PRICING BEHAVIOR OF MULTIPRODUCT RETAILERS

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ABSTRACT

While retail sales are an important economic phenomenon, previous research has not explored how the multiproduct nature of consumers' purchasing decisions affects the pricing dynamics of multiproduct retailers. In this paper, we first document the extent to which "sales", defined as temporary discounts in retail price, are a pervasive aspect of retailing and an important source of retail price variation. Using a large data set for 20 categories of grocery products across 30 U.S. metropolitan areas, we find that the majority of price changes and 25%-50% of retail price variation is the result of retail sales. We then develop a model describing the pricing dynamics of multiproduct retailers that is consistent with these empirical pricing regularities. Specifically, because consumers prefer to buy a bundle of goods from the same retailer, a given discount on any one good in the bundle will have a similar effect on consumers' likelihood of visiting that retailer. This implies that discounts on goods sold by a single retailer are substitutes instruments for retailers, and factors that influence one good's price will affect the pricing of other goods. Hence, if intertemporal price changes are a means of price discriminating (as suggested in the literature), the impact of these changes will be reflected in the prices of many goods, including even those for which discrimination is not feasible.

I. Introduction

As consumers, we all have some familiarity with the complex pricing strategies employed by supermarkets. For instance, among the more than 20,000 items they carry, supermarkets choose to offer only a small fraction of the items at a low "sale" price each week. Despite the high administrative costs of changing retail prices (Levy, et al.(1997)), retailers clearly find it profit maximizing to put different items on sale each week. The goal of this paper is to provide some insight into the extent of, and the reason for, this aspect of retailer behavior. First, we document the extent to which sales - defined as temporary price reductions - occur, and the relative importance of temporary versus permanent price changes.[1] Second, we develop a model to explain retailers' rationale for temporarily lowering prices. Specifically, we determine equilibrium pricing behavior in a dynamic model in which competing retailers each sell two goods, a non-perishable good which can be inventoried by consumers, and a perishable good which cannot.

To document the extent to which sales occur, we make use of a novel data set. The data, provided to us by the U.S. Bureau of Labor Statistics, consists of more than 300,000 monthly price quotes in 20 categories of grocery items collected from retailers in 30 metropolitan areas for the period 1988-1997. A desirable feature of this data set for studying sales is that it covers a wider range of products and geographic areas than the data used in previous studies of retail pricing. We observe multiple price series in each of the twenty categories of goods, where each price series represents a particular grocery item (e.g. an 18 ounce container of brand x's creamy peanut butter from retailer y) for up to 5 years.

Our data show that temporary price reductions are empirically-important phenomena that pervade retail pricing in the U.S. We find that the typical grocery product has a set "regular" price, and most deviations from that regular price are downward. The data also demonstrate that sales are an important source of retail price variation. Roughly 60% of price decreases are the result of sale behavior. Further, between 25% and 50% of the observed annual variation in retail prices is the

[1]This concept of a sale contrasts with other kinds of systematic price reductions that have been documented. One such pattern is that prices for goods with a "fashion" element often systematically decline over a fashion season (see, e.g., Pashigian (1988), Pashigian and Bowen (1991), Warner and Barsky (1995)), as retailers learn which styles are popular with consumers. We view this type of sale as a fundamentally different phenomenon.

result of retailers placing individual items on sale.

To understand these empirical findings, we develop a model of pricing dynamics for multiproduct retailers. The underlying logic of the model is that because consumers prefer buying a bundle of goods from the same retailer, a retailer's offer of a discount on any good in a bundle will have similar effects on a retailer's likelihood of attracting a given consumer. This implies that when a retailer lowers the price of one good, it will likely raise the price of another good its sells. At the same time, retailers can price discriminate against impatient, inelastic consumers by periodically reducing prices on non-perishable goods (those that can be inventoried by consumers -see, e.g., Conlisk, Gerstner and Sobel (1984)). In combination, these two factors imply that prices for many goods, including those for which price discrimination through intertemporal price changes is not feasible (which we call perishables), will change periodically. In addition, we show that in equilibrium, price movements will be quite different for perishable and non-perishable goods; non-perishable pricing will feature long periods of stable prices, following by significant but short-lived price reductions, whereas perishable prices will move more frequently, but by smaller amounts.

In the model, the desire of retailers to take advantage of differences in consumer's inventory costs is an important source of retail price variation. From this perspective, our empirical findings suggests that consumer inventorying is a sufficiently important phenomenon to motivate a large proportion of price movements. As has been noted elsewhere (e.g., Feenstra and Shapiro (2000)), consumer inventorying means that the response to temporary price changes (which lead to inventory behavior) may be substantially different than the response to permanent price changes (which do not). The recent empirical studies we are aware of which estimate demand elasticities do not control for the fact that much of the observed variation in retail prices is temporary, e.g. Hausman, Leonard, and Zona (1994) or Cotterill, Putsis, and Dhar (2000). Hence, if consumer inventorying is important, then the estimated demand elasticities do not correctly measure how consumers' purchases would change in response to permanent price changes.

II. Evidence Regarding Sales in the U.S.

This section provides systematic evidence regarding retail pricing regularities. While there is no single metric by which to measure (or even define) sales, the pricing patterns we find are consistent with our notion of sale behavior. Our results suggest that sales are a widespread feature of retail pricing in the U.S. Empirically, most grocery products have a “regular” price which is charged most of the time. When a price is not at its regular level, it is much more likely to be below the regular price than above it. The evidence suggests that sales do not appear to result primarily from unexpected changes in demand or supply. For example, they are not due to unexpectedly high inventories of perishable merchandise or short term changes in wholesale prices. Instead, retailers appear to systematically place the same items on sale over time, suggesting that sales are not a response to unexpected changes in inventory. Further, price changes across retailers within a market on any particular item are not highly correlated, suggesting that sales are not simply the result of short term changes in wholesale prices. Together, this evidence suggests that sales at least partially represent changes in retailer margins rather than changes in wholesale prices. Section IV presents a model of retailer sale behavior consistent with these empirical findings. The next two subsections discuss the data used and the specifics analyses performed.

Data Description

Our analysis makes use of two data sets. The first is a non-public data set we obtained from the U.S. Bureau of Labor Statistics (BLS), which to our knowledge has not been used to study retail pricing behavior. The second is a public data set provided by A.C. Nielsen.

The data provided to us by the BLS are collected for use in calculating the Consumer Price Index. Each month, the BLS samples food retailers in 88 geographic areas, collecting prices of specific items in up to 94 categories of goods.[2] Within each category, the BLS samples the price of a specific item at the same store for up to 5 years. For example, a price series for cola may consist of monthly prices of a 2-liter bottle of Coke in a particular retail outlet in Boston for a 5 year period. The data we use in this study consists of these individual price series for specific products. While most product categories have multiple price series in each geographic area, the price series provided to us do not contain information that identifies the specific product and package size sampled within each category. We only know that all of the prices within a price series correspond to prices for a

[2]A category is a fairly narrow classification of consumer goods, e.g. cola drinks, eggs, and white bread are BLS categories.

specific product at a specific store within a category. For example, we do not know whether that specific cola product in a price series is a 12-pack of Coke or a 2-liter bottle of Pepsi. We also cannot identify the store or chain associated with each price series. Hence, we cannot determine when two series are taken from the same store or chain.[3] The data we received from the BLS contains all of the price series the BLS collected on 20 categories of goods (see Table 1 for specific products) from 30 geographic areas for the period 1988-1997.[4]

To analyze the relationship between specific product prices across retailers at a point in time, e.g. the price of a 28 ounce bottle of Heinz ketchup across retailers within a city, we use a public use data set from A.C. Nielsen. This data set contains specific product and price characteristics for eight categories of goods at the individual store level for Springfield, MO and Sioux Falls, SD for the 124 week period beginning January 23, 1985.

Empirical Findings

Sales are a common feature of retail pricing. Table 1 provides summary statistics about the extent of "sales" in the BLS data set. The first column of Table 1 shows the number of price series in each category. The next two columns describe the sales phenomenon. Specifically, our concept of the phenomenon is that prices are at their "regular" level most of the time, and are significantly lower for brief periods. To examine whether this pattern exists, we begin by developing a measure of a product's regular price. We do this by dividing each price series in the data set into individual price series for each calendar year (e.g. the tenth price series for crackers in Chicago for 1996). From each calender year, we calculate the modal price for each annual price series, which we view as the regular price. Given this measure, we then determine the frequency with which prices in each

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[3]For this reason, we cannot use the BLS data to examine any implications regarding the relationship of prices movements on multiple products within a store.

[4]The geographic areas are: Atlanta, Boston, Buffalo, Chicago, Cleveland, Dallas, Dayton, Denver, Detroit, El Paso, Greater Los Angeles, Jacksonville, Kansas City, Los Angeles, Miami, Minneapolis, New Orleans, New York and Connecticut suburbs of New York City, Philadelphia, Portland, Richmond, St. Louis, San Diego, San Francisco, Scranton, Seattle, Syracuse, Tampa, Tucson, and Washington D.C. See Hosken, Matsa, and Reiffen (1999) for more details on this data set.

individual price series are equal to their annual modal values. Finally, we calculate the average frequency at the modal value over all of the series in each product category. For example, the average baby food price is at its modal value 72.05% of the time. With the exception of eggs and lettuce, the average product is at its modal price at least 50% of the time. We conclude that one feature of retail pricing is that most products have a "regular" price.

The second aspect of the sale phenomenon is evaluating what happens when price is not at its regular level. If sales are important, then when prices are not at their regular level, they should be more likely to be below the regular price than above it. Hence, we test for sales by comparing the percentage of deviations from the modal price that are below versus above the mode for each product category in our sample. In the third column of Table 1 we test this hypothesis by calculating the ratio of the number of observations below the regular price to the number above it. For every category of goods, the ratio is well above 1. Testing the hypothesis that the number of prices below the mode is larger than the number above the mode, we find that for every category we can reject the hypothesis of equality at the 1% confidence level.

While retailers certainly have incentives to place items on sale when their inventories unexpectedly increase, the evidence also suggests that is not the primary cause of retail sales. This conclusion follows from the observation that, unless retailers' excess inventories (e.g., due to forecasting errors) are systematically more common on specific products within categories, we would expect that sales due to excess inventories would be equally common on all items within a category. If instead retailers systematically place certain items within a category on sale quite often, while others rarely (if ever) go on sale, it would suggest other motivations for sales. We examine whether there are predictable patterns to which products go on sale by comparing the probability an item goes on sale[5] in year t conditional on the product going on sale or not going on sale in year t-1.[6]

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[5]We considered five definitions of a sale in performing this calculation: a price decrease in month t of at least 5, 10, 15, 20, or 25 percent followed by a price increase in month of t+1 of at least the same amount. In the interest of brevity, only the results for the 10% definition are presented here. The results for other sale definitions are quite similar.

[6]Specifically, we perform the following calculation. First, for every price series with at least 24 observations, we record whether that series experienced a sale during the first 12 months for which we have data. Next, we divide the sample into two parts: The first contains price series

If products are put on sale randomly (e.g., because of unexpectedly high inventories), then the two conditional probabilities should be the same. The results of these calculations appear in Table 2. For every product category in our sample, the conditional probability of observing a sale in year t is larger, often substantially larger, if the price series experienced a sale in year t-1. In fact, for all 20 comparisons shown in Table 2, we reject the null hypothesis with a z-statistic greater than 2.5.[7] This result is robust across 20 large categories of goods, over time, across the U.S. and for five different definitions of sales.

This finding that within categories, goods differ in their probability of going on sale is consistent with some recent findings. Hosken, Matsa and Reiffen (2000) show that within categories, goods that are more "popular" (e.g., those with higher market shares) are more likely to go on sale. They also present evidence consistent with that result. Hence, it appears to be the case that the sale pattern portrayed in Table 1 is more likely to be found for popular goods.

We also examined the extent to which retail price changes represent changes in retail margins, rather than in wholesale prices. Under the assumptions that (1) prices paid by retailers (wholesale prices) move together in each city, and (2) wholesale price changes are reflected in retail prices with a lag that is common across all retailers, we would expect that retail price changes for a given product would be highly correlated across retailers if sales were primarily driven by wholesale price changes.[8] Using the data from A.C. Nielsen, we calculate the correlations of price

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that have at least one sale in the first 12 months and the second contains those price series that do not have a sale. Within each product category we then calculate two conditional probabilities; the probability that a price series would experience a sale during the second year of the sample conditional on the product being in the first group (i.e., having a sale within the first 12 months), and the probability of a sale in the second year conditional on being in the second group.

[7]The corresponding number of z-statistics over 2.5 using all 5 sale definitions was 91 out of 100. For some of the comparisons of conditional probabilities, the number of price series is very small. In these cases it is incorrect to assume that the difference in proportions is approximately normal, and instead we simply interpret the computed z-statistics as measures of the size of the difference between conditional probabilities. Tables showing these tests for all five sale definitions are available from the authors on request.

[8]The assumption that in each city all retailers' wholesale prices move together is based on our understanding of industry practices, along with legal restrictions on differential pricing due to the Robinson-Patman Act. The Robinson-Patman Act makes it illegal for a firm in the U.S. to

changes across stores for the top three products (ranked by market share) in the eight categories of goods contained in the data set for two markets. We find that nearly half of the correlations are negative, and only 7% are greater than 0.3 (see summary in Table 3).[9] This finding suggests that retail price changes were not primarily driven by changes in wholesale prices.[10]

The evidence presented above conforms with our experience as consumers, as it shows that most goods have a "regular" price, and that most deviations from those regular prices are downward. A question that is perhaps less clear is the importance of sales as a source of price variation. That is, are sales an empirically important phenomenon? To evaluate this question, we present several empirical means of distinguishing between temporary and permanent changes in retail prices.

One way to examine the extent of temporary changes is to analyze the time series of first differences in price. Specifically, we examine the price changes between month t and t+1, conditional on price falling between months t-1 and t. If a price reduction is temporary, rather than permanent, then price would rise between month t and t+1. In contrast, if the price change between months t and t+1 is zero (or negative), it would suggest that the retail price movement reflects a permanent change in the retailer's cost (and/or the manufacturer's cost). In fact, as Table 4 indicates, across all categories of goods, 60% of price reductions are followed by a price increase, while only 23.3% remain at the new, lower level. This suggests that the majority of retail price reductions are temporary.

A more formal way to evaluate price movements is to statistically decompose the price variation. We compare the price variation due to temporary reductions to the price variation caused by other sources. In particular, for many products, there are predictable price changes due to harvest

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charge different prices to two buyers of a good, unless those buyers are end users (subject to some exceptions and defenses).

[9]Several of the correlation matrices are reported in Hosken and Reiffen (1999). The entire set of correlation matrices are available from authors.

[10]Some direct evidence on this point can be found in Chevalier, Kashyap, and Rossi (2000). Their data set includes both retail and wholesale prices for one U.S. supermarket chain. They find that most of the temporary reductions in retail prices for the goods they examine reflect retail margin changes, rather than wholesale price changes. Evidence reported in Levy, et al. (1997) also shows that most retail prices changes are actually changes in retail margins.

periods, or year-to-year changes in costs or demand. Thus, we first run a regression to determine the extent to which national shocks in each time period explain retail price variation. Specifically, for each product i, we regress its price in city j at time t on separate dummy variables for each of the 120 months in our data as depicted in equation (1) below.

$$(1)\quad p_{ijt} = \alpha_0 + \sum_{1}^{120} \beta_k * (Month_k) + e_{ijt}$$

The r-squared from these regressions tells us how much of the variation in retail price for each category is explained by national, time-specific shocks, such as supply changes. If all supply changes are national, and effect all products in a category, then the β_k will pick up the price changes due to changes in supply.

To determine the proportion of price variation accounted for by sales, we wish to design a model which will capture the variation in retail price caused by sales. We do this by adding a separate dummy variable to equation (1) which is equal to 1 each time a product goes on sale (equation (2) below).[11] The addition of a separate dummy variable for each observation that has a sale effectively controls for all of the price variation associated with observations that have sales. Thus, the residual from equation (2) is the result of "permanent" changes in retail prices, other than those associated with nation-wide time-specific shocks (e.g., seasonality) and sales. Hence, the difference in the r-squared between equations (1) and (2) has the interpretation of the additional portion of the retail price variation explained by sales (i.e., beyond that explained by time dummies).[12]

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[11]Where a product is defined to go on sale if its price falls by at least 5% between period t-1 and t, and then rises by at least 5% between period t and t+1.

[12]There are three additional details involved in estimating equations (1) and (2). First, because each price series within a product category corresponds to a unique product (e.g. within the cola category one price series could correspond to a 2 liter bottle and another to a case of cans), comparing the price level across products is not meaningful. For this reason we have scaled each price series by its mean value, so that the mean of every product's scaled price will be 1, and every price change can be interpreted as a proportional change. Second, because we are interested in describing the importance of relatively short term changes in price, we are restricting attention to one year time periods. In particular, when we scale price by a price series' mean, that mean is calculated over a 12 month period. Hence, our results only correspond to the

$$(2)\quad p_{ijt} = \alpha_0 + \sum_{1}^{120} \beta_k * Month_k + \sum_n \gamma_n * Sale_n + e_{ijt}$$

Figure 1 shows the r-squareds from equation (1) for each product category (the left-hand bars), and the difference between the r-squareds from equations (2) and (1). As one would expect, these calculations show that the price variation due to nation-wide shocks (such as supply changes) are largest for the three products - bananas, eggs, and lettuce - that are relatively unprocessed agricultural products. The time effects have little explanatory power for the remaining products. In contrast, for every product category, the incremental contribution of sales to explaining variance is at least 25%, and as much as 50%.

We also explored the question of whether the γ_n may be capturing sub-national (i.e., regional) supply shocks. To examine this question, we estimated equations (3) and (4), which generalize equations (1) and (2) by allowing the time effects (β_k) to vary across the 4 census regions.

$$(3)\quad p_{ijt} = \alpha_0 + \sum_{x=1}^{4} \sum_{k=1}^{120} \beta_{xk} * Month_k + e_{ijt}$$

$$(4)\quad p_{ijt} = \alpha_0 + \sum_{x=1}^{4} \sum_{k=1}^{120} \beta_{xk} * Month_k + \sum_n \gamma_n * Sale_n + e_{ijt}$$

The idea behind equations (3) and (4) is to allow for region-specific supply shocks. Figure 2 presents the percentage of retail price variation explained by the region- and time-specific shocks in equation (3), and the difference between the r-squareds from equations (3) and (4). Comparing Figures 1 and 2, we note that as one would expect, allowing the time effects to vary across census regions increases the percentage of variation explained by the time effects. However, for 18 of the 20 categories, the sale effect still explains *more* of the variation in retail prices than do the region specific time effects, while sales explain about the same amount as the region-specific time effects for the two remaining categories. The fact that regional shocks are more important for eggs and lettuce conforms with economic logic. These are the two products most likely to be produced within every region, and hence supply shocks for these products may well vary across regions. This provides some support for our premise that the B_k in equation (1) (and the B_{xk} in equation 3) are capturing supply shifts.

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importance of sales in explaining annual variation in price. Finally, because sale behavior differs across product categories, equations (1) and (2) are estimated separately by product category.

For every category, the incremental contribution of the sales effect is at least 25%, and exceeds 40% for six categories. While the regional supply effects have relatively greater explanatory power than the national shocks, the conclusion remains that temporary price reductions sales account for a substantial proportion of the annual variation in retail prices. This is consistent with our earlier conclusion that most deviations from the regular price are downward.

III. Existing Models of Sales

The previous section demonstrated that *sales*, in the sense of periodic, temporary reductions in specific product prices are a ubiquitous feature of retail competition. To understand this behavior we develop a model which draws primarily from work by Conlisk, Gerstner and Sobel (1984). The basic intuition in their model is that consumers differ in reservation values and in their willingness to wait (which is analytically similar to differences in costs of inventorying). Low-value consumers are more willing to wait for price reductions because the cost of waiting is higher for the high-value consumers, and hence only low-value consumers wait for the periodic price reductions. As a result, periodic price reductions allow a seller to charge a low price to all low-value customers, while most high-value customers purchase at a higher price.[13]

Sobel (1984) extends this model to the case of multiple retailers. High-value consumers are not only willing to pay more for the good and less willing to wait (as in Conlisk, Gerstner, and Sobel), but they also are store loyal. That is, each of these consumers buys from a specific preferred retailer if his price is below their reservation values, and do not buy at all if that retailer's price is above their reservation values. In contrast, low-value consumers are *shoppers*, buying from the retailer offering the lowest price (as long as that price is sufficiently low). The basic characteristic of the equilibrium in Sobel's model is that retailers charge a high price in periods in which the

[13]Lal and Matutes (1994) use a similar explanation for competing multiproduct retailers using different (static) pricing strategies for their array of goods. In their model, each retailer has a low price on a different good, which causes low transportation cost consumers to buy at more than one store each period, but allows the retailers to charge high prices on some items to high transportation cost/high reservation value consumers. Banks and Moorthy (1999) show that coupons can be another way of offering low prices to low reservation price/low search cost customers, while maintaining high prices to high reservation price/high search cost consumers.

aggregate purchases of shoppers would be small. As time passes, shoppers' potential aggregate purchases increase and it eventually becomes profitable to reduce price to compete for their business.

Pesendorfer (2002) both simplifies and generalizes the Sobel model. The simplification is that he assumes low-value customers do not behave strategically - which is to say that they buy whenever the price is below their reservation values.[14] The generalization is that Pesendorfer allows some portion of low-value consumers to be store-loyal. The Pesendorfer model is formally equivalent to a model in which both types of consumers consume one unit of the good in every period (rather than exit the market as soon they purchase one unit), but the low-value consumers consume from their own inventory whenever the price is above their reservation values.[15]

While Pesendorfer's model explains price discounts for goods that can be inventoried, or goods that are infrequently purchased, it does not explain discounts for perishable goods that are frequently purchased and not inventoried by consumers, such as dairy products and produce. However, the evidence (see Table 1) suggests that prices of these items also vary considerably over time. Varian (1980) provides a related explanation for price movements on products that are not typically inventoried (such as consumer durables or perishable food items). As in Sobel and Pesendorfer, Varian assumes that some customers do not compare prices across stores, but rather buy as long as the (randomly-chosen) retailer's price is below the consumer's reservation value, and others buy from the store with the lowest price.[16] Retailers then choose between obtaining a high

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[14] In contrast, in Sobel's model, low-value consumers may wait to buy, even if price is below their reservation values, if they expect price to fall further. Sobel shows that the expected price decline eventually dissipates, and that consumers rationally purchase the good. Sobel's equilibrium constitutes a perfect Nash equilibrium, while Pesendorfer's does not. However, the qualitative predictions of the Pesendorfer model are similar to Sobel's results in that both predict a mixed strategy equilibrium characterized by periodic sales. Because the model in Pesendorfer is more tractable and yields similar results, our approach follows Pesendorfer's.

[15] This formal equivalence require that low-value consumers have some inventory at the beginning of period 1, and that when price is below their reservation values, these consumers buy a sufficient quantity for storage to replace the inventory consumed since the previous sale. These assumptions are discussed further in Section IV.

[16] Given symmetry between retailers, Varian's assumption that some consumer randomly choose a retailer and do not compare prices across retailers is equivalent to Sobel's assumption that 1/J of these consumers are "loyal" to each of the J retailers.

price, and selling only to those customers who do not compare prices, or charging a "low" price, and potentially selling to shoppers as well. Varian shows that the only symmetric equilibrium features mixed strategies, where all retailers choose their price from a continuous distribution, so that each retailer changes his price each period.

Note that the reason for price movement in the Varian model is quite different from the reason in Conlisk, Gerstner, and Sobel. In Varian, these movements result from competition among imperfectly-competing retailers; a monopoly retailer would not vary price, given his demand assumptions. In contrast, the Conlisk, Gerstner, and Sobel model is a monopoly model, and sales are a means of price discrimination. Sales in the Sobel and Pesendorfer models combine elements of both explanations; price movements reflect both competition and a desire to price discriminate.

IV. A Model of Sales and Multiproduct Retailers

The models described in the previous section describe how and why a single-product retailer would adjust his price over time, even with unchanged costs. The phenomenon these models seek to explain is the pattern of prices illustrated in Figure 3. As shown there, peanut butter prices tend to remain constant for long periods, followed by brief periods of lower prices, followed by a return to their initial levels. Further, as Table 1 suggests, this pattern is common for many of the goods sold in supermarkets. One potentially-important abstraction in these models is that they consider the pricing behavior of retailers selling only one product. Real retailers, such as supermarkets, sell a variety of goods, and consumers prefer purchasing bundles of goods from a single retailer. In evaluating whether the existing models explain pricing behavior, it is important to consider how retailers' actions might change if they had a richer set of pricing alternatives available because of these two facts.

To evaluate this question, we analyze competition between retailers, all of whom sell the same two products, and those products have different storage characteristics. These two products correspond to the two types of products described in the previous literature on retailer sale behavior and reflect characteristics of products that supermarkets actually sell. The first good is a *non-perishable* (for which there is a potential for price discrimination) and the second is a *perishable* (for which there is no such potential). Selling two different types of goods allows retailers to use one product to compete with rivals in every period, while reserving the other for discriminating between high-value and low-value consumers in certain periods. We show that, as in the Sobel model, the desire to price discriminate results in periodic sales on the non-perishable product. We also find that, as in the Varian model, the desire to attract shoppers leads to price movements for perishable products. Moreover, we demonstrate that for multiproduct retailers, perishable prices move in response to changes in non-perishable prices. This implies that there are mass points in the pricing distribution for perishable products, an empirically-relevant result which does not follow from *any* of the models described in Section III.[17] Hence, incorporating the multiproduct aspect of retail competition helps explain a broader set of pricing relationships. In addition, modeling retailers as multiproduct sellers generates additional implications for prices. For example, we show that prices for perishable and non-perishable goods sold by a retailer should vary inversely over time.

As Bliss (1988) noted, an important aspect of multiproduct retailing is that most consumers buy an array of goods each time they visit certain kinds of retailers (especially supermarkets). Our model incorporates this feature by assuming that consumers know all of the relevant prices before visiting any store, and shop at no more than one store in each period. Thus, if a consumer purchases both goods in the same period, she buys both of them from the same retailer.[18] It follows that retailers compete for customers by attempting to offer the most attractive bundle of prices.

In analyzing what constitutes the most attractive bundle, it is necessary to consider the number of units of a good that a consumer might purchase during each visit. Of particular interest to us are non-perishable goods, which have the property that some consumers can practically buy more units of the good than they plan to consume in that period, inventorying a portion for later consumption. Hence, while consumers all have unit demand for *consuming* each good in each period, they do not necessarily all *purchase* one unit of the non-perishable each period.

Two key assumptions in the literature on intertemporal price discrimination described in the

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[17]As Table 1 shows, perishable goods such as lettuce, eggs, ground beef, and bananas all have mass points in their pricing distributions.

[18] Salop and Stiglitz (1982) make a similar assumption about the cost of visiting multiple retailers in their analysis of sales. As discussed in Section V, similar results can be obtained with weaker assumptions regarding the transactions cost of visiting multiple retailers.

previous section are that consumers are heterogeneous in both their valuations of the product and in their cost of search, and that these two values are positively correlated across individuals. We view such a positive correlation as plausible. For example, high-income consumers are likely to both have higher reservation values for many goods, and due to a higher shadow value of time, lower willingness to invest in learning about prices and taking steps to take advantage of that knowledge. There is reason to believe this positive correlation is more likely for non-perishable goods than perishables. For example, non-perishable goods typically have more value-added than perishable goods (e.g., breakfast cereals or canned soup as compared to bananas or ground beef). Products with considerable value-added by the manufacturer will typically be those for which brand names are important. Economic theory suggests that brand names will be more valuable for consumers who view search as particularly costly (see, e.g., Klein and Leffler, 1981, and Ward and Lee, 1999, for recent empirical evidence). It follows that there will be greater heterogeneity in reservation values for branded products than commodity products.[19] Relating this back to perishable and non-perishable products, we note that, consistent with the idea that brand names are more important for non-perishables, supermarkets typically carry a single product in many perishable categories, including several in our sample (bananas, lettuce, ground beef), while carrying multiple versions in the non-perishable categories.

Hence, following the literature described in Section III, we assume there are two kinds of consumers; those who are store loyal, and do not compare prices across stores (i.e., they have high search costs) and those who are shoppers, and evaluate stores on the basis of price. Following this literature, we assume that store loyals have higher reservation values and storage costs for the non-perishable product than do the shoppers. Specifically, store loyals have reservation values of α_H, which is higher than the value that shoppers (α_L) place on it. To isolate the incentive to price discriminate, we assume that store-loyals have infinite storage costs for the non-perishable, and shoppers have zero storage costs for it, although any significant difference in customers' storage costs would be sufficient for our purposes.

[19] These considerations also provide a justification for the assumption that reservation values and shopping costs are correlated for the non-perishable; consumers who find shopping particularly costly will have a high reservation value for branding.

For the reason discussed above, we assume that consumers are less heterogeneous with respect to their reservation values for perishable goods than for non-perishables. Specifically, we assume that all consumers have a common reservation value of β for the perishable (which is identical to the assumption in Varian). While this is a stronger assumption than needed for many of our results, it allows for a simple closed-form pricing equilibrium. In section V, we discuss the implications of more general assumptions regarding reservation values.

These assumptions imply the following about consumer behavior: Letting P_P be the price of the perishable and P_N be the price of the non-perishable at her preferred store, a store-loyal customer will make one of four choices in any period:

if $P_N > \alpha_H$ and $P_P > \beta$	buy nothing
if $P_N \leq \alpha_H$ and $P_P > \beta$	buy one unit of the non-perishable only
if $P_N > \alpha_H$ and $P_P \leq \beta$	buy one unit of the perishable only
if $P_N \leq \alpha_H$ and $P_P \leq \beta$	buy one unit of each good

Shoppers also make one of 4 choices. Suppose there are J retailers and let the superscript j index the specific store, then a shopper's four choices in any period are:

if $\min_j (P_N^j) > \alpha_L$ and $\min_j (P_P^j) > \beta$	buy nothing
if $\min_j (P_N^j) > \alpha_L$ and $\min_j (P_P^j) \leq \beta$	buy one unit of the perishable at lowest-priced store
if $\min_j (P_N^j) \leq \alpha_L$ and $\min_j (P_P^j) > \beta$	buy multiple units of the non-perishable at the lowest-priced store
if $\min_j (P_N^j) \leq \alpha_L$ and $\min_j (P_P^j) \leq \beta$	buy one unit of the perishable or multiple units of the non-perishable (or both) at whatever store offers the greatest consumer surplus.

The difference between the fourth option in the two cases illustrates an important component of shopping in our model. A store-loyal's decision rules regarding her purchases of the two products are independent; she buys one unit of good x at her preferred store if good x's price is below her reservation value for good x, without reference to good y's price. In contrast, a shoppers' decision rules for the two goods are linked. Since by assumption consumers visit at most one store per period, shoppers must consider the entire set of prices offered by each retailer, and determine the consumer surplus offered by each store based on the observed prices, and choose the store that offers the largest consumer surplus. Depending on the prices of the perishable and non-perishable items, they may buy one unit of the perishable, multiple units of the non-perishable, or both goods.

As long as $P_P^j \leq \beta$ and $P_N^j \leq \alpha_H$, customers loyal to retailer j will buy both products at that store. Indeed, if retailers only cared about selling to store-loyals, they would always charge $P_P = \beta$ and $P_N = \alpha_H$.[20] The reason that retailers might offer lower prices is that shoppers choose between retailers on the basis of the consumer surplus they can obtain.

Because shoppers purchase bundle of goods, they base decision on where to purchase on the consumer surplus (summed over all goods) offered by each retailer. It follows that competition between retailers can be described in terms of the consumer surplus offered and that retailers can offer shoppers that surplus using either or both of two instruments: perishable prices (P_P) and non-perishable prices (P_N). Define δ_j to be the consumer surplus retailer j offers shoppers. For example, if retailer j chooses to only offer the perishable good on "sale" (i.e., $P_N = \alpha_H$, $P_P < \beta$) then $\delta_j = \beta - P_p$. For the non-perishable, shoppers receive consumer surplus of $(\alpha_L - P_N)$ times the number of units purchased. To conform with the models described in the previous section, we assume that shoppers purchase a sufficient quantity of the non-perishable to replace the amount they consumed since the previous sale.[21] Letting M be the number of periods since the last sale, $\delta_j = \max\{[0,(M+1)(\alpha_L - P_N^j)]$

[20] If $P_P > \beta$ (or $P_N > \alpha_H$), then retailer j makes no sales of the perishable (non-perishable). Hence, we restrict the analysis to values of $P_N \leq \alpha_H$ and $P_P \leq \beta$.

[21] Following Pesendorfer, we assume that the decision rule of low-value consumers is to buy the non-perishable whenever $P_N < \alpha_L$. While the assumption that consumers exactly replace their depleted inventory is not derived from a model of optimal consumer inventory behavior, this omission is not critical for the question we are interested in studying. The only property of inventory behavior that is required for our results is that when a sale occurs, aggregate purchases of the good by low-value consumers is increasing in the length of time since the previous sale. This property holds for some simple inventory models that we investigated. For this reason, our model does not require identical inventorying behavior by all low-value consumers.

+ max[0, β- P_P^j]}. Whether retailer j makes any sales to shoppers depends on how δ_j compares to the consumer surplus (δ) offered by rival retailers.

To reduce notational complexity, we interpret α_L, α_H and β as the difference between the consumer's reservation value and the constant marginal cost of selling the good, so that we normalize the retailers' cost to zero. Additionally, we normalize the number of customers to one. We also assume that the seller cannot determine an individual consumer's type, so he cannot directly price discriminate on that basis. Given these assumptions, we can derive retailer profits from alternative pricing policies. Suppose that a portion, γ (where γ < 1) of customers are store-loyal, and (1-γ) are shoppers. Retailers are assumed to be symmetric, so that γ/J are loyal to each store. One strategy for retailer j is to charge $P_N^j = \alpha_H$ and $P_P^j = \beta$, which results in $\delta_j = 0$ for all shoppers. This yields profits of $\gamma(\alpha_H + \beta)/J + (1-\gamma)\beta/J$ if all rival retailers choose these same prices, and $\gamma(\alpha_H + \beta)/J$ if any rival retailer chooses to offer $\delta > 0$. An alternative strategy is to have a sale on the perishable only, so that $P_P^j < \beta$ and $P_N^j = \alpha_H$. This yields profits of $\gamma(\alpha_H + P_P^j)/J + (1-\gamma) P_P^j$ if retailer j offers the highest δ. Finally, he can only place the non-perishable on sale so that $P_N^j < \alpha_L$ and $P_P^j \leq \beta$, and retailer j will earn profits of $\gamma(P_P^j + P_N^j)/J + (1-\gamma)[(M+1)P_N^j + P_P^j)]$ if $\delta_j > \delta_{-j} \equiv \max_{i \neq j}(\delta_i)$ and $\gamma(P_N^j + P_P^j)/J$ otherwise.

We now proceed to derive equilibrium pricing for the two goods, under the assumption that retailers are all risk neutral. Our first result is that retailer j will at most, put one good on sale.

Proposition 1: It is more profitable for retailer j to place one good on sale than both goods (i.e., it is not profitable to charge $P_N < \alpha_H$ and $P_P < \beta$).

Proof: Retailer j's profit is

$$\frac{\gamma}{J}(P_P + P_N) + (1-\gamma)[Prob(\delta_j > \delta_{-j})](P_P + (M+1)P_N) \quad if\ P_N < \alpha_L$$
$$\frac{\gamma}{J}(P_P + P_N) + (1-\gamma)[Prob(\delta_j > \delta_{-j})]P_P \quad if\ P_N > \alpha_L$$

If retailer j chooses P_N and P_P such that $\alpha_H > P_N \geq \alpha_L$ and $P_P < \beta$ then $\delta = \beta - P_P$, and a small increase in P_N increases profits without reducing δ. Hence, $\alpha_H > P_N \geq \alpha_L$ and $P_P < \beta$ is not a profit-maximizing strategy, as retailer j would raise P_N to α_H. Conversely, if $P_N < \alpha_L$ and $P_P < \beta$, $\delta = \beta - P_P + (\alpha_L - P_N)(M+1)$, and an increase of ϵ in P_P accompanied by a decrease of $\epsilon/(M+1)$ in P_N

increases (strictly for M > 0) profits without changing δ, so he would set $P_P = \beta$. Hence, having only one good on sale dominates having both $P_P < \beta$ and $P_N < \alpha_H$. ■

The intuition behind Proposition 1 is that the cost of offering any given δ to shoppers is the foregone profits that could be obtained by selling to loyals only. It follows that for any given δ, retailer j wishes to offer it in a way that minimizes this cost. While it costs at least $\alpha_H - \alpha_L$ to offer any surplus through the non-perishable price, once P_N is less than α_L, the incremental cost of raising δ through a reduction in P_N is small. Hence, the average cost (per unit of δ) of offering surplus through the non-perishable price is decreasing in the level of δ, while the average cost of offering δ through lowering the perishable price is independent of the level of δ. Thus, for low levels of δ the retailer will put the perishable on sale and for high δ the non-perishable will be put on sale.

Proposition 1 shows that no more than one product will be on sale at any point in time. We now turn to the question of which, if either product, will be on sale. To characterize the equilibrium, note that $P_N = \alpha_H$, $P_P = \beta$ (or equivalently, $\delta = 0$) for all retailers is not an equilibrium, since any individual retailer can profitably offer $\delta > 0$ (by setting P_P slightly less than β), and make sales to all the shoppers. Varian (1980) formally shows that the only symmetric equilibrium features a mixed strategy in prices, or more generally, in δ. Because the distribution of prices has no mass points, it follows that at least one product will be on sale at all times. Varian's result (which we refer to as Lemma 1) combined with Proposition 1, implies that *exactly* one product will be on sale at each point in time. This suggests an interesting empirical implication: price movements for the perishable and the non-perishable goods should be negatively correlated. Specifically, in the symmetric equilibrium, whenever the non-perishable price changes, the perishable price will move in the opposite direction.[22]

Lemma 1: There are no point masses in the symmetric equilibrium distribution of δ.

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[22]The implication that no more than one product will be on sale at any point in time derives in part from the assumption that consumers necessarily visit no more than one retailer in each period. As we discuss in Section V, in a model in which consumers can (at some cost) visit multiple retailers, equilibrium might consist of multiple goods being on sale.

Proof: See Varian (1980), Proposition 3.

This generalizes the earlier intuition that $\delta = 0$ for all retailers cannot be an equilibrium. The idea is that if there were a specific $\tilde{\delta}$ which is offered with positive probability, a store would find it profitable to deviate by offering a slightly higher δ, say $\tilde{\delta} + \epsilon$, with that same probability. Such a store would expand its expected sales by a positive amount, since it would attract all of the shoppers when all of its rivals tied (which occurs with positive probability), while the loss due to the price reduction necessary to obtain the higher δ is arbitrarily small. Lemma 1 says that in the symmetric equilibrium, retailers do not offer any specific δ with a positive probability; instead in every period δ is drawn from a common distribution function. That is, the only symmetric equilibria involve mixed strategies in δ_j; whereby retailers choose δ according to some continuous distribution $G(\delta)$.[23] The remainder of this section is devoted to explicitly deriving the pricing equilibrium. The next two lemmas provide lower bounds for the pricing distributions for the two goods.[24]

Lemma 2: The lowest price any retailer will ever charge for the non-perishable is

$$\underline{P}_N = \frac{\gamma\alpha_H - J(1-\gamma)\beta}{\gamma + J(1-\gamma)(M+1)}.$$

Proof: Any sale price must yield profits at least as great as the profits from not having a sale. Note that Proposition 1 implies that if $P_N < \alpha_L$, then $P_P = \beta$, and the lowest price that retailer j will ever find it profitable to charge for the non-perishable solves

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[23]Note that $G(\delta)$ changes over time with changes in M, as detailed below.

[24]The analysis here does not require $P_N > 0$. Since we interpret P_N as the margin on the non-perishable, $P_N < 0$ does not imply a negative price. Moreover, given our assumption that shoppers buy both goods at the same store if $(\beta - P_P) \geq 0$ and $(\alpha_L - P_N) > 0$, P_N less than zero, but more than

$\dfrac{-P_P(1-\gamma+\gamma/J)}{(1-\gamma)(1+M)+\gamma/J}$ might be profitable. Hence, using the non-perishable as a "loss leader" may be profitable.

$$\frac{\gamma}{J}(\alpha_H + \beta) = \frac{\gamma}{J}(P_N + \beta) + (1-\gamma)[(M+1)P_N + \beta]$$

where the left-hand side is the profit from not having a sale, and the right-hand side is the profits from having a sale if he were certain that he would be offering the highest δ. Simplification yields

$$\underline{P}_N = \frac{\gamma\alpha_H - J(1-\gamma)\beta}{\gamma + J(1-\gamma)(M+1)} \blacksquare$$

It follows from Lemma 1 that a necessary condition for retailer j to place the non-perishable on sale is that

$$\alpha_L \geq \frac{\gamma\alpha_H - J(1-\gamma)\beta}{\gamma + J(1-\gamma)(M+1)}. \tag{3}$$

Since the right-hand side of equation (3) is decreasing in M, a sale on the non-perishable becomes profitable for a larger range of parameter values as M rises. The intuition is that as the number of periods since the last sale (M) grows, the ratio of the quantity of the non-perishable bought by loyals to the quantity bought by shoppers (who only buy the non-perishable during sales) falls. That is, the profit from selling to new customers increases with M, while the lost profit from not charging α_H to loyal customers is independent of M.

Lemma 3: The lowest price any retailer will ever charge for the perishable is $\underline{P}_P = \beta\frac{\gamma}{\gamma + J(1-\gamma)}$.

Proof: Following the same logic as Lemma 1, the lowest P_P that retailer j will ever choose solves

$$\frac{\gamma}{J}(\alpha_H + \beta) = \frac{\gamma}{J}(P_P + \alpha_H) + (1-\gamma)P_P$$

Solving for $\underline{P}_P$ yields $\underline{P}_P = \beta\frac{\gamma}{\gamma + J(1-\gamma)}$. $\blacksquare$

Proposition 1 along with Lemmas 2 and 3 implies that the maximum δ any retailer will offer is max{(M+1)(α_L - $\underline{P}_N$), β - $\underline{P}_P$ }. As shown in Lemma 4, there is no pure strategy equilibrium in pricing. Rather, in equilibrium, each retailer either chooses a δ by either setting $P_N < \alpha_L$ or by setting $P_P < \beta$ in order to attract shoppers. In addition, for any given δ, the retailer will choose the combination of P_N and P_P that maximizes his profits. The next lemma develops the properties of the profit-maximizing prices for any δ.

To determine which good is the more profitable means of offering any specific δ, it is useful to introduce some additional notation. Let $\pi_P(\delta)$ be the retailer's profit from placing the perishable on sale (i.e., setting $P_P = \beta - \delta$, which yields consumer surplus of δ) and let $\pi_N(\delta)$ be the retailer's profit from placing the non-perisable on place (i.e., setting $P_N = \alpha_L - \delta/(M+1)$, which also yields consumer surplus of δ). Recall that when consumer surplus is generated by lowering P_N the average cost (per unit of δ) is decreasing in the level of δ, while the average cost is constant if consumer surplus is generated by P_P. This implies that $\pi_N(\delta) - \pi_P(\delta)$ is increasing in δ. Because of this relationship, we can derive a $\bar{\delta}$, such that $\pi_N(\bar{\delta}) = \pi_P(\bar{\delta})$, which means that $\pi_N(\delta) > \pi_P(\delta)$ for all $\delta > \bar{\delta}$, with the reverse inequality for all $\delta < \bar{\delta}$. There are two kinds of solutions for the δ at which $\pi_N(\bar{\delta}) = \pi_P(\bar{\delta})$; interior and corner. $\bar{\delta}$ results in a corner solution if the δ that equates π_N and π_P is sufficiently large that it is not profitable to offer that δ (i.e., a corner occurs if $(M+1)(\alpha_L - \underline{P}_N) < \bar{\delta}$ -- in words, the δ associated with the maximum profitable discount on the non-perishable is less than $\bar{\delta}$). In that case, $\pi_N(\delta) < \pi_P(\delta)$ for all relevant δ, and only the perishable will be discounted. When there is a corner solution, pricing for the perishable is identical to the pricing that would occur if retailers sold only that product. An interior solution arises if $(M+1)(\alpha_L - \underline{P}_N) > \bar{\delta}$. In that case, either good may be offered for sale. Lemma 4 solves for $\bar{\delta}$ and provides the basis for determining the distribution function for δ.

Lemma 4: Let $\bar{\delta} \equiv \frac{(M+1)}{M}(\alpha_H - \alpha_L) - \frac{(M+1)^2}{M}\frac{J(1-\gamma)}{\gamma}\ Pr(\bar{\delta})\ \alpha_L$ where $Pr(\bar{\delta})$ is the probability that retailer j attracts shoppers when it offers $\delta_j = \bar{\delta}$ (for $M > 0$). When $M = 0$, $\bar{\delta}$ is implicitly defined in $Pr(\bar{\delta}) \equiv \frac{\gamma(\alpha_H - \alpha_L)}{J(1-\gamma)\alpha_L}$ if $\gamma(\alpha_H - \alpha_L) < J(1-\gamma)\alpha_L$, and $\bar{\delta} = (\beta - \underline{P}_P)$ otherwise. Then

a. $\bar{\delta} \geq 0$,

b. $\pi_P(\delta) > \pi_N(\delta)$ for all $\delta < \bar{\delta}$,

c. If $(M+1)(\alpha_L - \underline{P}_N) > \bar{\delta}$ (i.e., an interior solution exists) then $\pi_P(\delta) < \pi_N(\delta)$ for δ such that $(M+1)(\alpha_L - \underline{P}_N) > \delta > \bar{\delta}$.

Proof: See appendix.

Corollary: In the symmetric equilibrium,

a. $\bar{\delta} = (M+1)/(\gamma M)[\gamma(\alpha_H - \alpha_L) - \alpha_L J(1-\gamma)(M+1)(G(\bar{\delta}))^{J-1}] > 0$ for $M > 0$. $\bar{\delta}$ is also positive for $M = 0$.

b. As long as β is greater than 0, $\pi_P(\delta) > \pi_N(\delta)$ for δ sufficiently small. That is, there will always be a positive probability of a sale on the perishable for $\beta > 0$.

Proof: See appendix

Lemma 4 and its corollary indicate that in the symmetric equilibrium, $\bar{\delta}$ is always positive, which means that there is always a range of δ for which putting the perishable on sale is the most profitable means of offering that δ to shoppers. If there is a corner solution (for some M), then only the perishable will be on sale for that M. However, because $(M+1)(\alpha_L - \underline{P}_N)$ is increasing in M, there may be an interior solution for sufficiently large M. When there is an interior solution, then the retailer will offer the non-perishable on sale when it offers a large amount of consumer surplus to non-loyals ($\delta > \bar{\delta}$) and place the perishable on sale for $\delta < \bar{\delta}$ (Lemma 4, results b and c). The logic for why sales on the non-perishable will be associated with large δ is that it is costly (in terms of foregone profit from store loyals) to offer any consumer surplus to shopper by setting P_N below α_L, since the retailer has to sacrifice at least $\gamma(\alpha_H - \alpha_L)/J$. In order for the retailer to find such a price reduction profitable, he must have a large probability of winning. In contrast, offering small δ by setting P_P below β entails only a small reduction in profits from loyals, and consequently small δ is always offered through discounts on the perishable.

In terms of deriving equilibrium prices, Lemma 4 means that $G(\delta)$ can be decomposed into two cumulative distribution functions; $G(\delta) = 1 - F_1(P_N)$ for $\delta \geq \bar{\delta}$ and $G(\delta) = (1-\rho)(1 - F_2(P_P))$ for

$\delta<\bar{\delta}$. Proposition 2 derives these two distribution functions.

Proposition 2. Let $F_1(P_N)$ be the distribution of non-perishable prices and $F_2(P_P)$ be the distribution of perishable prices in the symmetric equilibrium.

a. If $(M+1)(\alpha_L - \underline{P}_N) > \bar{\delta}$, then retailer j puts the non-perishable on sale with probability $\rho = 1 - G(\bar{\delta})$.

$$\rho = 1 - \left[\frac{\frac{\gamma}{J}(\alpha_H - P_N(\bar{\delta}))}{(1-\gamma)[(M+1)P_N(\bar{\delta}) + \beta]} \right]^{\frac{1}{J-1}}$$

here $P_N(\bar{\delta}) = \alpha_L - \bar{\delta}/(M+1)$. When the non-perishable is on sale, $P_P = \beta$.

b. If $(M+1)(\alpha_L - \underline{P}_N) > \bar{\delta}$, then the cumulative distribution function for P_N is

$$F_1(P_N) = \begin{bmatrix} 1 - \left[\frac{\gamma(\alpha_H - P_N)}{J(1-\gamma)[(M+1)P_N + \beta]} \right]^{\frac{1}{J-1}} & \text{for } P_N \in [\underline{P}_N, \alpha_L - \frac{\bar{\delta}}{M+1}], \\ \rho & \text{for } P_N \in (\alpha_L - \frac{\bar{\delta}}{M+1}, \alpha_H), \\ 1 & \text{for } P_N = \alpha_H \end{bmatrix}$$

c. If $(M+1)(\alpha_L - \underline{P}_N) < \bar{\delta}$, then $\rho = 0$ and $F_1(P_N) = 0$ for $P_N < \alpha_H$ and $F_1(\alpha_H) = 1$.

d. With probability $1-\rho$ retailer j sets $P_N = \alpha_H$, and chooses P_P according to the distribution

function $F_2(P_P) = 1 - \left[\frac{(\beta - P_P)\gamma}{J(1-\gamma)P_P} \right]^{\frac{1}{J-1}} (1-\rho)^{-1}$.

Proof: a and b. From Lemmas 1-3, we know that δ is randomly drawn from a continuous distribution with support $(0, \max\{\beta - \underline{P}_P, (M+1)(\alpha_L - \underline{P}_N)\})$. In equilibrium, the profits from charging each price for which the density function is positive must be equal to the profits from charging $P_N = \alpha_H$ and $P_P = \beta$, which are equal to $\gamma[\beta + \alpha_H]/J$. To calculate $G(\delta)$, note that by Proposition 1, retailer j will put at most one good on sale. If $(M+1)(\alpha_L - \underline{P}_N) > \bar{\delta}$, then retailer j will sometimes put the non-perishable on sale. Specifically, Lemma 4 implies that whether P_N or P_P will be lowered in order to generate consumer surplus of δ depends on δ. For $\delta > \bar{\delta}$, δ is obtained by setting $P_N < \alpha_L$. Given this result, when retailer j chooses a $\delta > \bar{\delta}$, the probability that a rival offers more consumer surplus is equivalent to the probability the rival offers a lower P_N. Hence for $\delta > \bar{\delta}$, $G(\delta) = 1 - F_1(P_N)$, where $F_1(P_N)$ is the common c.d.f. for P_N. To determine $F_1(P_N)$, note that any P_N for which the density function is positive must yield the same profits as can be obtained by not holding a sale. Hence, the distribution function for P_N, conditional on a sale occurring on the non-perishable must solve

$$\frac{\gamma}{J}(\alpha_H + \beta) = \frac{\gamma}{J}(P_N + \beta) + (1-\gamma)[(M+1)P_N + \beta](1 - F_1(P_N))^{J-1}$$

Solving for $F_1(P_N)$ yields

$$F_1(P_N) = 1 - \left[\frac{\frac{\gamma}{J}(\alpha_H - P_N)}{(1-\gamma)[(M+1)P_N + \beta]}\right]^{\frac{1}{J-1}}$$

The lower bound for the support is the lowest price the retailer could profitably charge for the non-perishable item. As Lemma 2 shows, this price is

$$\underline{P}_N = \frac{\gamma\alpha_H - J(1-\gamma)\beta}{\gamma + J(1-\gamma)(M+1)}$$

The highest P_N for which $G(\delta) = 1 - F_1(P_N)$ corresponds to the δ for which it is equally profitable to have a sale on the perishable and non-perishable, or $P_N = \alpha_L - \bar{\delta}/(M+1)$. By Lemma 4, for any $\delta < \bar{\delta}$, it will be more profitable to lower P_P rather than P_N, so that letting $\rho \equiv F_1\left(\alpha_L - \frac{\bar{\delta}}{M+1}\right)$, we know that $F_1(P_N) = \rho$ on the open interval $\left(\alpha_L - \frac{\bar{\delta}}{M+1}, \alpha_H\right)$, and $F_1(\alpha_H) = 1$. By Proposition 1, when $P_N < \alpha_L$, $P_P = \beta$.

c. If $(M+1)(\alpha_L - \underline{P}_N) < \bar{\delta}$, then it is more profitable to put the perishable on sale than the non-perishable for all relevant δ. In that case, retailer j will set $P_N = \alpha_H$.

d. From Lemma 1, we know that there is not a point mass at $\delta = 0$, so that the perishable must be on sale whenever $P_N = \alpha_H$. To solve for $F_2(P_P)$, the c.d.f. of P_P, first note that expected profits when the perishable is on sale at $P_P = \beta - \delta$ are $\gamma(\beta- \delta + \alpha_H)/J + (1-\gamma)\, G(\delta)^{J-1}(\beta - \delta)$. In equilibrium, this must equal the expected profits from not having a sale so that

$$G(\delta) = \left[\frac{\delta\gamma}{J(1-\gamma)(\beta-\delta)}\right]^{\frac{1}{J-1}} \tag{4}$$

To relate $F_2(P_P)$ to $G(\delta)$, note that if retailer j puts the perishable on sale, a rival might offer more consumer surplus either by putting the non-perishable on sale, or by offering a lower perishable price. This means that the probability that any one rival offers more consumer surplus than retailer j is $1 - G(\delta) = \rho + (1 - \rho)(F_2(P_P)) \Rightarrow G(\delta) = (1 - \rho)(1 - F_2(P_P))$. Using (4) this implies

$$F_2(P_P) = 1 - \left[\frac{(\beta - P_P)\gamma}{J(1-\gamma)P_P}\right]^{\frac{1}{J-1}}(1-\rho)^{-1}. \blacksquare$$

Proposition 2 shows that the profitability of alternative prices for the non-perishable depends on M, the length of time since the previous sale by any retailer on that product. As long as $\alpha_L > \underline{P}_N$, the probability of a sale and the cumulative distribution function for any $P_N < \alpha_L$ is strictly increasing in M. Additionally, Lemma 2 implies that the lower bound for the support of the distribution of P_N declines as M rises. These results parallel the results in the Conlisk et al., Sobel and Pesendorfer models. The primary difference is that sales on the non-perishable are more likely in the multi-product environment (to see this, note that F_1 is increasing in β, and the single-product models are equivalent to $\beta = 0$). The implications for perishable prices are similar to those in Varian; the primary differences are that F_2 implies the distribution of prices will vary over time and may feature a mass point at β. Example 1 presents an illustration of the equilibrium, and how the price distributions change over time.

Example 1: Suppose that $\alpha_H = 5$, $\alpha_L = 2$, $\beta = 1.5$, $\gamma = .75$ and $J = 2$. This implies that the perishable price will be at least .9 (i.e., $\underline{P}_P = .9$), while the lower bound on the support for the non-perishable price distribution ($\underline{P}_N$) depends on M. At M = 0 or 1, it turns out that there is a corner solution, so that the non-perishable will never be discounted. For example, for M = 1, $\bar{\delta} = 0.605$ and $\underline{P}_N = 1.71$, which means that the P_N which generates $\bar{\delta}$ (1.697) is below the lowest price a retailer could ever profitably charge for the non-perishable when M= 1. Hence, for M = 0 or 1, δ takes on a value between 0 and 0.6, and δ is always created by setting P_P below β.

As M increases, the profitability of putting the non-perishable on sale rises. For example, for M = 2, $\underline{P}_N =$ 1.33, and a sale on the non-perishable would be profitable if $\bar{\delta} < (M+1)(\alpha_L - \underline{P}_N)$ = 3 * .67 = 2. As shown in Table 5, for M = 2 $\bar{\delta} = 0.46$, so that this inequality is satisfied. That is, for δ between 0.46 and 2, δ is created by lowering P_N, and for δ between 0 and 0.46, δ is created by lowering P_P. The probability of a sale on the non-perishable (ρ) is .327 when M = 2. If there is no sale on the non-perishable when M = 2, then since $\bar{\delta}$ is decreasing in M, and $(M+1)(\alpha_L - \underline{P}_N)$ is increasing in M, $\bar{\delta}$ will be less than $(M+1)(\alpha_L - \underline{P}_N)$ for all M > 2. In fact, the probability of holding a sale on the non-perishable is nearly 50% for M=3, almost 59% for M=4, and about 65% for M=5.

Table 5

M	$\underline{P}_N$	$\bar{\delta}$	ρ	$P_N(\bar{\delta})$
2	1.333	.464	.327	1.845
3	1.091	.38	.491	1.905
4	.923	.323	.589	1.935
5	.8	.281	.654	1.953

Figure 4 portrays the c.d.f. for these four values of M. As the corollary to Lemma 4 implies, ρ is strictly less than 1, so that for any M, there is a positive probability that the perishable will go on sale. The c.d.f. for the perishable price changes with M though its effect on ρ. The c.d.f. for P_P for values of M between 1 and 5 is portrayed in Figure 5. ■

The example illustrates several implications of the model. First, the probability of a sale on a non-perishable is an increasing function of the elapsed time since the previous sale on that good by any retailer. Second, F_1 implies that the probability and depth of a sale of the non-perishable is increasing in J, the number of retailers. Similarly, F_2 implies that the depth of sale on the perishable is increasing in J. Hence, other things equal, markets with fewer retailers will have fewer and shallower sales. Third, a price reduction on a perishable will be less likely in a period in which there is a price reduction on the non-perishable. Finally, Proposition 2 implies that a non-perishable is more likely to have the same price in consecutive periods than a perishable, and conditional on a price reduction occurring, the average change will be larger for the non-perishable.

V. Discussion

The model in Section IV explains some features of the observed pricing behavior of food retailers. In this section, we discuss the robustness of these results to alternative assumptions, and explore some of the empirical implications of the model.

One important assumption in the model is that the degree of heterogeneity in consumer valuation for the perishable is lower (or less correlated with type) than the heterogeneity for the non-perishable. To simplify the presentation in Section IV, we make the extreme assumption that consumers are homogeneous in their valuations of the perishable. Similar results to those found in Section IV can be obtained with less restrictive assumptions. One such alternative assumption is that average reservation values for the perishable are the same as for the non-perishable, but unlike the case for the non-perishable, the reservation values for the perishable are uncorrelated with whether the consumer is a store-loyal or a shopper. Hence, for perishables, γ% of both shoppers and store-loyals have a reservation value of α_H and $(1-\gamma)$% of both groups have a reservation value of α_L. A second alternative assumption is that consumers are heterogeneous with respect to reservation values for the perishable, and those reservation values are perfectly correlated with whether the consumer is a store-loyal or a shopper, but the degree of heterogeneity is lower for the perishable (e.g., the value that shoppers place on the perishable is of β_L, which is equal to α_L but store loyal customers' value of the perishable is $\beta_H < \alpha_H$).

The main change associated with using the second alternative assumption, rather than the assumption in Section IV, is that Lemma 1 no longer holds. Instead, there is a mass point in the distribution of δ at $\delta = 0$, or equivalently at $P_N = \alpha_H$, $P_P = \beta_H$. Lemma 1 does hold under the first alternative assumption, but we cannot calculate a closed-form equilibrium price distribution under that assumption.

However, under either of these two alternative modeling assumptions, most of the important results from section IV still hold. For example, Proposition 1 continues to hold under either of these two assumptions. Significantly, under either of these assumptions, the basic finding that small δ are offered by placing the perishable on sale still holds, as does the finding that the probability of a sale on the non-perishable increases over time.

Another important assumption in the model is that consumers necessarily visit no more than one retailer in each period. This is an important assumption in establishing Proposition 1. If consumers can (at some cost) visit multiple retailers, equilibrium might consist of multiple goods being on sale. For example, in Lal and Matutes' (1994) model, if rival retailers put different goods on sale and transportation costs are low enough, then consumers will "cream skim" by buying some items at each retailer. Under those circumstances, it may be profitable for a retailer to offer a given δ by placing several goods on sale to deter such cream skimming.

Even if retailers place multiple products on sale (whether to prevent cream-skimming or otherwise), the logic of our analysis suggests that for a retailer selling a large number of products, the number of items on sale will be fairly stable, but the composition of the sale items will change over time. In particular, the price-discrimination motive implies that specific non-perishables will go on sale periodically. In some periods, it will be profitable to put a relatively large number of such products on sale. In those periods, relatively few perishable products will be on sale. Conversely, in periods in which there are relatively few non-perishables that can profitably be put on sale, a larger number of perishables will be on sale.

For this reason, we suspect that the model's implication for the correlation of non-perishable and perishable prices does generalize to a model in which multiple goods are placed on sale each period. Because the BLS data do not identify the supermarket associated with each price series, we cannot test this implication using the BLS data. However, a data set which consisted of a sufficiently large number of items from individual supermarkets would allow for a test of this prediction. In particular, such data would enable us to determine if price movements for perishable products were negatively correlated with price movements for non-perishable products, as predicted by the theory.

Another implication of the model concerning perishable and non-perishable prices also generalizes to an environment with multiple goods of each kind. Because the motivation for sales differs between perishable goods and non-perishables goods, pricing behavior will differ between these types of goods; price movements would be more frequent, but perhaps smaller, for perishable goods. To empirically examine that implication, one needs to operationalize the idea of perishability. We view perishability as related to storage costs; goods with higher storage costs are more perishable. Hence, although frozen orange juice will last months in a freezer, it is more costly to store frozen orange juice than it is to store canned tuna fish, because the latter requires a less-costly storage facility. For this reason, the notion of perishability is not as stark as the distinction made in the model.

With this definition in mind, we present the following suggestive evidence. Consider the 20 categories of goods discussed in Section II. Of these goods, we view four as clearly the most difficult to store for long periods; eggs, lettuce, bananas and ground beef. At the other extreme, several of the products can be readily stored for long periods of time. Such products include baby food, soap and detergent, canned soup, peanut butter and paper towels.

Table 6 presents the frequency of non-zero price changes between consecutive months for each of the 20 categories in our sample. Consistent with our expectation, price changes fairly frequently for the four highly-perishable categories of goods. For example, lettuce prices are unchanged in consecutive months less than 10% of the time. To be sure, lettuce (and perhaps the three other products as well) is more likely to be subject to supply shifts than other products in our sample. Such shifts change wholesale prices and hence retail prices, and for all of our products some of the observed price movements are not "sale" behavior in the sense used here.

Ideally, we would control for these shifts using wholesale price data. Because we lack that data, we instead capture aggregate cost shifts using time-specific indicator variables. Consistent with our expectation (see Figure 1),[25] bananas, eggs, and lettuce are the three products with the greatest

[25]By allowing separate time effects for each of the 120 months in our data set, we are implicitly controlling for all national time-specific supply and demand shocks for each of the 20 product categories we study. Hence, the left bar for each product in Figure 1 shows an upper bound on the proportion of retail price variation that can be accounted for by national changes in wholesale prices. This is an upper bound because this technique will overstate the role of wholesale price changes and understate the role of retailer behavior. For example, if there is a

proportion of their price variation explained by these time-specific effects (approximately 13%, 26%, and 22% respectively). In terms of testing the theory, what is noteworthy is that sales appear to account for an *additional* 44%, 31%, 25%, and 44% of retail price variation for bananas, eggs, lettuce, and ground beef. This suggests that for these products retail margin changes play at least as important a role in price variation as do supply changes.

At the other extreme, the data are less consistent with our expectations. While baby food has the greatest percentage of months with no change, some of the other products that we viewed as good examples of non-perishables have more frequent price movement than goods that we view as more perishable. For example, white bread's (a fairly perishable product) price moves less often than prices for products like canned soup, peanut butter, soap and detergent, and paper towels. To examine this question more rigorously would require a more objective measure of perishability and better data (especially data on wholesale prices).

Of course, other factors are important in determining which specific grocery items go on sale. For example, Hosken, Matsa and Reiffen (2000) develop a model, supported by empirical evidence, which shows that if advertising a sale is costly, then retailers will choose to put on sale those products that are popular with the largest fraction of the population. In terms of the model in Section IV, this result implies that among perishables, more popular products (those that offer the most δ per advertising dollar) such as lettuce and yogurt will be put on sale, rather than less popular ones like radishes.

VI. Conclusion

With the increasing availability of high-quality data on retail prices and quantities, economists (as well as marketing professionals and others) have enthusiastically begun to estimate economic magnitudes, such as demand elasticities.[26] It is well understood that identifying these

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common national demand shift (e.g., hot dogs for the 4th of July holiday) and this shift leads to more sales (consistent with the findings in Hosken, Matsa, and Reiffen (2000) and Chevalier, Kashyap, and Rossi (2000)), our time dummies will attribute the reduction in price to changes in wholesale price, rather than retail changes.

[26] As an example, there was a recent NBER conference on the use of high-frequency data, such as that from supermarket scanners.

magnitudes requires variation in some independent variable, such as price. What is perhaps less well appreciated is the relevance of the source of this variation. This paper shows that the majority of retail price changes are actually sales; that is, temporary reductions in retail prices. These sales account for 25-50% of the annual price variation for the grocery products we study. Because these temporary reductions are such an important source of price variation, understanding why these changes occur is critical to interpreting econometric estimates which use this data.

This paper explores the pricing behavior of multiproduct retailers, and it provides an explanation for some observed retail pricing regularities. An implication of our analysis is that an important source of price variation is the desire of retailers to take advantage of differences in inventory costs across consumers. As noted above, if differences in consumer inventory costs are an important determinant of the consumers' purchasing patterns of, then estimated elasticities may not correspond to the experiment of interest to the analyst. In fact, given the multiproduct nature of a retailer's offerings, it is by no means clear that a retailer's unit sales of a particular class of products will vary inversely with its price. For example, in our model, unit sales of the perishable at an individual store are highest when the non-perishable is on sale (because more consumers are in the store); in that instance, however, the perishable price is high.[27]

More generally, taking account of the multiproduct nature of a grocery retailer's offerings yields a richer set of implications than can be derived from models of single-product retailers. For example, the model implies that individual perishable product prices vary inversely with individual non-perishable product prices. The model also implies that perishable and non-perishable products should have different pricing dynamics; perishable products will go on sale more often, but at less dramatic discounts than non-perishables. This last point is supported by our empirical analysis which shows that the most perishable products in our sample are the most likely to go on sale.

We view the multiproduct nature of consumers' purchases as an important aspect of the demand facing retailers. The model presented here shows how this aspect makes the two-product retailer choose different prices than two single-product retailers. In this sense, the model helps

[27]This is not simply an artifact of our assuming that consumers have unit demand for both goods. More generally, a retailer having a very successful sale, i.e. attracting many shoppers, will also sell more of the goods not on sale because consumers purchase bundles of goods.

explain some observed pricing regularities. Of course, goods sold by a single retailer differ in ways other than those modeled here, and consequently retailers have even richer pricing alternatives than our model suggests. Future research that analyzes the impact of these differences across products (e.g., differences in likelihood of purchase) would help develop a more complete understanding of the observed pricing behavior of multiproduct retailers.

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Appendix

Lemma 4: Let $\bar{\delta} \equiv \frac{(M+1)}{M}(\alpha_H - \alpha_L) - \frac{(M+1)^2}{M}\frac{J(1-\gamma)}{\gamma}\, Pr(\bar{\delta})\ \alpha_L$ where $\Pr(\bar{\delta})$ is the probability that retailer j attracts non-loyals when it offers $\delta_j = \bar{\delta}$ (for $M > 0$). When $M = 0$, $\bar{\delta}$ is implicitly defined in $Pr(\bar{\delta}) \equiv \frac{\gamma(\alpha_H - \alpha_L)}{J(1-\gamma)\alpha_L}$ if $\gamma(\alpha_H - \alpha_L) < J(1 - \gamma)\alpha_L$, and $\bar{\delta} = (\beta - \underline{P}_P)$ otherwise. Then

a. $\bar{\delta} \geq 0$,

b. $\pi_P(\delta) > \pi_N(\delta)$ for all $\delta < \bar{\delta}$ (i.e., an interior solution exists).

c. If $(M+1)(\alpha_L - \underline{P}_N) > \bar{\delta}$ then $\pi_P(\delta) < \pi_N(\delta)$ for δ such that $(M+1)(\alpha_H - \underline{P}_N) > \delta > \bar{\delta}$.

Proof: a. To see that $\bar{\delta}$ must be non-negative (for $M > 0$), note that since $\alpha_H > \alpha_L$, $\bar{\delta} < 0$ would imply that $\Pr(\bar{\delta}) > 0$; i.e., firm j can attract shoppers by offering negative consumer surplus. This violates consumer rationality. For $M = 0$, $\bar{\delta}$ must be greater than 0, since both $\gamma(\alpha_H - \alpha_L)$ and $J(1 - \gamma)\alpha_L$ are positive

b. By Proposition 1, retailers will never put both products on sale. The P_P required to generate consumer surplus of δ is $\beta - \delta$. Hence, the profits from putting the perishable on sale to generate δ are

$$\Pi_P = \frac{\gamma}{J}(\beta + \alpha_H - \delta) + (1-\gamma)Pr(\delta_j)(\beta - \delta)$$

Where $\Pr(\delta_j)$ is the probability that retailer j attracts non-loyals when it offers δ_j. Since the non-perishable price which yields consumer surplus of δ equals $\alpha_L - \delta/(M+1)$, the profits from putting the non-perishable on sale to generate δ are

$$\Pi_N = \frac{\gamma}{J}(\beta + \alpha_L - \frac{\delta}{M+1}) + (1-\gamma)Pr(\delta_j)(\beta + (M+1)\alpha_L - \delta)$$

First note that $\pi_N(\delta) - \pi_P(\delta)$ is strictly increasing in δ. Hence, if $\pi_N(\delta) - \pi_P(\delta)$ is positive for some $\tilde{\delta}$, it will be positive for all $\delta > \tilde{\delta}$, and if $\pi_N(\delta) - \pi_P(\delta)$ is negative for some $\tilde{\delta}$, it will be negative for all $\delta < \tilde{\delta}$. Solving for the δ at which $\pi_N(\delta) = \pi_P(\delta)$ allows us to divide the set of all possible δ into two mutually exclusive sets; one in which lowering P_P is a more profitable way to generate δ and one in which lowering P_N is more profitable . Specifically, $\pi_P(\delta) > \pi_N(\delta)$ if

$\delta < \bar{\delta} \equiv \frac{(M+1)}{M}(\alpha_H - \alpha_L) - \frac{(M+1)^2}{M}\frac{J(1-\gamma)}{\gamma}Pr(\delta_j)\alpha_L$ (for M > 0) For M = 0, the same calculation

yields the implicit solution to $Pr(\bar{\delta}) \equiv \frac{\gamma\ (\alpha_H - \alpha_L)}{J(1-\gamma)\alpha_L}$ if $\gamma(\alpha_H - \alpha_L) < J(1-\gamma)\alpha_L$.

c. By construction, $\pi_N > \pi_P$ if $\delta > \bar{\delta}$. In addition, if $(M+1)(\alpha_H - \underline{P}_N) > \bar{\delta}$, then offering a sale on the non-perishable yields higher profits to retailer j than having a sale on neither good, assuming no other retailer offers more than $\bar{\delta}$ in consumer surplus. ■

Corollary: In the symmetric equilibrium,

a. $\bar{\delta} = (M+1)/(\gamma M)[\gamma(\alpha_H - \alpha_L) - \alpha_L J(1-\gamma)(M+1)(G(\bar{\delta}))^{J-1}] > 0$ for M > 0. $\bar{\delta}$ is also positive for M = 0.

b. As long as β is greater than 0, $\pi_P(\delta) > \pi_N(\delta)$ for δ sufficiently small. That is, there will always be a positive probability of a sale on the perishable for $\beta > 0$.

Proof: a. In the symmetric equilibrium, $Pr(\delta_j) = Prob(\delta_j < \delta_{-j}) = (G(\delta))^{J-1}$, where G( ) is a distribution function common to all retailers. By Lemma 4.a, $\bar{\delta}$ is non-negative. Using the definition of $\bar{\delta}$, $\bar{\delta}$ = 0 would imply $\gamma(\alpha_H - \alpha_L) = \alpha_L J(1-\gamma)(M+1)(G(0))^{J-1}$, and hence a mass point at G(0). As Varian (1980) showed, the equilibrium distribution cannot have a mass point (see Lemma 1). Hence, $\bar{\delta} > 0$ for M > 0. For M = 0, since both $\gamma(\alpha_H - \alpha_L)$ and $J(1-\gamma)\alpha_L$ are positive, the implies $Pr(\bar{\delta})$ is strictly positive, and less than 1 if $\gamma(\alpha_H - \alpha_L) < J(1 - \gamma)\alpha_L$.

b. First note that β is necessarily greater than $\underline{P}_P$. This means that there is always a profitable discount that can be offered on the perishable, as long as the probability of winning is sufficiently high. At the same time, part a. shows that $\bar{\delta} > 0$, which implies that $\pi_P(\delta) > \pi_N(\delta)$ for δ sufficiently small.■

Table 1: Proportion of Observations at the Mode and Proportion of Deviations above and Below Mode by Product Category

Product	Number of Price Series	Percent of Obs. At Mode	Ratio of the # of Prices Below Mode to the # of Prices Above Mode*
Baby Food	299	72.05	1.74
Bananas	1142	56.19	2.01
Canned Soup	1310	67.06	1.93
Cereal	1631	65.16	1.74
Cheese	1233	65.35	1.53
Snacks	1288	74.05	2.46
Cola Drinks	1116	62.53	2.23
Cookies	750	71.47	2.37
Crackers	311	64.75	3.29
Eggs	905	39.07	1.26
Frozen Dinners	561	68.08	2.77
Frozen Orange Juice	491	57.78	2.23
Ground Beef	909	60.36	2.16
Hotdogs	471	63.06	2.39
Lettuce	672	20.98	3.57
Margarine	477	62.81	2.11
Paper Products	620	66.87	2.41
Peanut Butter	342	64.13	1.93
Soap and Detergents	820	68.04	2.40
White Bread	1043	69.05	1.69
All Products	16391	59.37	2.09

*In all cases the null hypothesis that the proportion of price increases equals the proportion of price decreases can be rejected at a confidence level of α=.0001.

Table 2 - Percent of Price Series Experiencing at Least One Sale in the Second Year of the Sample, Conditional on Whether there is a Sale within the First Year

Product	Conditional on at least one sale within the First Year (number of price series)	Conditional on no Sale within the First Year (number of price series)	Z-Statistic (p-value)
Baby Food	26.7% (15)	3.7% (82)	3.17 (.0016)
Bananas	84.0% (401)	52.9% (87)	6.41 (0)
Canned Soup	51.8% (110)	17.4% (265)	6.81 (0)
Cereal	53.2% (77)	22.0% (259)	5.29 (0)
Cheese	56.1% (139)	21.0% (257)	7.07 (0)
Snacks	68.5% (124)	25.8% (151)	7.08 (0)
Cola Drinks	72.0% (157)	25.4% (122)	7.72 (0)
Cookies	66.7% (63)	20.0% (115)	6.18 (0)
Crackers	84.9% (53)	25.5% (51)	6.10 (0)
Eggs	63.5% (244)	38.5% (218)	5.37 (0)
Frozen Dinners	60.9% (46)	34.2% (38)	2.43 (.015)
Frozen Orange Juice	64.6% (113)	36.4% (118)	4.28 (0)
Ground Beef	70.3% (246)	36.1% (216)	7.37 (0)

Hot Dogs	65.1% (83)	37.5% (56)	3.20 (.0014)
Lettuce	96.1% (417)	70.0% (40)	6.59 (0)
Margarine	66.2% (74)	32.1% (109)	4.54 (0)
Paper Products	76.5% (17)	32.3% (31)	2.93 (.0034)
Peanut Butter	49.0% (51)	17.4% (109)	4.17 (0)
Soap and Detergent	64.5% (31)	21.2% (33)	3.51 (.0004)
White Bread	60.9% (151)	15.0% (233)	9.34 (0)

Sale defined as a 10% or more reduction in price in month t, followed by an increase of similar magnitude in month t+1.

Table 3: Summary of Correlations of First Differences in the Prices of Three Leading Brands Across Grocery Chains in Springfield, Missouri and Sioux Falls, South Dakota

	Sioux Falls	Springfield	Total
Number of Correlations	238	121	359
Percent Positive	51.2%	52.9%	51.8%
Percent Larger than .3	7.1%	6.6%	7.0%
Percentage Positive and Significant at 10% level	14.3%	16.5%	15.0%
Percentage Positive and Significant at 5% level	10.9%	13.2%	11.7%

The table is a summary of the correlations of first differences in price of specific brands across chains. For example, we calculate the correlation of the first difference in price of Skippy Peanut Butter across all the grocery chains in Springfield, Missouri. A similar calculation is done for each of the three leading brands for eight categories of consumer goods in the data set (peanut butter, tub margarine, stick margarine, ketchup, sugar, light tuna, dark tuna, and tissue). The difference in the number of correlations between Sioux Falls and Springfield reflects differences in the products carried across chains and differences in the number of chains.

Table 4: Proportion of Price Increases or Decreases Following a Price Decrease

Product	Observations	Price Increase	Price Decrease
Baby Food	329	46.2%	4.9%
Bananas	5970	58.7%	16.8%
Canned Soup	2370	60.0%	6.6%
Cereal	2246	62.5%	6.5%
Cheese	3124	57.8%	9.8%
Snack Food	2254	66.6%	7.1%
Cola Drinks	3123	57.2%	13.4%
Cookies	1596	73.9%	6.1%
Crackers	1077	71.7%	7.1%
Eggs	7330	55.1%	21.8%
Frozen Dinners	946	65.4%	7.4%
Frozen Orange Juice	2306	52.1%	13.7%
Ground Beef	5006	60.8%	13.5%
Hot Dogs	1301	64.2%	10.1%
Lettuce	10328	60.7%	34.3%
Margarine	1752	58.6%	11.1%
Paper Towels	463	60.3%	8.6%
Peanut Butter	1303	53.5%	10.4%
Soap and Detergent	612	60.0%	7.8%
White Bread	2032	68.2%	6.3%
All Products	55468	60.0%	16.7%

Table 6: Proportion of Price Changes by Product

Product	Observations	Percentage of Price Changes
Baby Food	6058	17.8%
Bananas	24567	52.3%
Canned Soup	24254	26.1%
Cereal	24067	27.0%
Cheese	24871	31.2%
Snack Food	19310	27.4%
Cola Drinks	17754	39.0%
Cookies	12792	30.0%
Crackers	6404	39.3%
Eggs	26196	63.7%
Frozen Dinners	6531	33.6%
Frozen Orange Juice	12809	38.6%
Ground Beef	26013	42.4%
Hot Dogs	8692	35.7%
Lettuce	24158	90.0%
Margarine	11017	36.0%
Paper Towels	3225	34.5%
Peanut Butter	8602	33.3%
Soap and Detergent	4610	31.2%
White Bread	22674	24.9%
All Products	314604	40.6%

Figure 1: Proportion of Price Variance Explained by Monthly Time Effects and the Incremental Effect of Sales

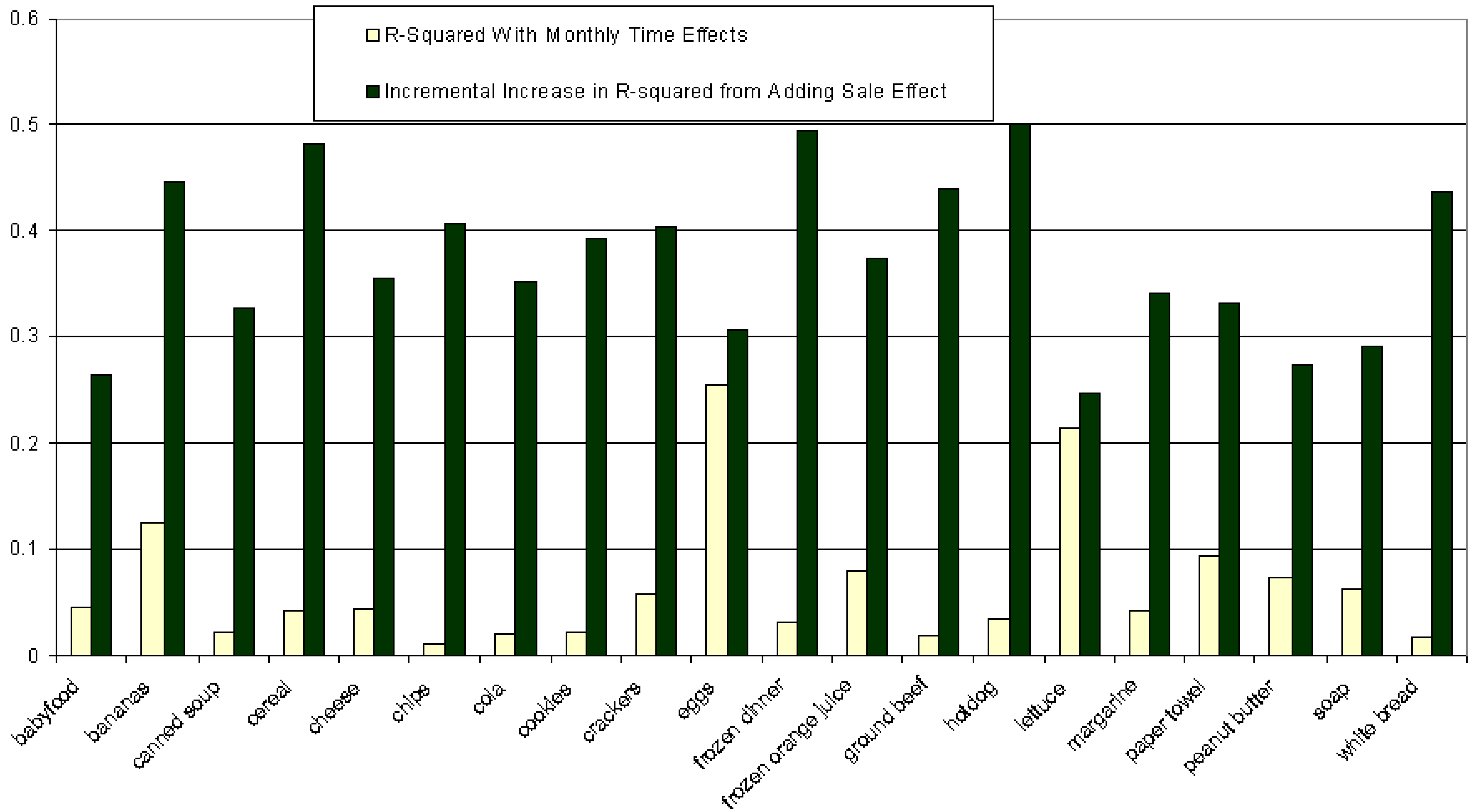

Figure 3: Time Series of Shelf Prices of Peter Pan Peanut Butter in Springfield, MO

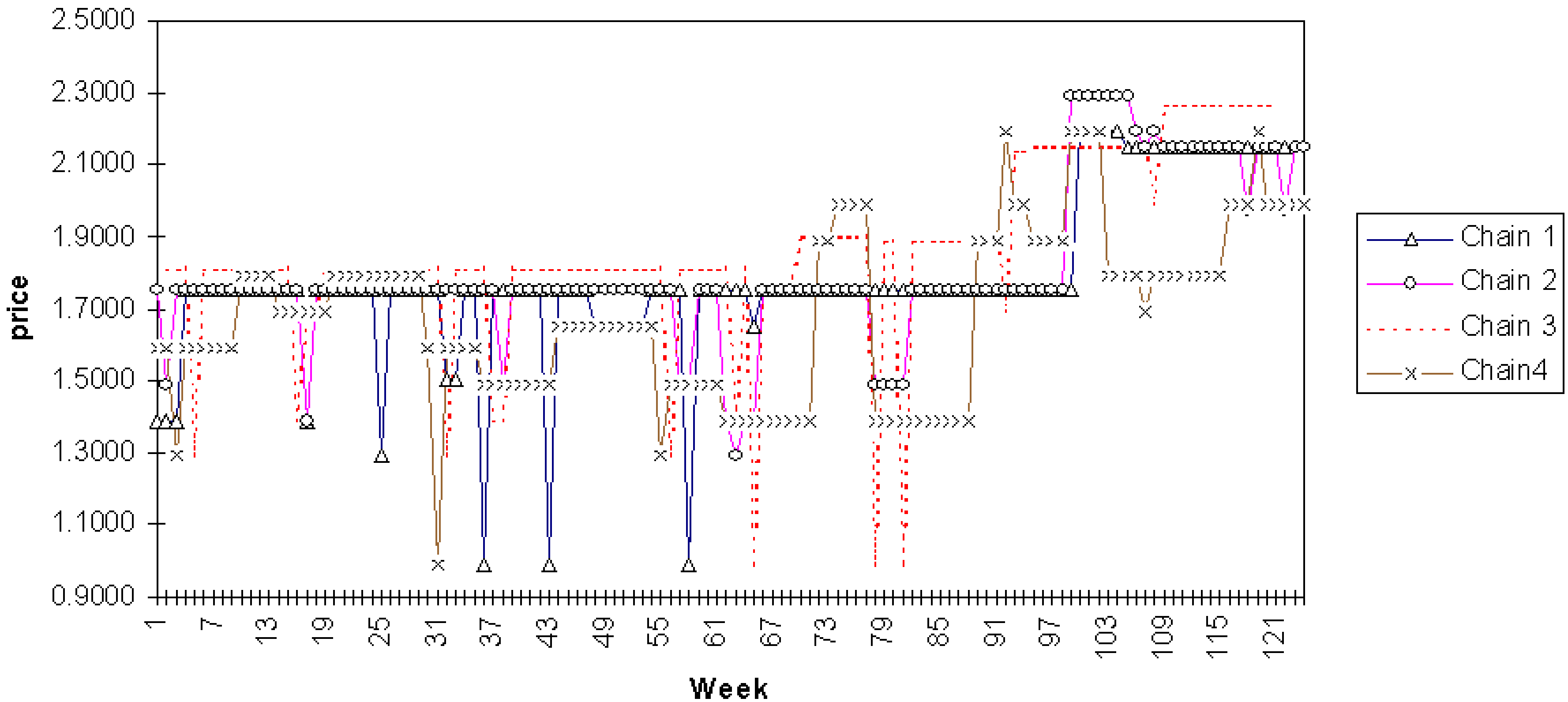

Figure 4

F 1(Pn) V alues for Alternative M

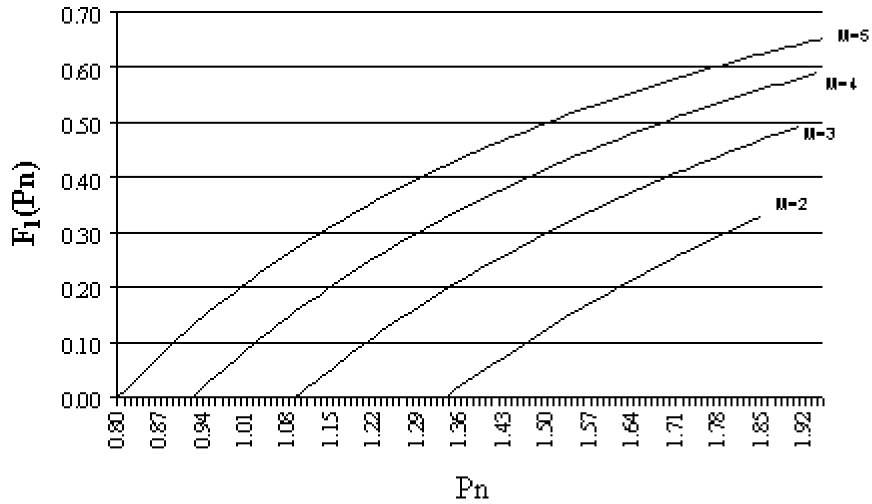

Figure 5

F 2(P p) V alues for Alternative M