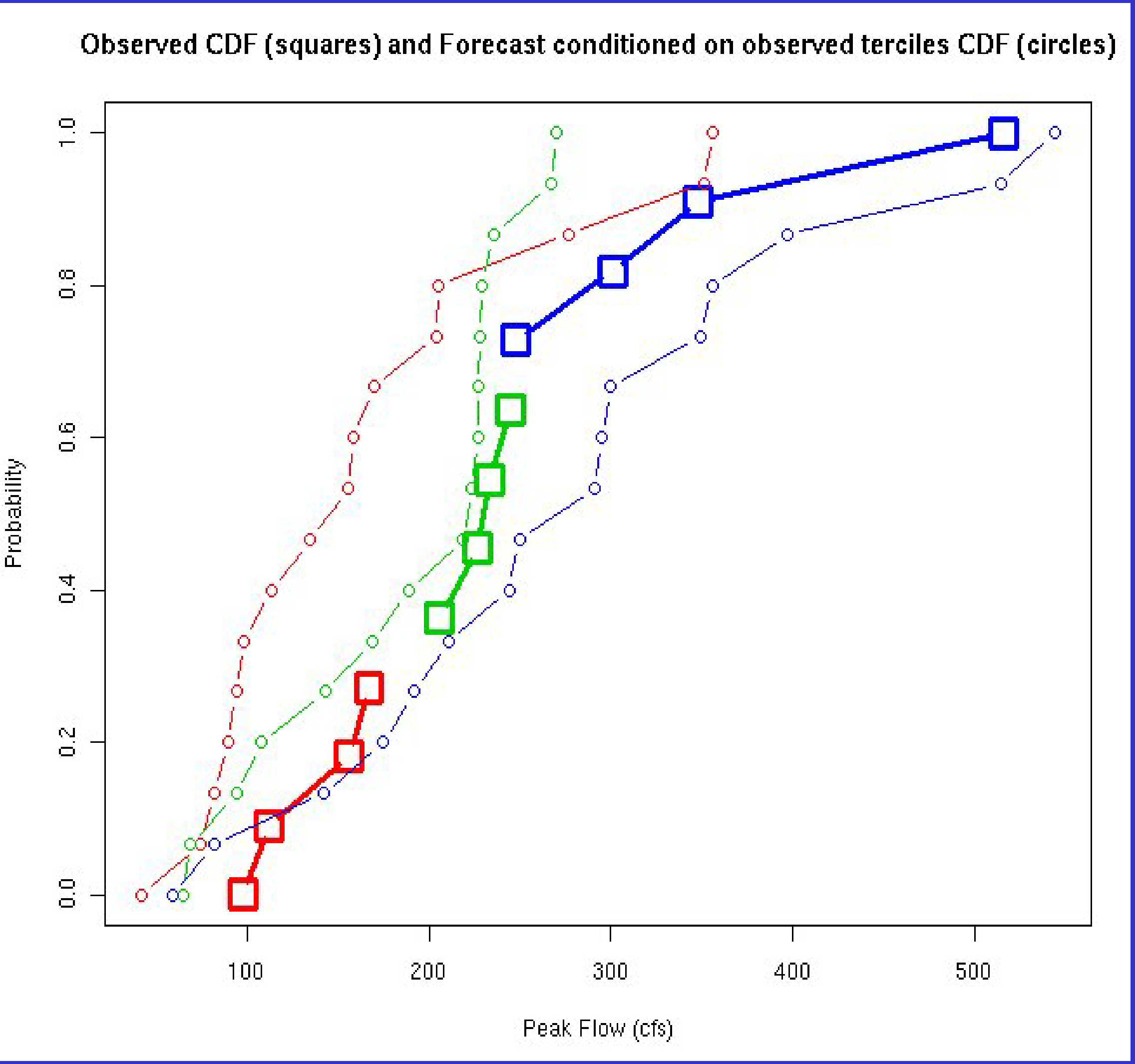
Hi OBS Forecasts:
82, 192, 295, 300,
211, 397, 514, 544,
142, 291, 349, 356,
59, 175, 244, 250



Discrimination with sample data set

5. Discrimination is shown by the degree to which the conditional forecast CDFs are separated from each other.

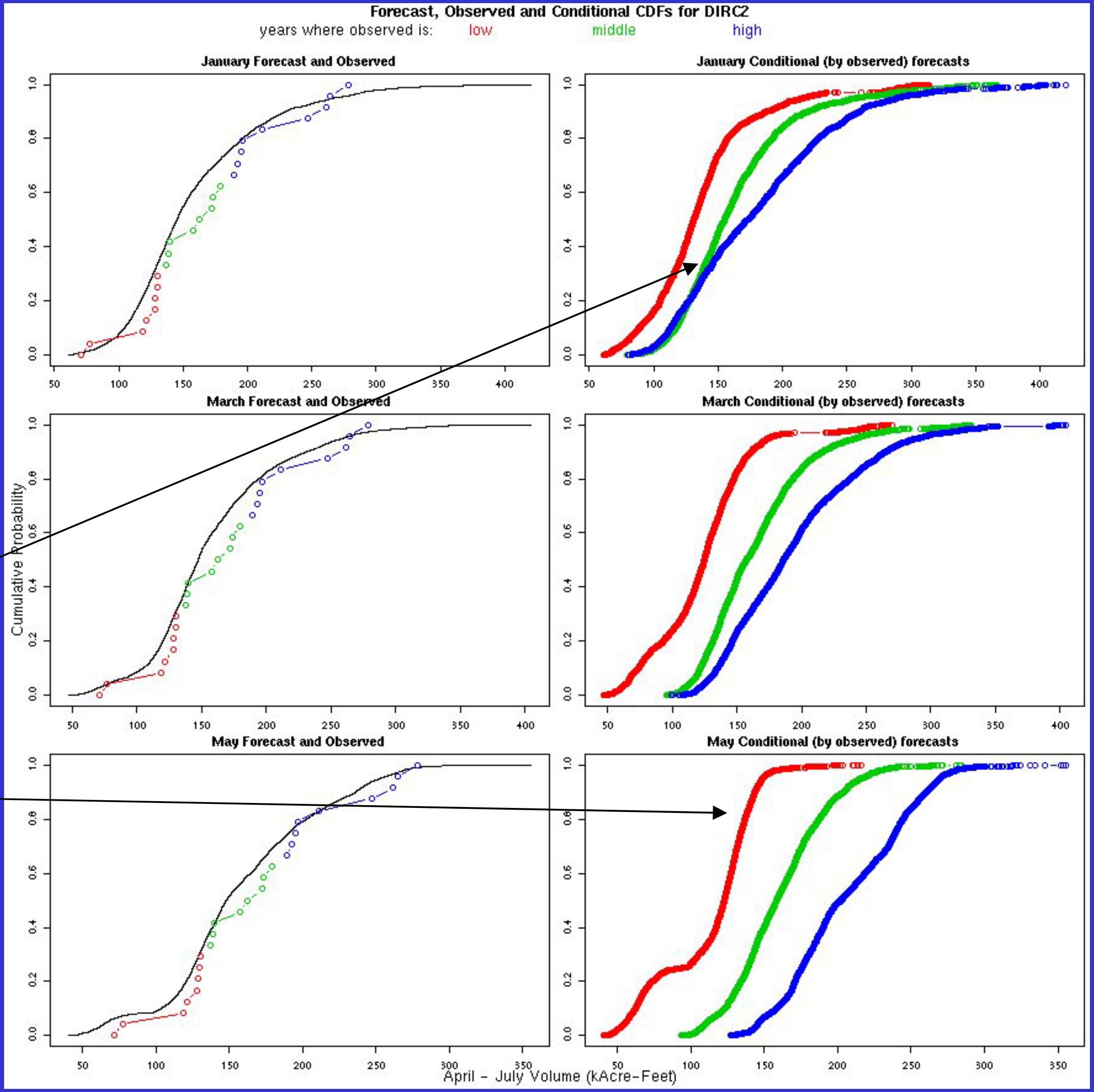
In this case, high forecasts discriminate better than mid and low forecasts.



DISCRIMINATION

How well do April – July volume forecasts discriminate when they are made in Jan, Mar, and May?

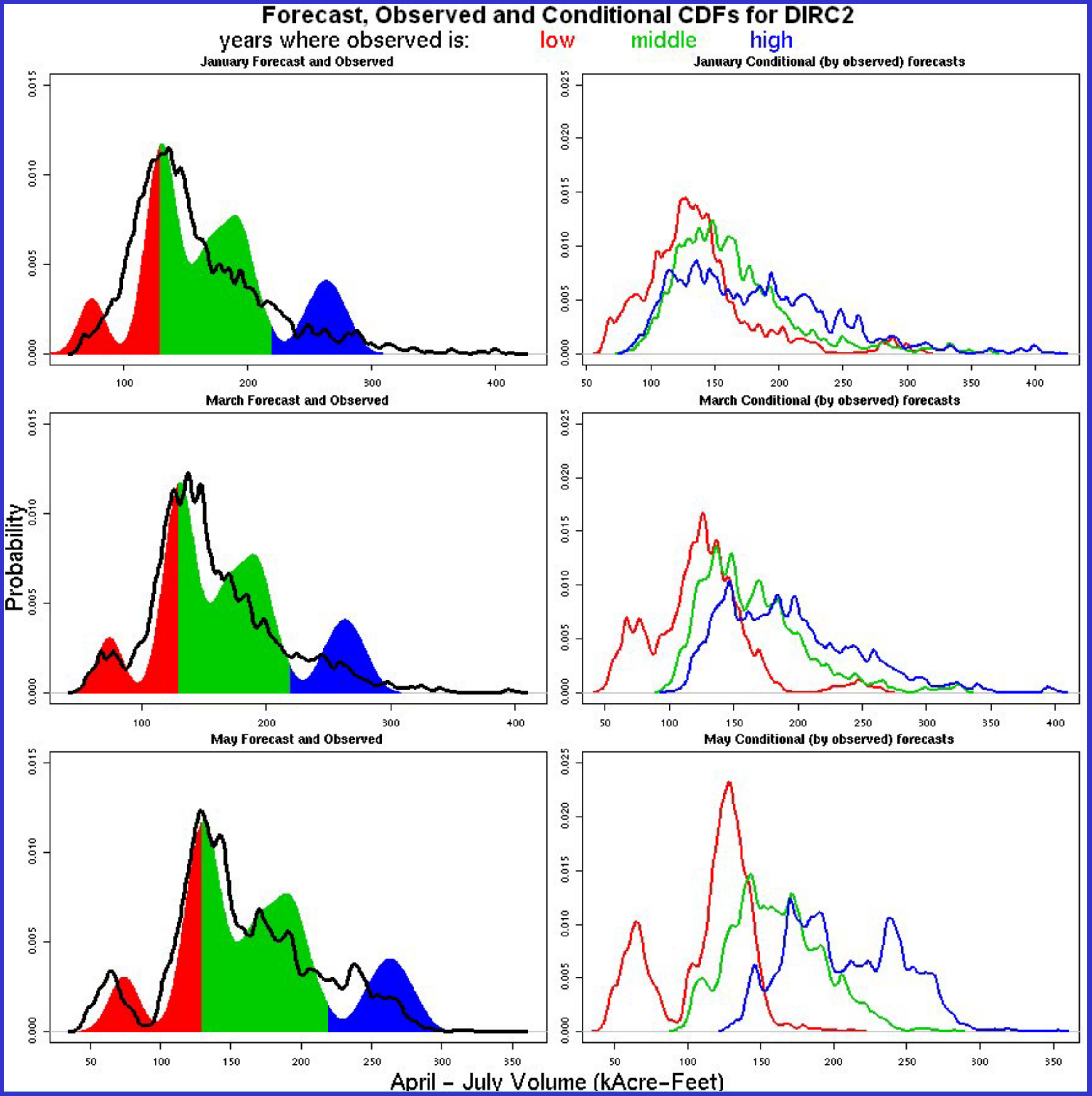
Poor discrimination in Jan between forecasting high and medium flows. Best discrimination in May.



Discrimination

Another way to look at discrimination using PDF’s in lieu of CDF’s.

The more separation between the PDF’s the better the discrimination.




TWO DRAMATIC SCIENTIFIC FINDINGS ARE ANNOUNCED ON THE SAME DAY...
THERE IS ENOUGH WATER ON MARS TO SUPPORT HUMAN COLONIZATION!
Mars
H_2O
DNA Code
WE HAVE MAPPED THE HUMAN GENOME AND WILL SOON RID THE PLANET OF DISEASE AND THE RAVAGES OF AGE!
...WHICH, A COUPLE OF CENTURIES FROM NOW, PROVES TO BE FORTUITOUS...
THIS IS CONTROL. WE HAVE JUST LAUNCHED ANOTHER 250,000 THREE-HUNDRED-YEAR-OLDS TO MARS.
LAUNCH HIKE
GEEZER 174
HORSEY

Reliability

"Reliability pertains to the relationship of the forecast to the average observation for specific values of the forecast. Reliability measures sort the forecast/observation pairs into groups according to the value of the forecast variable, and characterize the conditional distributions of the observations given the forecasts." Wilkes (1995)

Whereas discrimination examines the relationship between given observations and the subsequent forecasts, reliability examines the relationship between forecasts and the subsequent observations.

Reliability Diagram

Reliability measures sort the forecast/observations pairs into groups according to the value of the forecast variable relative to an arbitrary value, and characterize the conditional distributions of the observations given the forecasts.

Traditional reliability diagrams transform a probabilistic forecast into a forecast of probability that an arbitrary value, such as flood stage or normal or …, will be exceeded. On one hand this limits the robustness of reliability as a verification measure. On the other, if the threshold value is of paramount importance, traditional reliability diagrams may be the most important verification measure.

Reliability Diagram with sample data set

1. Choose threshold value to base probability forecasts on. For simplicity we'll choose the mean forecast over all years and all ensembles.

YEAR	E1	E2	E3	E4	OBS
1981	42	74	82	90	112
1982	65	143	223	227	206
1983	82	192	295	300	301
1984	211	397	514	544	516
1985	142	291	349	356	348
1986	114	277	351	356	98
1987	98	170	204	205	156
1988	69	169	229	236	245
1989	94	219	267	270	233
1990	59	175	244	250	248
1991	108	189	227	228	227
1992	94	135	156	158	167

Mean(E1,E2,E3,E4) = 208

Reliability Diagram with sample data set

2. Choose the number of categories to group forecasts into. This will depend on the total number of forecasts as well as the number of ensembles. Something like (total number of forecasts) / 10 will assure an average of ten forecasts in each category. With large a large number of forecasts it is usual to choose ten categories. Since the sample data set is small, we'll use five categories. Since we have only four ensembles and we are assuming an empirical distribution there are only five possible probability forecasts: 0/4, ¼, 2/4, ¾, 4/4. In our small case study, these five numbers will make up the five categories.

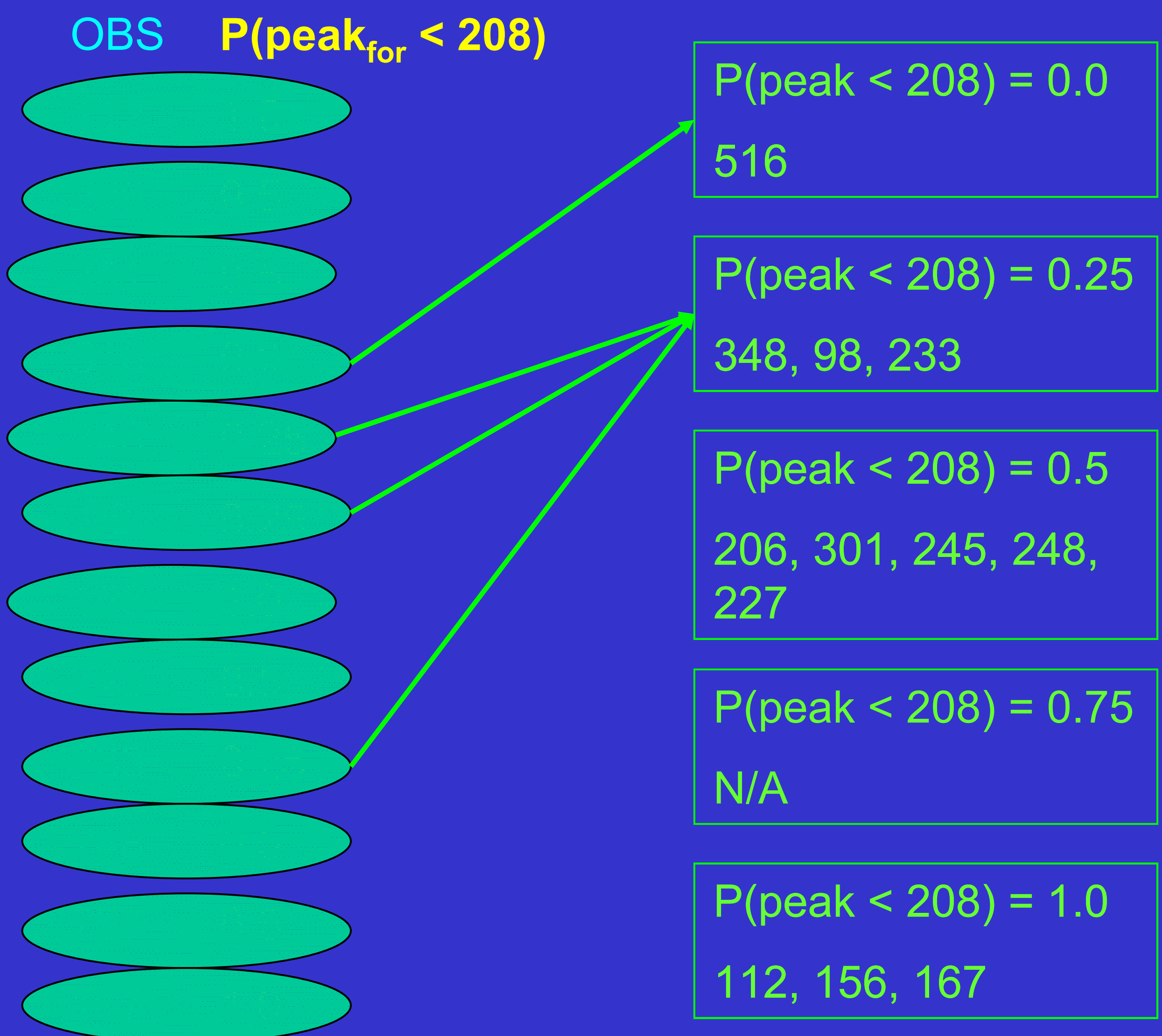
Reliability Diagram with sample data set

3. For each forecast, calculate the forecast probability below the threshold value.

YEAR	E1	E2	E3	E4	OBS	**$P(peak_{for} < 208)$**
1981	42	74	82	90	112	1.0
1982	65	143	223	227	206	0.5
1983	82	192	295	300	301	0.5
1984	211	397	514	544	516	0.0
1985	142	291	349	356	348	0.25
1986	114	277	351	356	98	0.25
1987	98	170	204	205	156	1.0
1988	69	169	229	236	245	0.5
1989	94	219	267	270	233	0.25
1990	59	175	244	250	248	0.5
1991	108	189	227	228	227	0.5
1992	94	135	156	158	167	1.0

Reliability Diagram with sample data set

4. Group the observations into groups of equal forecast probability (or, more generally, into forecast probability categories).



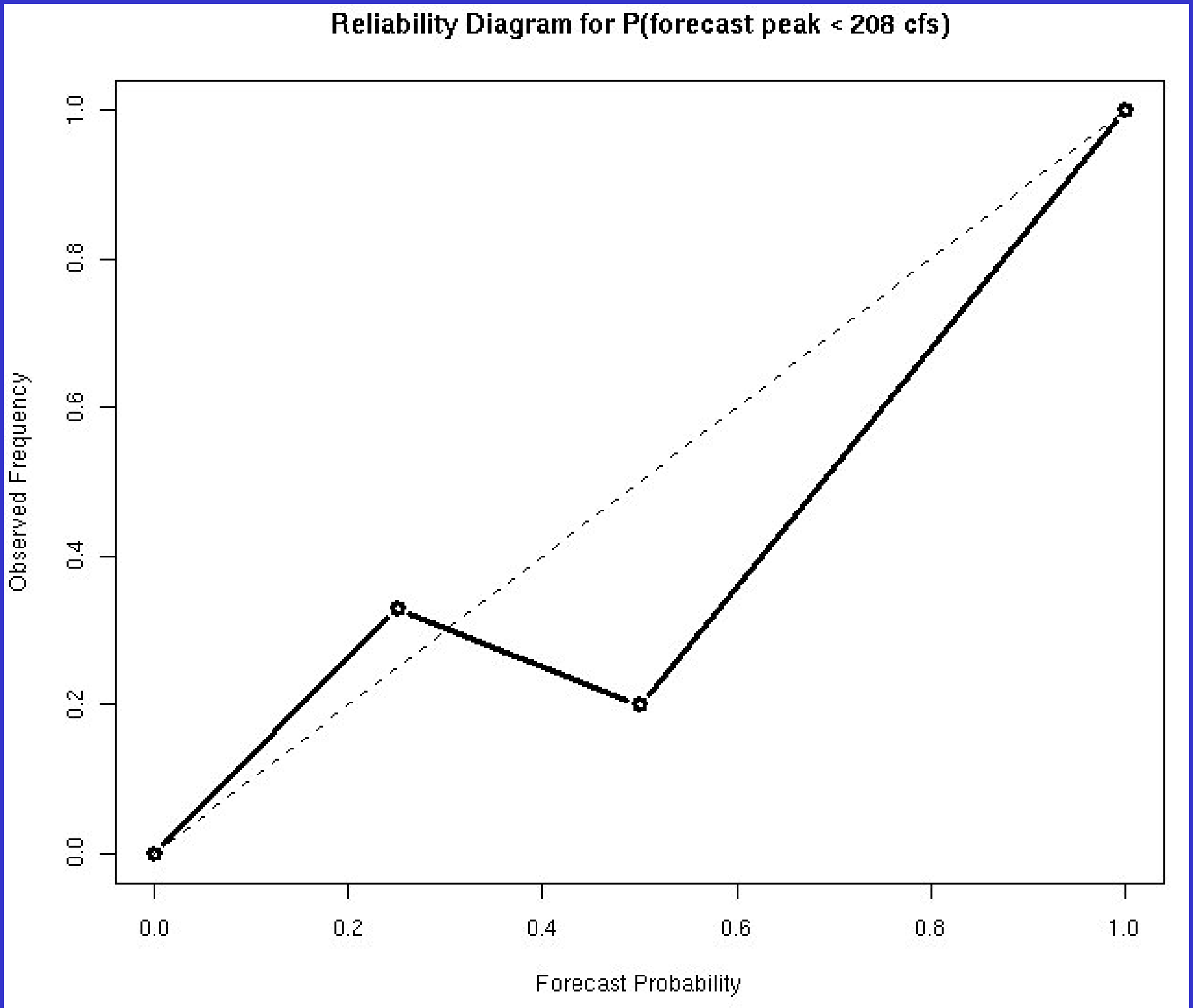
Reliability Diagram with sample data set

5. For each group, calculate the frequency of observations above the threshold value, 208 cfs.

$P(peak < 208) = 0.0$ 516	$P(\text{obs peak} < 208 \text{ given } [P(peak_{for} < 208) = 0.0]) = 0/1 = 0.0$
$P(peak < 208) = 0.25$ 348, 98, 233	$P(\text{obs peak} < 208 \text{ given } [P(peak_{for} < 208) = 0.25]) = 1/3 = 0.33$
$P(peak < 208) = 0.5$ 206, 301, 245, 248, 227	$P(\text{obs peak} < 208 \text{ given } [P(peak_{for} < 208) = 0.5]) = 1/5 = 0.2$
$P(peak < 208) = 0.75$ N/A	$P(\text{obs peak} < 208 \text{ given } [P(peak_{for} < 208) = 0.75]) = 0/0 = NA$
$P(peak < 208) = 1.0$ 112, 156, 167	$P(\text{obs peak} < 208 \text{ given } [P(peak_{for} < 208) = 1.0]) = 3/3 = 1.0$

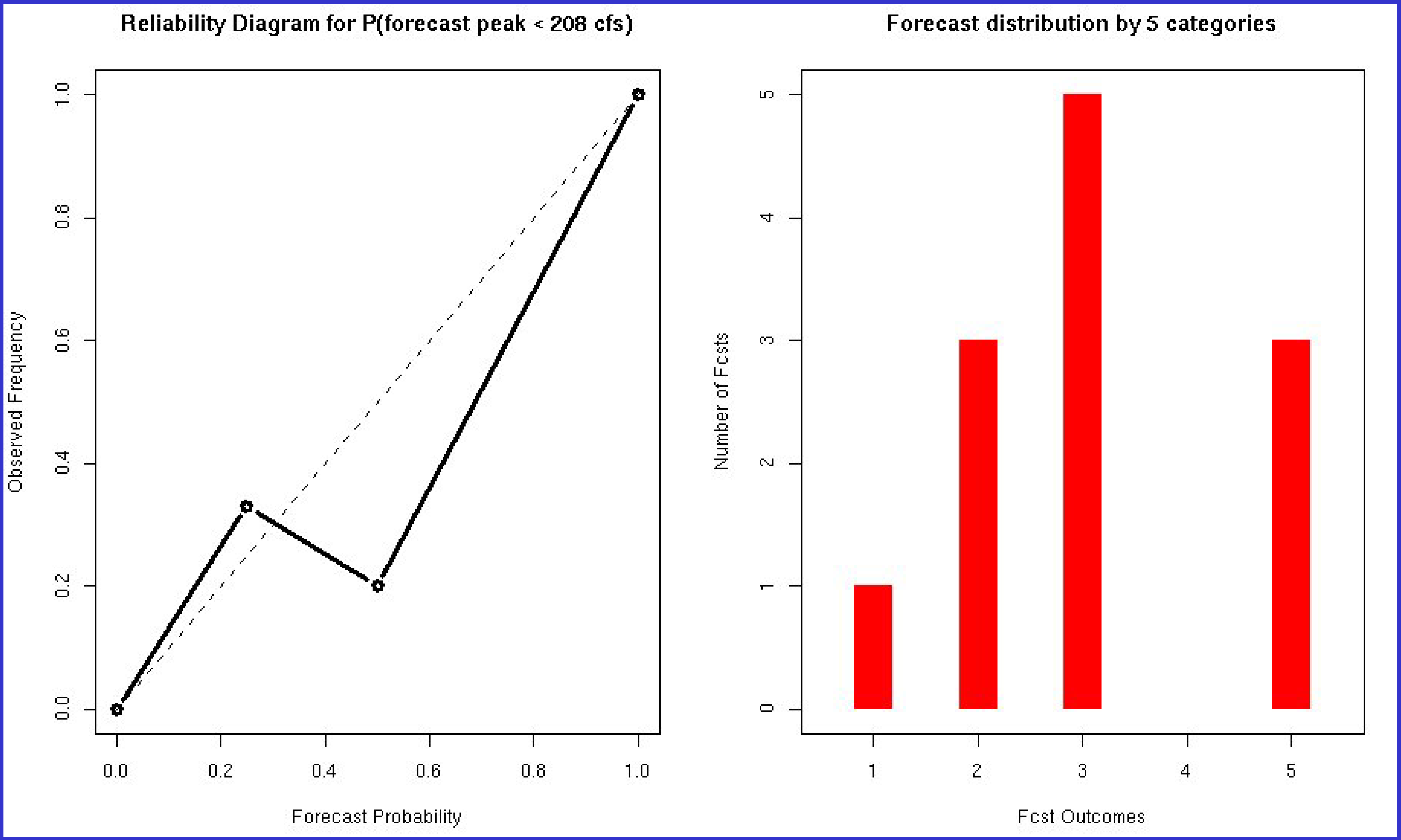
Reliability Diagram with sample data set

6. Plot centroid of the forecast category (just points in our case) on the x-axis against the observed frequency within each forecast category on the y-axis. Include the 45 degree diagonal for reference.

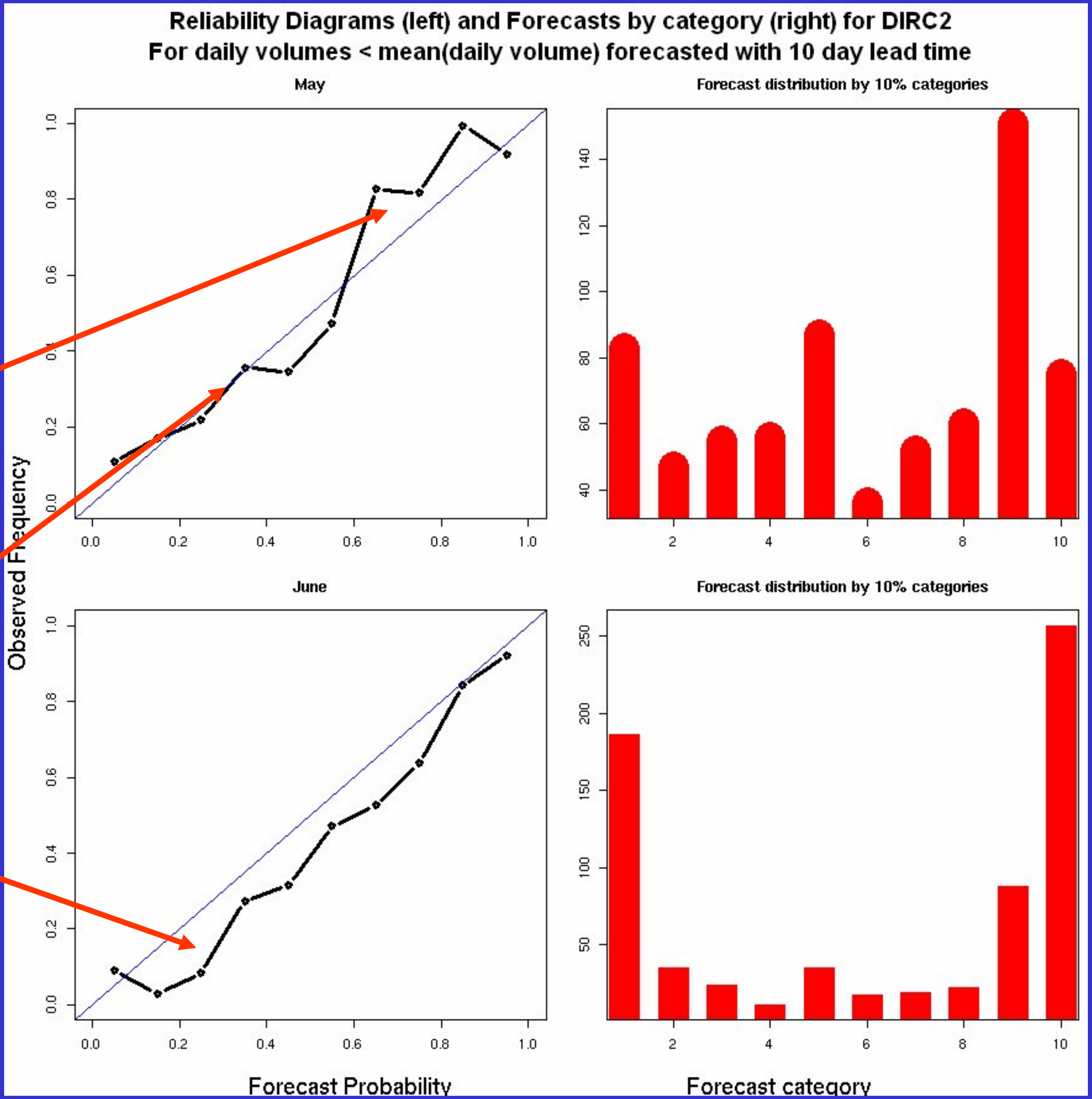


Reliability Diagram with sample data set

7. Include bar plot showing the number of observation/forecast pairs in each category.



Reliability Diagram Example



Under Forecasting if area is above the diagonal

Perfect if on the diagonal

Over Forecasting if area is below the diagonal

Multi-Category Reliability Extension

A major constraint of reliability diagrams is the requirement to define an event to construct the probabilities on.

Recent work from Hamill (1997) demonstrated a multi-category extension to reliability diagrams. Although the arbitrary selection of categories remains, the inclusion of multiple categories may make reliability diagrams a more robust verification measure.

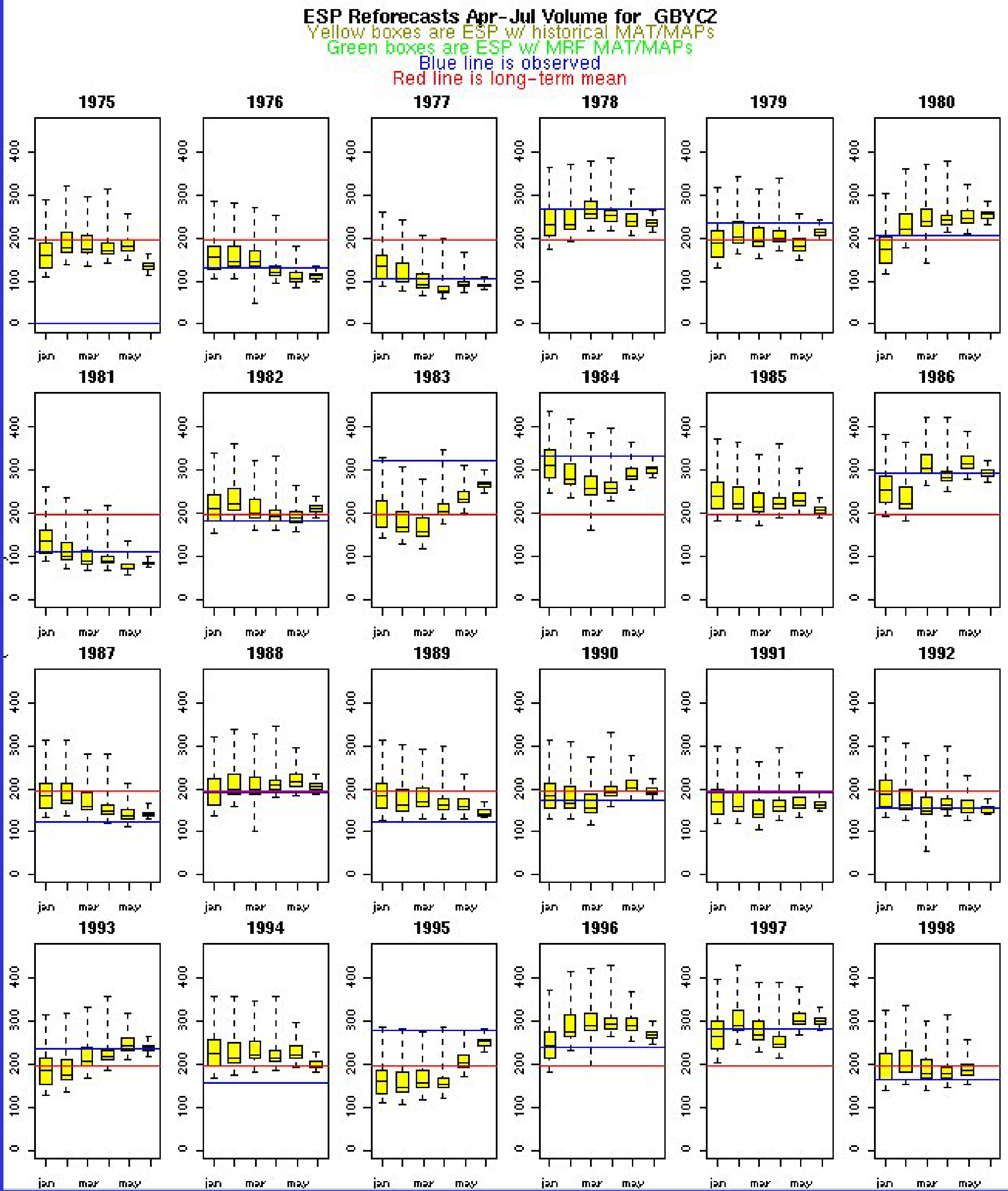
Other Verification Tools

Verification measures beyond what was presented here exist. Their exclusion here is not meant to diminish their usefulness.

Statistical verification is not meant as a substitute for examination of the actual forecasts and observations. An inspection of the actual forecasts and their corresponding observations can be invaluable. The next slide illustrates this.

Ensemble Forecast Analysis:

Forecasts for April-July volume for a particular basin (Granby, CO) are depicted with box and whisker plots here. The observation is a blue line.



Credits:

Franz, Kristie and Sorooshian, Soroosh, 2002: Verification of NWS Probabilistic Hydrologic Forecasts, M.S. Thesis, Univ of Ariz.

Hamill, T.M., 1997: Reliability Diagrams for Multicategory Probabilistic Forecasts. Wea. Forecasting, 12, 736-741.

Hersbach, Hans, 2000: Decomposition of the Continuous RPS for Ensemble Prediction Systems, Wea. Forecasting, 15, 559-570.

Wilks, D.S., 1995: Statistical Methods in the Atmospheric Sciences: An Introduction. Academic Press, 467 pp.