A STOCHASTIC MODEL OF THE LONG-RANGE FINANCIAL STATUS OF THE OASDI PROGRAM

ACTUARIAL STUDY NO. 117

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FOREWORD

The 2004 Annual Report of the OASDI Board of Trustees includes a brief description of the OCACT Stochastic Model (OSM, Version 2004.1) and projections produced using it. Actuarial Study No. 117 is intended to provide more extensive details of this model and its results.

The purpose of the OSM is to provide probability distributions for the range of possible future experience of the OASDI program under present law. This probabilistic representation of uncertainty augments the presentations of low and high cost alternatives and sensitivity analyses that have traditionally been included in the Trustees Report. It should be noted that this model is in its first stage of development. Future improvements and refinements to the model are expected. In particular, future revisions are expected to more fully reflect the range of uncertainty about future experience.

Anthony W. Cheng and Michael L. Miller developed the equations for the OSM, Version 2004.1. Programmers of the individual modules are identified below:

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Seung An and Sue Kunkel of OCACT provided direction and support throughout the development process. Sheldon Baker and Lise Holler assisted in the publication of the study. In addition, an acknowledgement must be made of the Long Term Modeling Group of the Congressional Budget Office and the Office of Policy of the Social Security Administration, both of which provided consultation during the early development of the OSM.

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September 30, 2004

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TABLE OF CONTENTS

III. Documentation of the Computer Program

IV. Results

V. Appendices

LIST OF FIGURES

II. Equation Selection and Parameter Estimation

IV. Results

A STOCHASTIC MODEL OF THE LONG-RANGE FINANCIAL STATUS OF THE OASDI PROGRAM

I. INTRODUCTION

Each year the Board of Trustees of the Federal Old-Age and Survivors Insurance (OASI) and Disability Insurance (DI) Trust Funds provides three separate sets of long-range (75-year) assumptions for key demographic and economic variables that affect the future financial status of the combined OASI and DI (OASDI) programs. The intermediate (alternative II) set of assumptions represents the Trustees' best estimate for future experience, while the low cost (alternative I) and high cost (alternative III) sets of assumptions are more and less favorable, respectively, from the perspective of the trust funds' future financial outlook. The Office of the Chief Actuary (OCACT) of the Social Security Administration (SSA) uses the three sets of assumptions to project the principal factors affecting the financial status of the OASDI program. Taken together, these three projections give policy makers a sense of the range of variation in the assumptions and in the financial status of the OASDI program. However, this deterministic approach makes no attempt to assign values to the likelihood of these sets of assumptions.

This Actuarial Study documents the OCACT Stochastic Model (OSM), Version 2004.1. The OSM assigns random variation for some of the key demographic and economic assumptions. These include the total fertility rate, rates of change in mortality, levels of immigration, emigration, and net other immigration, unemployment rate, inflation rate, real interest rate, growth rate in the real average wage, and disability incidence and recovery rates. The OSM is designed such that the projected values for each variable are centered on the intermediate assumptions of the 2004 Trustees Report (TR04II).

Stochastic variation is introduced by developing equations based on standard time-series models. Generally, an equation may include the following: the variable's prior-period values, prior-period error terms, and other variables. In addition, each equation includes a random error term. The ranges of the regressions depend on the nature and quality of the historical data. Projected values for each variable in each year are computed using Monte Carlo techniques to assign the degree of stochastic variation around the Trustees' intermediate assumptions. Each simulation projects annual values for each variable over the 75-year period, in addition to summary measures of the financial status of the combined OASDI Trust Funds (e.g., the long-range actuarial balance).

It is important to note that the results presented here should be interpreted with caution and with a full understanding of the inherent limitations of the process. If certain changes are made to the model specifications, then the projections could be significantly altered. For example, if any one equation is respecified, or if the degree of interdependency among variables is modified, or if the historical period used in fitting any equation is changed, the results would be different. In addition, if variables other than those mentioned above (such as labor force participation rates, retirement rates, marriage rates, and divorce rates) were included in the stochastic modeling, the results would differ. Finally, additional variability would be expected to result from incorporating statistical approaches that would allow for potential structural shifts in the long-range central tendencies (i.e., parameter uncertainty). In conclusion, the current OSM's projected variation is likely to be narrower than the true range of uncertainty for the future.

The remaining chapters of this Actuarial Study provide detailed information from the OSM. Chapter II presents the equations used to model random variation in the assumptions. Chapter III explains the overall structure of the OSM and its modules. Chapter IV presents projection results, including the projected probability distributions for the stochastic assumptions and the summary actuarial measures used to assess the long-range financial status of the OASDI program. Chapter IV also presents a sensitivity analysis for each stochastic assumption.

Six appendices to this Actuarial Study are included. Appendix A contains background material on various financial estimates of the OASDI program. Appendices B and C contain introductions to time-series modeling and Monte Carlo simulation, respectively. Appendix D provides additional details of the time-series equations used in the OSM. Appendix E provides a glossary of terms used in this study, and appendix F is a bibliography of references cited in this study.

II. EQUATION SELECTION AND PARAMETER ESTIMATION

Equations were selected for a set of assumption variables that include the total fertility rate, changes in mortality, levels of immigration, emigration, and net other immigration, unemployment rate, inflation rate, real interest rate, growth rate in the real average wage, and disability incidence and recovery rates. The parameters of the equations were estimated using standard time-series modeling techniques, and then modified so that the projected variation was centered on the TR04II. Appendix B discusses the theory behind this procedure. This chapter briefly describes and presents each equation, while appendix D provides more detailed information and statistics. Historical time-series data can be obtained from OCACT.[1]

A. FERTILITY

The *total fertility rate* is the sum of age-specific birth rates[2] for women aged 14 through 49. Thus, the total fertility rate for a given year may be interpreted as the average number of children that would be born to a woman throughout her lifetime if she were to survive the entire childbearing period and experience the observed age-specific birth rate each year of her life.

Historical values for the total fertility rate in the U.S. for 1917 through 2002 are available from the National Center for Health Statistics[3] and the U.S. Census Bureau.[4] The total fertility rate ranged from a minimum of 1.74 in 1976 to a maximum of 3.68 in 1957, and has remained relatively stable, near 2.00, since 1990. The rate was 2.01 in 2002.

Using time-series analysis, an ARMA(4,1) equation was selected and parameters were estimated using the entire range of data. Figure II.1 presents the actual and fitted values. The R-squared value was 0.98. The modified equation is:

$$F_t = F_t^{TR} + 1.99 f_{t-1} - 1.51 f_{t-2} + 0.91 f_{t-3} - 0.42 f_{t-4} + \varepsilon_t - 0.67 \varepsilon_{t-1}. \tag{1}$$

In this equation, F_t represents the total fertility rate in year t; F_t^{TR} represents the projected total fertility rate from the TR04II in year t; f_t represents the deviation of the total fertility rate from the TR04II total fertility rate in year t (i.e. $f_t = F_t - F_t^{TR}$) and ε_t represents the random error in year t.

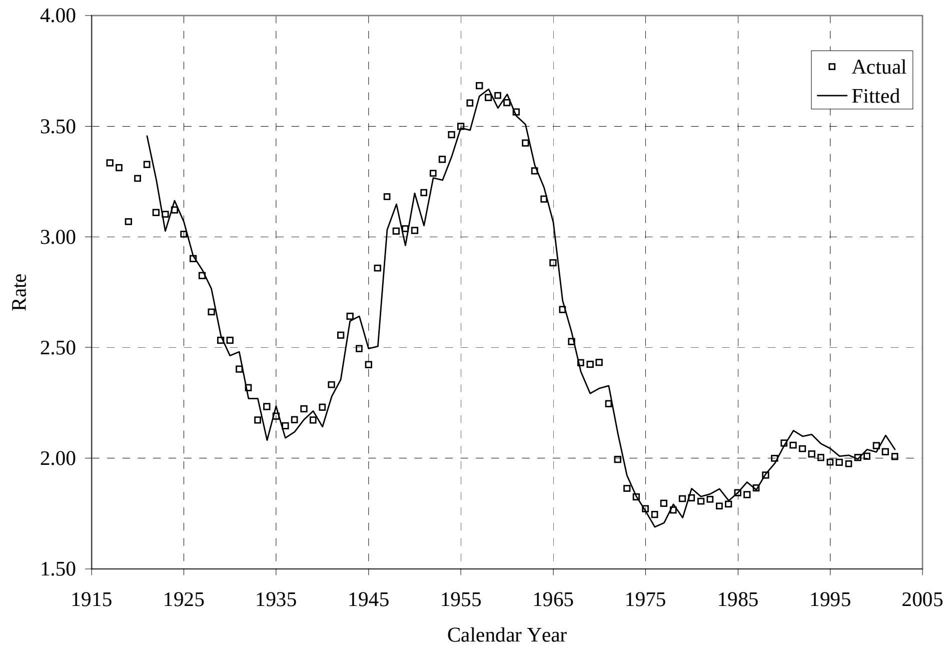
1 Contact OCACT (actuary@ssa.gov).

2 Age-specific birth rates are defined as the number of live births to women of a given age divided by the estimated female population of the given age at midyear.

3 www.cdc.gov/nchs

4 www.census.gov

Figure II.1—U.S. Total Fertility Rate, Calendar Years 1917-2002



B. MORTALITY

The annual rate of decrease in the *central death rate*[5] (which is sometimes referred to as the annual rate of improvement in mortality) is calculated as the negative of the percent change in the central death rate for a given year. Thus, a positive value represents a decrease in the central death rate from one year to the next.

Central death rates were calculated for 42 age-sex groups (under 1, 1-4, 5-9, 10-14, …, 85-89, 90-94, and 95+; male and female) for the period 1900 through 2000.[6] Data for the annual numbers of deaths and the U.S. resident population are from the National Center for Health Statistics and the U.S. Census Bureau, respectively. For the population aged 65 or older, annual deaths and enrollments are from the Centers for Medicare & Medicaid Services.

Using the approach of other researchers (Congressional Budget Office, 2001), an AR(1) equation was selected for the annual rate of decrease in the central death rate for each age-sex group. The general form of the modified equation is:

$$MR_{k,t} = MR_{k,t}^{TR} + \phi_k mr_{k,t-1} + \varepsilon_{k,t}. \qquad (2)$$

In this equation, $MR_{k,t}$ represents the annual rate of decrease in the central death rate for group k in year t; $MR_{k,t}^{TR}$ represents the projected annual rate of decrease from the TR04II for group k in year t; $mr_{k,t}$ represents the deviation of the annual rate of decrease from the TR04II value for group k in year t; and $\varepsilon_{k,t}$ represents the random error for group k in year t. Appendix D contains the estimates of the parameters, ϕ_k, in Equation (2).

A Cholesky decomposition was performed using the residuals from the 42 fitted equations. Appendix B discusses this technique. The Cholesky matrix used was 42×42 with the age groups in ascending order with alternating male and female groups.

5 The central death rate is defined as the annual number of deaths for a particular group divided by the estimated population of that group at midyear.

6 A detailed description of the methodology used in calculating death rates by age and sex can be found in Actuarial Study No. 116. www.socialsecurity.gov/OACT/NOTES/as116/as116_Foreword.html

C. IMMIGRATION

Total immigration is defined here as legal immigration minus legal emigration plus net other immigration. Each component is modeled separately.

1. Legal Immigration

Legal immigration is defined as persons lawfully admitted for permanent residence into the United States.[7] The level of legal immigration largely depends on legislation which basically serves to define and establish limits for certain categories of immigrants. The Immigration Act of 1990, which is currently the legislation in force, establishes limits for three classes of immigrants: family-sponsored preferences, employment-based preferences, and diversity immigrants. However, no numerical limits currently exist for immediate relatives of U.S. citizens.

Historical data for legal U.S. immigration for years 1901 through 2002 are from the U.S. Citizenship and Immigration Services.[8] Legal immigration averaged nearly one million per year from 1900 through 1914, then decreased substantially to about 23,000 in 1933. Since the mid-1940s, legal immigration increased steadily to over one million in 2002.

An ARMA(4,1) equation was selected and parameters were estimated using the entire range of historical data. The R-squared value was 0.92. Figure II.2 presents the actual and fitted values. The modified equation is:

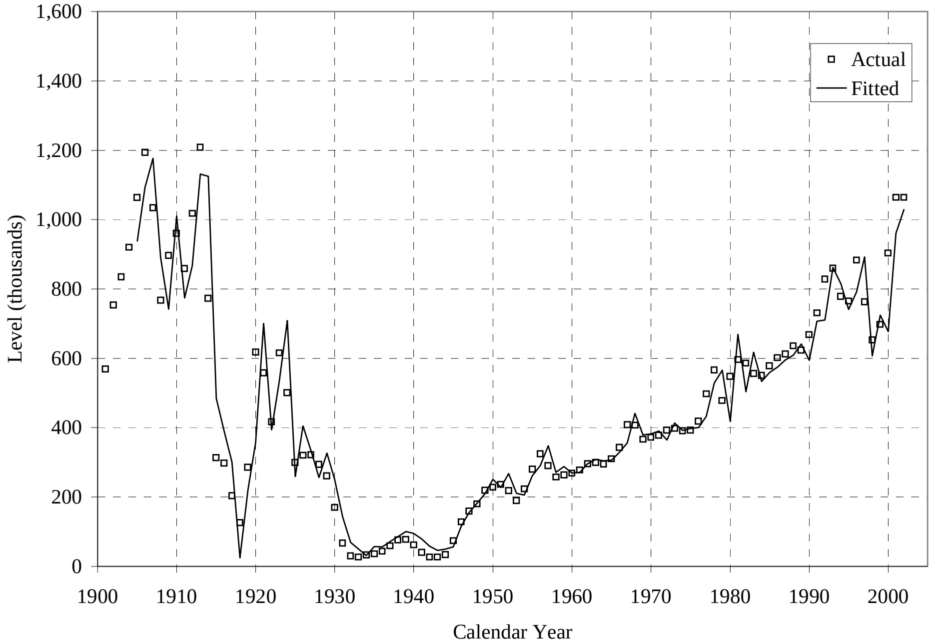
$$IM_t = IM_t^{TR} + 1.08im_{t-1} - 0.54im_{t-2} + 0.69im_{t-3} - 0.31im_{t-4} + \varepsilon_t + 0.49\varepsilon_{t-1}. \tag{3}$$

In this equation, IM_t represents the annual level of legal immigration in year t; IM_t^{TR} represents the projected level of legal immigration from the TR04II in year t; im_t represents the deviation of the annual level of legal immigration from the TR04II value in year t; and ε_t represents the random error in year t.

7 For more detailed information, refer to the Yearbook of Immigration Statistics - uscis.gov/graphics/shared/aboutus/statistics/ybpage.htm.

8 Formerly known as the Immigration and Naturalization Service (INS).

Figure II.2—U.S. Legal Immigration, Calendar Years 1901-2002



2. Legal Emigration

Legal emigration is defined as the number of persons who lawfully leave the United States, and are no longer considered to be a part of the Social Security program. Although annual emigration data are not collected in the United States, the U.S. Census Bureau estimates that the level of emigration for the past century roughly totaled one-fourth of the level of legal immigration.

Using the Census estimates as an approximate guide, the parameters of Equation (3) are multiplied by one-fourth.[9] The modified equation is:

$$EM_t = EM_t^{TR} + 0.27em_{t-1} - 0.13em_{t-2} + 0.17em_{t-3} - 0.08em_{t-4} + \varepsilon_t + 0.12\varepsilon_{t-1}. \tag{4}$$

In this equation, EM_t represents the annual level of legal emigration in year t; EM_t^{TR} represents the projected annual level of legal emigration from the TR04II in year t; em_t represents the deviation of the annual level of legal emigration from the TR04II value in year t; and ε_t represents the random error in year t.

3. Net Other Immigration

Net other immigration is defined as the annual flow of persons into the United States minus the annual flow of persons out of the United States who do not meet the above definition of legal immigration or legal emigration. Thus, net other immigration includes unauthorized persons and those not seeking permanent residence.

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9 It is important to note that legal emigration is simulated independently from legal immigration.

Since complete data does not exist for net other immigration, we rely on indirect measurements from the U.S. Census Bureau for our estimate. The Census Bureau accomplishes this by comparing two consecutive decennial census populations, applying known components of change, and assigning the residual to net other immigration. The annual level of net other immigration is assumed to follow a random walk. The modified equation is of the form:

$$\Delta O_t = \Delta O_t^{TR} + \varepsilon_t; \quad O_{2003} = O_{2003}^{TR}. \tag{5}$$

In this equation, ΔO_t represents the change in the annual level of net other immigration in year t; ΔO_t^{TR} represents the projected annual change in the level of net other immigration consistent with the TR04II in year t; and ε_t represents the random error in year t. The equation is initialized with O_{2003}^{TR}, the level of net other immigration in 2003 from the TR04II.

D. RELATED ECONOMIC VARIABLES

The unemployment rate, inflation rate and real interest rate are simulated together using a vector autoregression, in order to capture the relationship among the three variables that economic theory suggests are related.[10] In the vector autoregression, each variable is regressed on the prior-period values of all three variables. Vector autoregressions of different prior-period lengths were tested and it was determined that a vector autoregression including 2 prior years provided a reasonable fit. The historical period considered was 1960 to 2002.

For the vector autoregression, the unemployment rates (as defined in section II.D.1) were expressed as log-odds ratios[11] to bound the values between 0 and 100 percent. In addition, the adjusted inflation rates (as defined in section II.D.2) had a logarithmic transformation[12] applied to give them a lower bound for the vector autoregression. Instead of simply log-transforming the inflation rate series, 3.0 percent was added to the inflation rate series prior to the log-transformation. This gave the inflation rates a lower bound of -3.0 percent. For the remainder of this section and for section II.E, references to the unemployment rates and inflation rates refer to the transformed rates.

1. Unemployment Rate

The *unemployment rate* is the number of unemployed persons seeking work as a percentage of the civilian labor force. Historical values are published by the Bureau of Labor Statistics (BLS).[13] The annual levels are an average of the 12 monthly rates.

Between 1960 and 1974, the unemployment rate was relatively stable, ranging from a low of 3.5 percent in 1969 to a high of 6.7 percent in 1961. Between 1975 and 1994, the unemployment rate moved to higher levels, and peaked at 9.7 percent in 1982. From 1994 to 2000, a rapid economic expansion resulted in unemployment rates falling to 4.0 percent.

For the unemployment rate equation, the R-squared value was 0.85. The actual and fitted values are shown in figure II.3. The modified equation is:

$$U_t = U_t^{TR} + 0.96u_{t-1} - 0.30u_{t-2} + 0.40i_{t-1} - 0.08i_{t-2} + 0.75r_{t-1} + 0.61r_{t-2} + \varepsilon_{1t}. \qquad (6)$$

In this equation, U_t represents the unemployment rate in year *t*; U_t^{TR} is the unemployment rate from the TR04II in year *t*; u_t represents the deviation of the unemployment rate from the TR04II unemployment rate in year *t*; i_t is the deviation of the inflation rate from the TR04II inflation rate in year *t*; r_t is the deviation of the real interest rate from the TR04II real interest rate in year *t*; and ε_{1t} is the random error in year *t*.

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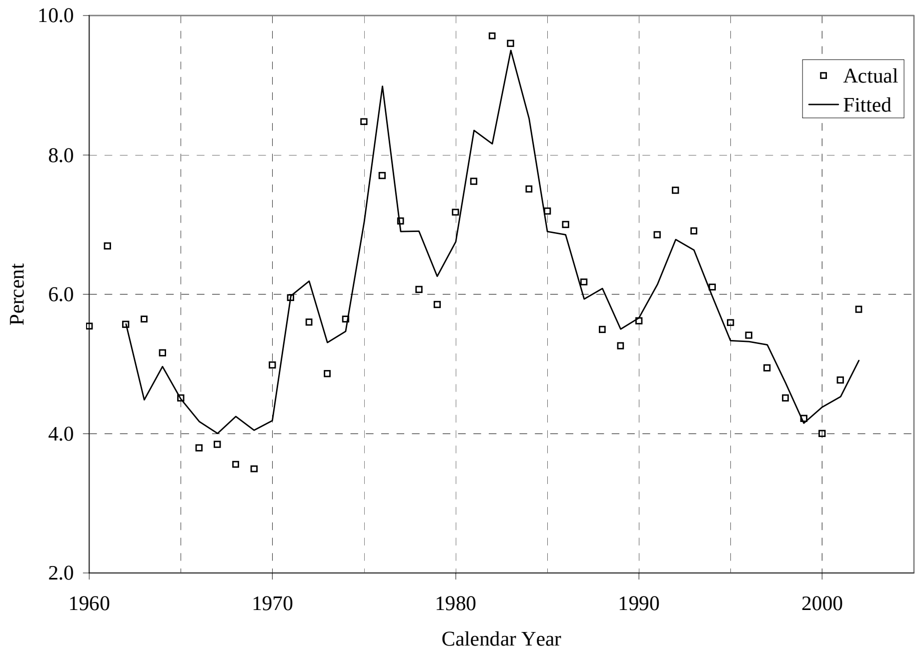
10 Foster (1994) suggested that a multivariate approach might capture a more appropriate range of variability for these economic variables. CBO implemented this approach in their stochastic model.
11 $U_t = \log[RU_t/(1 - RU_t)]$, where RU_t is the unemployment rate in year *t* expressed as a decimal.
12 $I_t = \log(\pi_t + 0.03)$, where π_t is the percent change in the adjusted inflation rate in year *t* expressed as decimals.
13 www.bls.gov/cps/home.htm

Figure II.3—Unemployment Rate, Calendar Years 1960-2002



2. Inflation Rate

The *inflation rate* is defined as the annual growth rate in the Consumer Price Index for Urban Wage Earners and Clerical Workers (CPI). The BLS publishes historical values for the CPI.[14] The BLS periodically introduces improvements to the CPI that affect its annual growth rate but does not revise earlier values. Consequently, OCACT has adjusted the CPI by back-casting the effects of the improvements on earlier values to improve consistency. The inflation rate is important because it determines the annual cost-of-living adjustment (COLA) for OASDI benefits.

Over the historical period from 1960 to 2002, the adjusted inflation rate ranged from a low of 0.8 percent in 1961 and 1962 to a high of 10.9 percent in 1980, and was 1.4 percent in 2002.

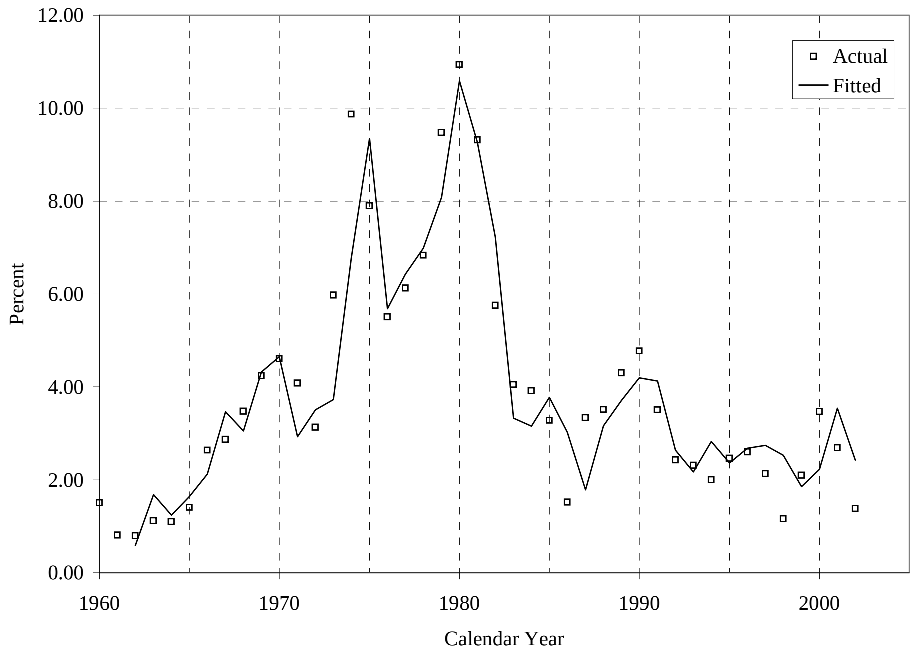
For the inflation rate equation, the R-squared value was 0.83. The actual and fitted values are shown in figure II.4. The modified equation is:

$$I_t = I_t^{TR} - 0.77u_{t-1} + 0.72u_{t-2} + 0.60i_{t-1} + 0.30i_{t-2} - 4.85r_{t-1} + 1.80r_{t-2} + \varepsilon_{2t}. \tag{7}$$

In this equation, I_t is the CPI inflation rate in year t; I_t^{TR} is the CPI inflation rate from the TR04II in year t; u_t represents the deviation of the unemployment rate from the TR04II unemployment rate in year t; i_t is the deviation of the inflation rate from the TR04II inflation rate in year t; r_t is the deviation of the real interest rate from the TR04II real interest rate in year t; and ε_{2t} is the random error in year t.

[14] www.bls.gov/cpi

Figure II.4—Inflation Rate (CPI), Calendar Years 1960-2002



3. Real Interest Rate

All securities held by the OASI and DI Trust Funds are issued by the Federal Government. Almost all of these securities are special issues (i.e., securities issued only to the trust funds). Historical data on actual nominal interest rates of new purchases of these securities are published by OCACT.[15] The nominal interest rate on new purchases of these securities for a given month is set equal to the average market yield on all marketable Federal obligations that are not callable and do not mature within the next 4 years.[16] Annual nominal interest rates are the average of the 12 monthly rates, which in practice are compounded semiannually.[17] The *real interest rate* earned on these obligations is equal to the annual (compounded) *nominal yield* divided by the inflation rate.

Looking at the period from 1960 to 2002, real interest rates on new purchases of special issues rose to much higher levels in the 1980s, as investors demanded higher risk premiums for increased uncertainty surrounding the unexpectedly high rates of inflation. Since then, the rate of inflation and the real interest rate have declined.

For the real interest rate equation, the R-squared value was 0.81. The actual and fitted values are shown in figure II.5. The modified equation is:

$$R_t = R_t^{TR} + 0.06u_{t-1} - 0.05u_{t-2} + 0.03i_{t-1} - 0.03i_{t-2} + 1.23r_{t-1} - 0.32r_{t-2} + \varepsilon_{3t}. \quad (8)$$

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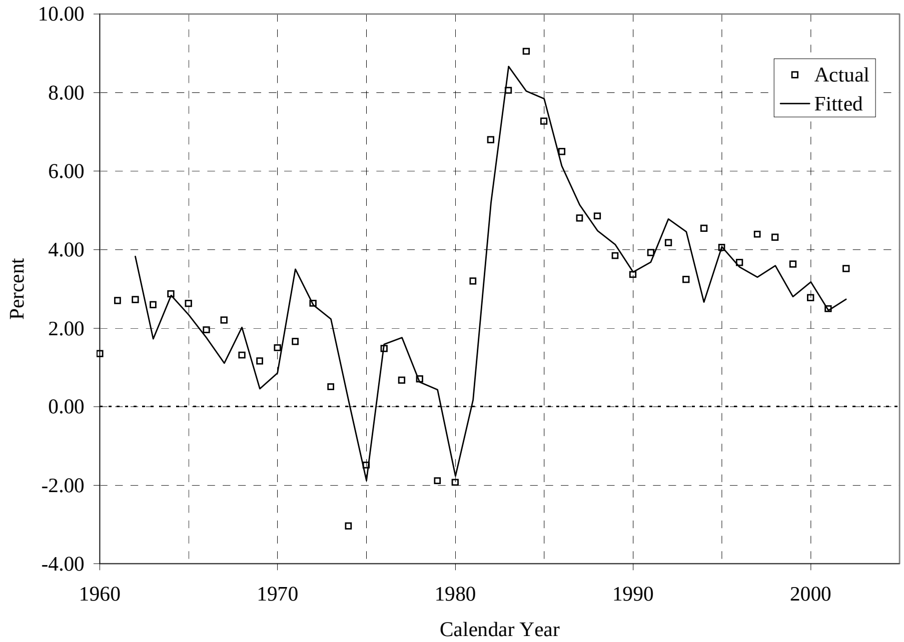
[15] www.socialsecurity.gov/OACT/ProgData/newIssueRates.html

[16] For more details on the history of trust fund investment policy, see Actuarial Note 142, *Social Security Trust Fund Investment Policies and Practices*, by Jeff Kunkel, www.socialsecurity.gov/OACT/NOTES/note142.html.

[17] For example, the annualized nominal yield on a special issue with a 6.0-percent nominal interest rate is equal to $(1+.06/2)^2$ - 1 = 6.09 percent.

In this equation, R_t represents the real interest rate in year t; R_t^{TR} represents the real interest rate from the TR04II in year t; u_t represents the deviation of the unemployment rate from the TR04II unemployment rate in year t; i_t represents the deviation of the inflation rate from the TR04II inflation rate in year t; r_t represents the deviation of the real interest rate from the TR04II real interest rate in year t; and ε_{3t} is the random error term in year t.

Figure II.5—Real Interest Rate, Calendar Years 1960-2002



E. REAL AVERAGE COVERED WAGE (PERCENT CHANGE)

The *real average covered wage* is defined as the ratio of the average nominal OASDI covered wage to the adjusted inflation rate. Because of the expansion of covered employment, the annual growth rate in the real average covered wage differs significantly from the annual growth rate in a real average economy-wide wage series. In the future, however, the annual growth rates in the two measures are expected to be approximately identical since projected coverage changes are insignificant. Hence, the historical variation of the annual percent change in the real average economy-wide wage is used to model the future variation of the annual percent change in the real average covered wage.

The real average economy-wide wage is the ratio of the average nominal wage to the adjusted CPI. The nominal wage is the ratio of wage disbursement as published by the Bureau of Economic Analysis' (BEA) National Income and Product Accounts (NIPA) to civilian employment. Civilian employment is the sum of total wage employment, as published by the BLS from its Household Survey, and total U.S. Armed Forces from the Census Bureau. The BLS periodically introduces improvements to its employment data but does not revise earlier data. However, the BLS has developed adjustment factors to improve the comparability of employment data with earlier years.[18] OCACT has used these factors to adjust the wage employment data.

The formula for calculating the annual percent change in the real average wage, given a nominal wage series, is:

$$W_t = (NW_t / NW_{t-1}) / (CPI_t / CPI_{t-1}) - 1.$$

W_t is the annual percent change in the real average wage expressed in decimals in year t; NW_t is the level of the nominal average wage in year t; and CPI_t is the level of the CPI in year t.

The model estimates the annual percent changes in the real economy-wide wage as a function of the current unemployment rate and the unemployment rate of the previous year, expressed as log-odds ratios, over the period from 1968 to 2002. The value for 1974 was an outlier and therefore was excluded in the development of the equation. The R-squared value was 0.53. The actual and fitted values are shown in figure II.6.

The estimated coefficients and standard error of the regression are then used to simulate the percent change in the real average covered wage. The modified equation is:

$$W_t = W_t^{TR} - 0.06u_t + 0.04u_{t-1} + \varepsilon_t. \quad (9)$$

In this equation, W_t represents the percent change in the real average covered wage in year t; W_t^{TR} represents the percent change in the real average covered wage from the TR04II in year t; u_t represents the deviation of the (log-odds transformed) unemployment rate from the TR04II unemployment rate in year t; and ε_t represents the random error in year t.

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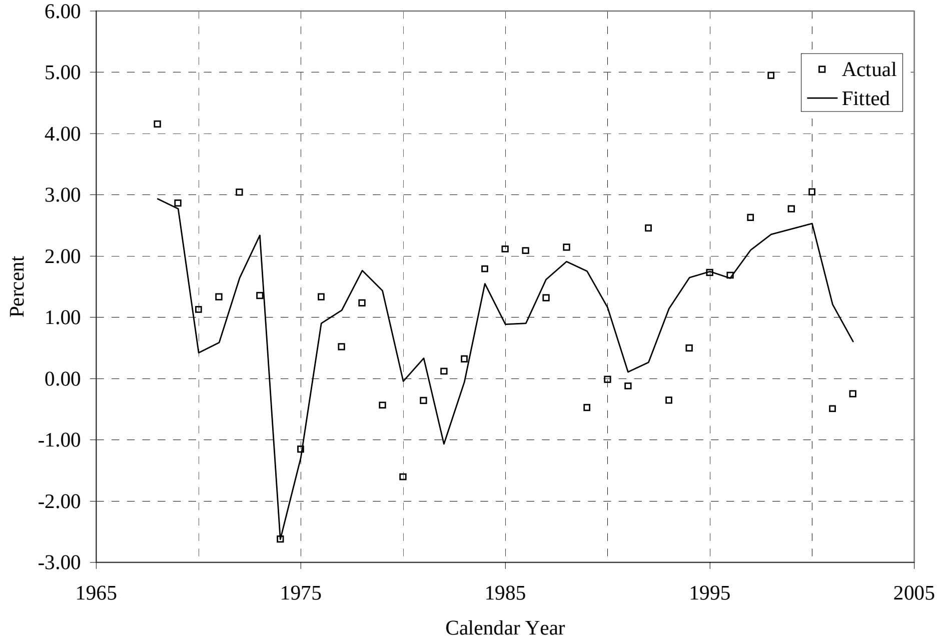
[18] For a detailed description of the methodology, refer to the article titled Creating Comparability in CPS Employment Series - www.bls.gov/cps/cpscomp.pdf.

Figure II.6—Real Average Wage (Percent Change), Calendar Years 1968-2002



F. DISABILITY INCIDENCE RATE

The *disability incidence rate* for a given year is the proportion of the exposed population at the beginning of that year who become newly entitled to disability benefits during the year. The exposed population is comprised of workers who are disability insured but not entitled to disability benefits. The historical disability incidence rates used to fit the equations are age-adjusted to the 1996 exposed population. The age-adjusted disability incidence rates (male and female) are the crude rates that would occur in the disability exposed population as of January 1, 1996, if that population were to experience the observed or assumed age-sex specific disability incidence rates in the selected year.

Data on disability incidence are obtained from SSA administrative records and the age-adjusted disability incidence rates are computed by OCACT. Over the historical period from 1970 to 2003, disability incidence rates have varied widely due to changes in legislation and program administration as well as economic and demographic factors.

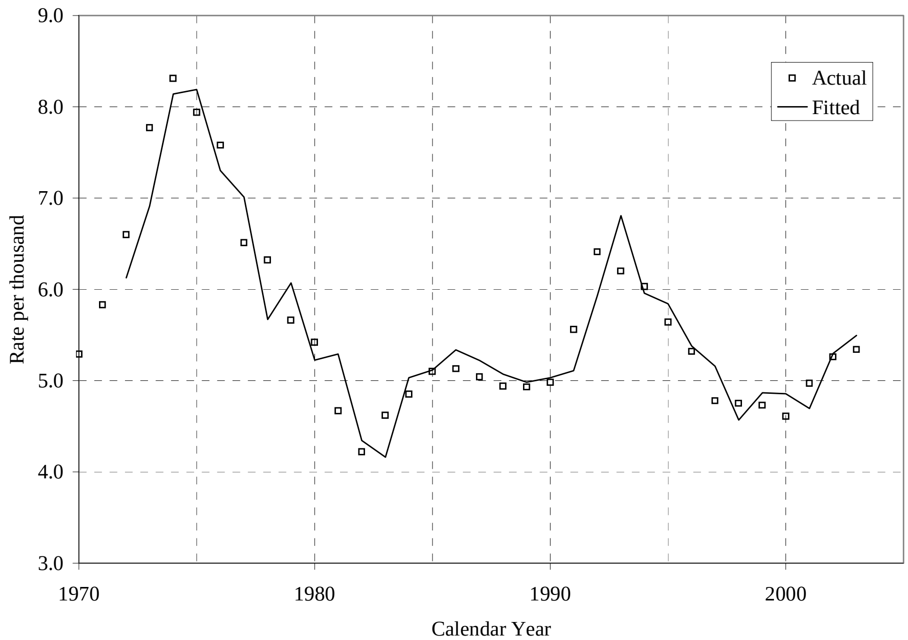
The equations for disability incidence rates are selected separately for males and females. Over the historical period of 1970 through 2003, both the male and the female series fail their tests for stationarity. However, in this model it is assumed that the nonstationarity in these series is due to the various changes in the law over the historical period. Therefore, both series were modeled without correcting for nonstationarity. Using time-series analysis, both series were modeled individually as AR(2) processes. The R-squared values for the male and female disability incidence rate equations were 0.89 and 0.87, respectively. The actual and fitted values for males and females are shown in figures II.7 and II.8, respectively.

The modified male disability age-adjusted incidence rate equation is:

$$DIM_t = DIM_t^{TR} + 1.47 dim_{t\text{-}1} - 0.63 dim_{t\text{-}2} + \varepsilon_t . \tag{10}$$

In this equation, DIM_t represents the male disability incidence rate in year *t*; DIM_t^{TR} represents the male disability incidence rate from the TR04II in year *t*; dim_t represents the deviation of the male disability incidence rate from the TR04II male disability incidence rate in year *t*; and ε_t is the random error in year *t*.

Figure II.7—Male Disability Incidence Rate, Calendar Years 1970-2003

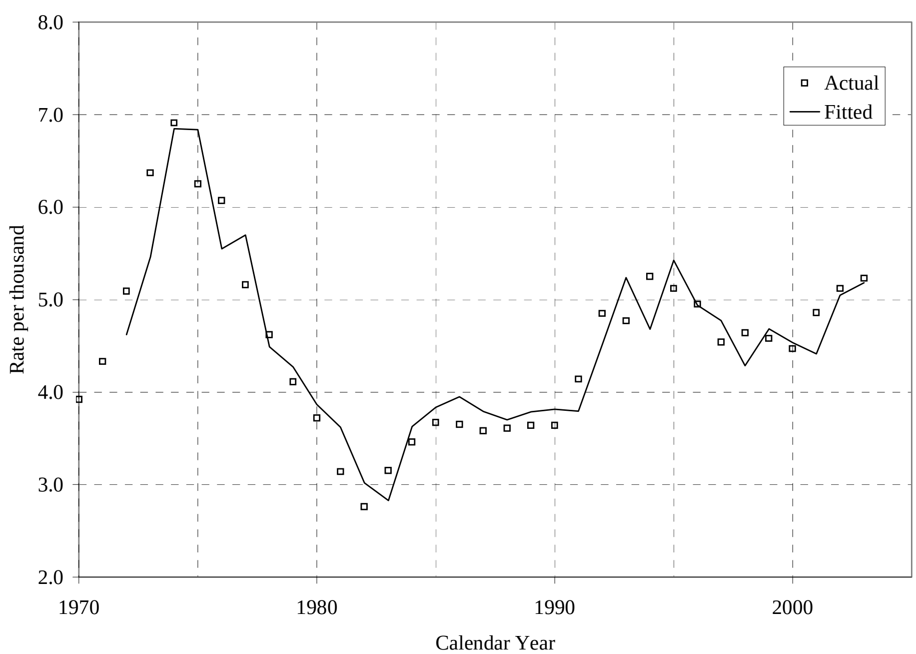


The modified female age-adjusted disability incidence rate equation is:

$$DIF_t = DIF_t^{TR} + 1.45dif_{t-1} - 0.62dif_{t-2} + \varepsilon_t. \tag{11}$$

In this equation, DIF_t represents the female disability incidence rate in year t; DIF_t^{TR} represents the female disability incidence rate from the TR04II in year t; dif_t represents the deviation of the female disability incidence rate from the TR04II female disability incidence rate in year t; and ε_t is the random error in year t.

Figure II.8—Female Disability Incidence Rate, Calendar Years 1970-2003



G. DISABILITY RECOVERY RATE

The *disability recovery rate* for a given year is the proportion of disabled-worker beneficiaries whose disability benefits terminate as a result of the individual's recovery from disability. The age-adjusted disability recovery rates (male and female) are the crude rates that would occur in the in-current-payment population as of January 1, 1996, if the population were to experience the observed or assumed age-sex specific disability recovery rates in the selected year.

Data on disability recovery are obtained from SSA administrative records and the age-adjusted disability recovery rates are computed by OCACT. Over the historical period from 1970 to 2003, there has been substantial variation in the age-adjusted disability recovery rates. This variation is believed to be mostly due to changes in the law. For example, the age-adjusted disability recovery rate for males jumped from 10.3 per thousand in 1996 to 24.4 per thousand in 1997, largely as a result of the effects of a provision in Public Law 104-121. This provision prohibited benefit payments to individuals where drug addiction and/or alcoholism was material to the determination of disability.

The equations for disability recovery rates were modeled separately for males and females. The historical disability recovery rates used to fit the equations were age-adjusted to the 1996 in-current-payment population. Due to the frequent changes in the law, the time period considered was narrowed to 1985 through 2003. The value for 1997 was excluded in the development of the equation due to the change in the law described above. The AR(1) model provided the best fit of the models that were tested. The R-squared values for the male and female disability recovery rate equations were 0.83 and 0.50, respectively.

The modified male age-adjusted disability recovery rate equation is:

$$DRM_t = DRM_t^{TR} + 0.58drm_{t-1} + \varepsilon_t. \tag{12}$$

In this equation, DRM_t represents the male disability recovery rate in year *t*; DRM_t^{TR} represents the male disability recovery rate from the TR04II in year *t*; drm_t represents the deviation of the male disability recovery rate from the TR04II male disability recovery rate in year *t*; and ε_t is the random error in year *t*.

The modified female age-adjusted disability recovery rate equation is:

$$DRF_t = DRF_t^{TR} + 0.57drf_{t-1} + \varepsilon_t. \tag{13}$$

In this equation, DRF_t represents the female disability recovery rate; DRF_t^{TR} represents the female disability rate from the TR04II in year *t*; drf_t represents the deviation of the female disability recovery rate from the TR04II female disability recovery rate in year *t*; and ε_t is the random error in year *t*.

III. DOCUMENTATION OF THE COMPUTER PROGRAM

This chapter describes the details of the computer program used to run the OSM. The model was written in Fortran 90/95 and compiled using the Compaq Visual Fortran compiler, version 6.1.A. The program has about 26,000 lines of source code and was written in modular format with 20 source code files. It uses over 160 data files as input and about 425 MB of RAM. On a personal computer with a 2.8 GHz Intel Pentium 4 processor, running 5,000 simulations takes about 34 hours.

A. ORGANIZATION

The OSM contains nine modules. They are executed sequentially in the following order: Assumptions, Population, Economics, Insured, DIB (Disability Insurance Beneficiaries), OASIB (Old-Age and Survivors Insurance Beneficiaries), Awards, Cost, and Summary Results. In a sequential model, the output from an earlier module may become input to a later module. The flow of data among the OSM modules is summarized in table III.1. The first column lists the nine modules in the order in which they are executed. For each module, the second column lists the modules from which it receives input, while the third column lists the modules to which it provides input. For example, the Population Module receives input from only one module (i.e., Assumptions) and provides output (that then becomes input) to six modules (i.e., Economics, Insured, DIB, OASIB, Awards, and Cost). The Assumptions Module does not receive data from any of the other modules, while the Summary Results Module does not send data to any of the other modules. It is important to note that in this table the only instance in which a module sends data to an earlier solved module is the DIB Module sending data to the Economics Module. This is possible because the DIB data sent there is from the prior (not current) year.

The computer program used to solve the modules is organized to go through three main phases: initialization, simulation, and wrap-up. In the initialization phase, the program prepares input and output files and variables needed by each module. In the simulation phase, the program solves the first eight modules using two nested loops. The first (outermost) is the run number loop. It loops once for each simulation. The second is the year loop. It loops from the first year of the simulation (the current Trustees Report year) through the last year of the simulation (the Trustees Report year plus 75). Thus, for the 2004 Trustees Report, the year loop starts in 2004 and ends in 2079 (a total of 76 years for each simulation). In the wrap-up phase, the program sorts and prints the final output results.

Table III.1—Module Dependencies

Module	Input Modules	Output Modules
Assumptions	N/A	Population Economics DIB Cost Summary Results
Population	Assumptions	Economics Insured DIB OASIB Awards Cost
Economics	Assumptions Population DIB	Insured Awards Cost
Insured	Population Economics	DIB OASIB
DIB	Assumptions Population Insured	Economics OASIB Cost
OASIB	Population Insured DIB	Cost
Awards	Population Economics	Cost
Cost	Assumptions Population Economics DIB OASIB Awards	Summary Results
Summary Results	Assumptions Population Cost	N/A

B. MODULES

All of the modules, with the exception of the Assumptions and Summary Results Modules, are adapted from the deterministic computer model used to prepare the 2004 Trustees Report. The modules are written so that the set of nonstochastic inputs required to begin the projections is identical with the input assumptions used when running the deterministic model under the TR04II. Moreover, the mean value for each stochastic variable is assumed to be the same as the value assumed for the variable under the TR04II.

1. Assumptions

The Assumptions Module contains 54 equations, one for each stochastic variable. These equations are described in detail in chapter II, and include ones for the total fertility rate, rates of mortality improvement (21 male age groups and 21 female age groups), immigration level, emigration level, net other immigration level, unemployment rate, inflation rate, real interest rate, percent change in real average covered wages, disability incidence rates (male and female), and disability recovery rates (male and female).

The equations are used to set the annual values for the stochastic variables. In any particular year, the value for a stochastic variable is determined, in part, by the equation's error term. If the error term for an equation is not dependent on the error terms of other equations, then a random number is drawn for each year from a normal distribution with mean zero and standard deviation equal to the estimated standard error for the equation. If the error term for an equation is dependent on the error terms of other equations, then a Cholesky decomposition is used to assign the appropriate level of covariance. See appendix B for more details on this process.

The final step of this module is to use the error terms to calculate the results of each equation. Chapter II provides more details on specific equations.

For the mortality equations, an additional step decomposes the annual rates of decrease in the central death rates by age group into single years of age.

2. Population

The Population Module projects the Social Security area population by sex, single year of age, and marital status. The components of change—fertility, mortality, and immigration—are applied each year throughout the projection period based on levels generated in the Assumptions Module. The population is grouped by marital status using the relative proportions for each age-sex group projected under the TR04II.

The population is projected by starting with the beginning of the year population, adding births and immigration, and subtracting deaths and emigration. The total fertility rate is distributed among women of childbearing age using the relative proportions of age-specific birth rates for each year from the TR04II. The age-specific birth rates are then applied to the midyear population to calculate the number of births. For the mortality projection, central death rates are computed by applying the rates of decrease in the single year of age central death rates to the previous year's central death rates. Death probabilities are derived from the central death rates by assuming a uniform distribution of deaths for each age. The death probabilities are then applied to the beginning of the year population to calculate the number of deaths for each single year of age and sex group. For each type of immigration, the annual levels are distributed among the age-sex groups by using the relative proportions from the TR04II. The resulting population is then distributed by marital status using the relative martial proportions for each age-sex group projected in the TR04II.

3. Economics

The Economics Module receives data from the Assumptions, Population, and DIB Modules. The Assumptions Module passes civilian unemployment, and inflation rates, along with the growth rate in the real average covered wage. The Population Module passes the age-sex levels of the Social Security area population and their life expectancies. The DIB Module passes age-sex levels of disabled-worker beneficiaries in current-payment status.

For employment-related variables, the module projects various measures for the total U.S. economy and then converts them to OASDI covered concepts. For the earnings variables, the module initially projects OASDI covered wages then converts them to a U.S. economy-wide concept. The module estimates levels for most key variables by projecting deviations from values produced for the TR04II.

Labor Force Participation

Future civilian labor force participation rates by age and sex are influenced by projected disability prevalence ratios, business cycles, life expectancies, and Social Security area population. For a given year, the civilian labor force is summed from the products of the age-sex civilian labor force participation rates and civilian noninstitutionalized populations. For each age-sex group, the civilian noninstitutionalized population is the product of the Social Security area population and the ratio of the noninstitutional to Social Security area population from the TR04II.

Total Employment

The civilian unemployment rates by age and sex are projected by distributing the stochastically projected aggregate rate to its age-sex components. Projected total economy-wide employment is summed from age-sex components derived from the corresponding components of the civilian labor force and unemployment rates. The concept of total economy-wide employment is consistent with the Bureau of Labor Statistics' Current Population Survey, and thus represents an "average" level of employment for a particular year. The projected total economy-wide employment by age and sex is then used to estimate the number of workers with employment at-any-time during the year, a concept closer to OASDI covered employment. Total at-any-time employment by age and sex is influenced by the relative number of illegal immigrants in the workforce and by the proportion of the population employed.

Covered Employment

Total OASDI covered employment by age and sex is projected by removing noncovered workers, including those assumed illegal, from the total at-any-time employment. Total OASDI covered employment is then distributed to those with wages and to those with self-employed net income only.

Covered Wages

Total OASDI covered wages are the product of the number of covered wage workers and their average nominal covered wage. The average nominal covered wage is determined using the annual inflation rate and the real average OASDI covered wage from the Assumptions Module. Total U.S. economy-wide wages are projected as a ratio to OASDI covered wages, adjusted for relative differences in illegal immigration. Total compensation for wage workers, total and covered self-employed income, taxable wages, and taxable self-employed net income are all derived from assumed relationships from the TR04II. Multi-employer refund wages are projected as a ratio to OASDI covered wages, adjusted for relative differences in the unemployment rate.

Taxable Payroll, Average Wage Index, and COLA

The OASDI taxable payroll is the sum of taxable wages and self-employed income, less one-half of multi-employer refund wages. The average wage index is determined using the annual growth rate in the economy-wide average wage, defined as the ratio of total U.S. economy-wide wages to total at-any-time wage employment. The COLA is determined by the inflation rate.

4. Insured (Fully Insured and Disability Insured)

Fully insured status is required to receive worker benefits and is determined by a worker's accumulation of quarters of coverage (QCs). Prior to 1978, one QC was credited for each calendar quarter in which at least $50 was earned. Quarterly reporting was replaced by annual reporting in 1978. The minimum annual required amount, starting with $250 for each QC in 1978, is adjusted each year according to the average wage index. This value for 2004 is $900. Thus, if a worker earns at least $3,600 in covered employment anytime during 2004, then the worker receives credit for four quarters of coverage.

Fully insured status is determined by the number of earned QCs and the worker's age. To be fully insured, a worker must have a total number of QCs greater than or equal to the number of years elapsed after attaining age 21 (with a minimum of six QCs required). Once reaching 40 QCs, the worker remains permanently fully insured. Disability insured status is acquired by any fully insured worker over age 30 who has accumulated 20 QCs during the 40-quarter period ending with the quarter in which the disability began. A fully insured worker aged 24-30 needs to accumulate at least one-half of the quarters elapsed after attaining age 21. A fully insured worker under age 24 needs to have accumulated six QCs during the 12-quarter period immediately before becoming disabled.

In the TR04II, projections of the fully insured population, as a percentage of the Social Security area population, are made by age and sex for each birth cohort beginning with 1900. These percentages are based on 30,000 simulated work histories for each sex and birth cohort. The simulated work histories are constructed to reproduce fairly closely the historical insured percentages from 1990 to date, using the historical portions of the following data:

- Median earnings, by age and sex,
- Covered workers and Social Security area population, by age and sex, and
- Net legal immigrants and other immigrants, by age and sex.

The projected portions of the above data are then used in order to continue the simulation process of work histories of the Social Security area population throughout the projection period. Projected fully insured percentages for each sex and birth cohort are then determined by identifying all simulated work histories that meet the QC requirement for fully insured status as a percentage of the 30,000 simulated cases which represent the Social Security area population. A similar process is applied to produce the disability insured percentages.

In the OSM, the Insured Module projects the percentages of the population that will be fully insured and disability insured for each birth-sex cohort. Projections of fully insured percentages are based on the baseline projection in the TR04II and an adjustment that accounts for the difference between the 10-year moving averages of the covered worker rates[19] from the Economics Module and the TR04II. Projections of the disability insured percentages are modeled in a similar manner. Finally, these percentages are multiplied by the Social Security area population from the Population Module to produce the numbers of insured.

5. Disability Insurance Beneficiaries (DIB)

The DIB Module begins with projections of the disabled-worker beneficiaries in current-payment status. The projections are based on the age-sex specific disability insured population received from the Insured Module and the age-adjusted male and female incidence and recovery rates passed from the Assumptions Module. Additionally, the module uses an estimate[20] of those currently entitled (as of the beginning of the projection period) to a disabled-worker benefit as a starting value.

Disabled Workers

The number of disabled-worker beneficiaries at the end of a year is calculated by adding those newly entitled to a disabled-worker benefit during the year to those currently entitled at the beginning of the year and subtracting those who recover, die, or convert to a retirement benefit upon reaching normal retirement age during the year. New entitlements are calculated by multiplying the incidence rate by the exposed population (disability insured less those currently entitled). For

[19] Covered worker rates are defined as the number of covered workers, expressed as a percentage of the Social Security area population.

[20] The current number of entitled disabled-worker beneficiaries is not completely known because of the time lag between entitlement to and receipt of benefits.

each sex, the future age-specific incidence rates are assumed to grow at the same rate as the growth in the age-adjusted incidence rate.[21]

Deaths and recoveries are calculated by applying the death and recovery rates to the number of people who are currently entitled at the beginning of the year and to the number of people who are newly entitled during the year. Death rates[22] by age, sex, and duration since entitlement are projected to improve at the same rate as the general population aged 19 through 64. For each sex, the future age-specific recovery rates are assumed to grow at the same rate as growth in the age-adjusted recovery rate.[22] The number of disabled-worker beneficiaries in current-payment status is then estimated by reducing those currently entitled by those for whom payment has not yet begun.

Dependents of Disabled Workers

The projected number of auxiliary beneficiaries of disabled workers basically depends on the projections of disabled workers and the Social Security area population. Minor child beneficiaries of disabled workers are projected as the product of the child population, and factors which represent the probabilities that a worker is under normal retirement age, is disability insured, and is disabled, and a statistical residual factor. Student and disabled-adult-child beneficiaries are calculated similarly. Married aged-spouse beneficiaries of disabled workers are projected as a percentage of disabled-worker beneficiaries. This percentage is set as in the TR04II and a factor that adjusts for differences between the OSM and the TR04II projected distributions of the age 62 or older married population. Young-spouse and divorced aged-spouse beneficiaries are calculated similarly, but with their respective populations.

6. Old-Age and Survivors Insurance Beneficiaries (OASIB)

The OASIB module receives variables passed from the Population, Insured, and DIB Modules. The Population Module passes the Social Security area population by age, sex, and marital status. The Insured Module passes the number of fully insured persons, also by age, sex, and marital status. The DIB Module passes disability prevalence rates,[23] and the numbers of disabled-worker and converted disabled-worker beneficiaries, by age and sex. Using the data received, the OASIB Module estimates the number of retired-worker beneficiaries, along with five categories of auxiliary and survivor beneficiaries who are eligible to receive benefits based on the earnings of a retired or deceased worker (also referred to as the primary account holder). These categories are aged-widow(er), aged-spouse, disabled-widow(er), children (minor, student, and disabled adult), and young-spouse beneficiaries.

Aged Widow(er)s

Aged widow(er)s are divided into two subcategories: insured and uninsured. The number of insured aged-widow(er) beneficiaries is projected as the product of the widowed and divorced population aged 60 or older, and the probability that:

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[21] Growth is determined from a 1994-96 base period.
[22] Growth is determined from a 1991-95 base period.
[23] A disability prevalence rate is the ratio of the number of disabled workers to the number of disability insured workers.

- The primary account holder is deceased,
- The primary account holder was fully insured at death, and
- The aged-widow(er) is fully insured but, if at least age 62, did not apply for a retired-worker benefit based on his/her own earnings (assuming that his/her own retired-worker benefit is less than his/her widow(er) benefit).

The number of uninsured aged-widow(er) beneficiaries is projected as the product of the widowed and divorced population aged 60 or older, and the probability that:

- The primary account holder is deceased,
- The primary account holder was fully insured at death,
- The aged widow(er) is not fully insured,
- The aged widow(er) is not receiving a young-spouse benefit for the care of a child, and
- The aged-widow(er)'s benefits are not withheld because of receipt of a significant government pension based on earnings in noncovered employment.

For both the insured and uninsured categories, an additional probability is applied which accounts for other conditions not previously mentioned. For example, in the case of an aged widow(er), the additional factor includes the probability that the widow(er) did not remarry before age 60. In the case of a divorced widow(er), the factor includes the probability that the marriage to the primary account holder lasted at least 10 years.

Retired Workers

To calculate the number of retired-worker beneficiaries, the population aged 62 or older is multiplied by the probability that:

- The worker is fully insured,
- The worker is not receiving disability benefits, and
- The worker is not an insured aged widow(er).[24]

Retirement prevalence rates[25] used in the TR04II are then applied to calculate the number of retired-worker beneficiaries.

Aged Spouses

The number of aged spouses of retired workers is projected as the product of the married and divorced population aged 62 or older, and the probability that:

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[24] In this case, the worker is fully insured and, therefore, eligible to receive his/her own retired-worker benefit. Instead, he/she has decided not to apply for a retired-worker benefit, and is receiving only an aged-widow(er) benefit.

[25] A retirement prevalence rate is the ratio of the number of retired workers to the number of fully insured workers (not receiving disability or widow(er)'s benefits).

- The primary account holder is alive and fully insured,
- The primary account holder is receiving a retirement benefit (not required for divorced spouses),
- The aged spouse is not receiving a young-spouse benefit for the care of a child,[26]
- The aged-spouse is not insured, and
- The aged-spouse's benefits are not withheld because of receipt of a significant government pension based on earnings in noncovered employment.

In addition to the stated conditions, an adjustment is made for other requirements. One such requirement is that the aged spouse has been married to the primary account holder for at least 1 year. In the case of a divorced aged spouse, the requirement is that their marriage had lasted at least 10 years. As is the case with many of the listed requirements, there are exceptions to this requirement.

Disabled Widow(er)s

To calculate the number of disabled-widow(er) beneficiaries, the widowed and divorced population ages 50 through 64 is multiplied by the probability that:

- The primary account holder is deceased,
- The primary account holder was fully insured at time of death,
- The surviving spouse is disabled, and
- The disabled widow(er) is not receiving another type of benefit.

Finally, an additional factor is applied to account for other eligibility requirements. For example, there is a 7 year deadline for surviving spouses to qualify for benefits on the basis of disability.

Children

The OASIB Module calculates the number of child beneficiaries for three different child categories: minor, student, and disabled adult. Child beneficiaries are estimated separately for retired and deceased primary account holders. The population of potential beneficiaries for minor children includes children under age 18, while student status includes children of age 18 (and occasionally also age 19). Disabled adult status includes all persons over age 17 who were disabled prior to age 22.

To calculate the number of minor children of retired workers, the population of children under age 18 is multiplied by the probability that:

[26] The upper age limit to be eligible for a young-spouse benefit is 69, as long as there is a dependent child under 16 or disabled. Since the minimum age to receive an aged-spouse benefit is 62, there is a chance that some spouses between the ages of 62 and 69 are still receiving young-spouse benefits.

- The parent is fully insured,
- The parent is indeed receiving a retired-worker benefit, and
- The child is not receiving a benefit based on his/her other parent's earnings.

For minor children of deceased workers, the same population is multiplied by the probability that:

- The parent is deceased,
- The parent was fully or currently insured at time of death, and
- The child is not receiving a benefit based on his/her other parent's earnings.

Student and disabled adult children of retired and deceased workers are calculated similarly using their respective age-specific populations.

For each child category, an adjustment is made for other conditions, such as the marital status of the child (more common in the case of a disabled adult child) or the dependency status of the child. For example, if a child marries, he/she is no longer entitled to a benefit. Also, if it is determined that the child is not dependent upon the parent (or was not at the time of the parent's death) then he/she is not entitled to receive benefits.

Young Spouses

Young-spouse beneficiaries are broken into two categories, young spouses of retired workers and young spouses of deceased workers (also referred to as mother-survivor and father-survivor beneficiaries). In order to estimate the number of young spouses of retired workers, the married population under age 65 is multiplied by the probability that:

- The primary account holder is age 62 or older,
- The young spouse has a child (under age 16 or a disabled adult) in his/her care, and
- The child is entitled to a child benefit.

To estimate the number of young spouses of deceased workers, the population of widowed and divorced spouses under age 65 is multiplied by the probability that:

- The primary account holder is deceased,
- The young spouse has a child in his/her care (under age 16 or a disabled adult),
- The child is entitled to a child benefit, and
- The young spouse has not remarried.

As with all categories, an additional factor is applied to account for other eligibility requirements, such as ensuring that the young spouse is not entitled to a widow(er) benefit, or is not receiving a retired-worker benefit based on his/her own earnings.[27]

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[27] The eligible age requirement to receive a young-spouse benefit is 69 or younger. Therefore it is necessary to exclude those who are of age to receive a young-spouse benefit, but are receiving their own retired-worker benefit and are counted in the retired-worker calculation.

7. Awards

The Awards Module uses a stratified sample of newly entitled worker beneficiaries. A one-percent sample is used for OASI accounts and a five-percent sample is used for DI accounts. In addition, the Awards Module is passed historical and projected covered workers and population from the Economics and Population Modules, respectively. The module also utilizes historical and projected data from the Economics Module such as average wage, and average taxable earnings in order to produce the modified earnings levels and the earnings history of the sample for the representation of future awards. The module ultimately produces projected levels of benefits, in terms of Average Indexed Monthly Earnings (AIME), for those beneficiaries newly awarded by age, sex, and trust fund. These projected levels are passed to the Cost Module.

Awards Sample

The sample of worker beneficiaries who are newly awarded in 2003 is the foundation of the Awards Module. The sample contains a total of 29,002 newly awarded beneficiaries, 14,412 retired-worker beneficiaries and 14,590 disabled-worker beneficiaries. This sample is a subset of the 10-percent sample used for the TR04II.

Each record in the sample includes the worker's history of taxable earnings under the OASDI program as well as additional information such as birth date, sex, type of benefit, and month of entitlement of the worker. This information allows us to compute the benefits and classify each beneficiary in the sample. Some preliminary calculations made on the sample are utilized within the model. These include the sample's *covered worker rates* calculated separately for retired-worker and disabled-worker beneficiaries. These rates are determined using the sample's earnings histories for 1951 through 2002. The rates are defined for each age group and sex as the ratio of (1) the number of beneficiaries with earnings to (2) the total number of beneficiaries.

Breakdown of the Awards Module

A goal of the Awards Module is to adjust earnings histories and earnings levels in the sample to represent those of future awards. The Awards Module is composed of three distinct components:

- The *Coverage Loads* component applies an adjustment to the earnings histories to reflect the projected changes in covered worker rates.
- The *Base Loads* component adjusts the earnings levels to incorporate earnings above the historical maximum taxable earnings, or wage base.
- The *AIME* component computes an AIME level for each beneficiary in the sample. A distribution of these AIME levels is the basic input for determining average benefits by the Cost Module.

Coverage Loads

In order to estimate future benefit levels, the earnings histories in the sample are modified to represent those of the sample in a given projection year. First, the covered worker rates of current and future sample cohorts are calculated based on sex and age group using data provided by both the Economics Module and the Population Module. The percentage changes from these rates[28] are then applied to the corresponding *sample's* covered worker rates by randomly removing or adding earnings in such a way as to maintain the levels in the sample's average taxable earnings for each year by sex.

Base Loads

The earnings posted for beneficiaries in the sample are limited by the historical wage base. Prior to 1975, the maximum annual amount of earnings on which OASDI taxes were paid was determined by ad hoc legislation. After 1974, however, the annual maximum level was legislated to be determined automatically, based on the increases in the Social Security Average Wage Index (AWI). Prior to these automatic wage base increases, a larger portion of workers earned income at or above the base. Additional ad hoc legislation raising the annual maximum taxable level occurred in 1979, 1980, and 1981. In addition, the AWI used in the automatic calculation of the annual taxable maximum was modified in the early 1990s to include deferred compensation amounts. Hence, an adjustment must be made to incorporate earnings above the historical wage base in the sample to reflect the earnings levels for future samples.

AIME

Average taxable earnings may grow at a different rate than the AWI. Therefore, in the AIME component, all earnings levels in the sample are adjusted to reflect the overall increase in the average taxable earnings of future cohorts, relative to that in the AWI, as projected by the Economics Module.

Based on the modified earnings levels and earnings histories, the new AIMEs can be obtained for the future samples. The module then calculates the distribution of the AIME levels.

8. Cost

The Cost Module serves two broad purposes. The first is to compute the year-by-year progress of the combined OASDI Trust Funds for a 75-year projection period.[29] The second is to produce the summary measures used to assess the long-range financial status of the OASDI program for the 75-year projection period.

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[28] Changes in rates are determined as the ratio of the absolute difference in the rates to the potential difference in the rates.

[29] A projection for the 76th year is required to estimate the target fund, equal to the present value of the outgo in the 76th year. See appendix A.5 for more information.

Progress of Trust Funds

In order to determine the progress of the trust funds, the dollar amounts of income and outgo are computed for each year. Income includes payroll taxes, the taxation of benefits, and interest. Outgo consists of scheduled benefits, administrative expenses, and the railroad retirement interchange. Once the values of income and outgo have been determined for a given year, the amount of end-of-year assets is determined by starting with beginning-of-year assets, adding income and subtracting outgo. Each simulation of the model projects the progress of the trust funds for 75 years. For any simulation, full scheduled benefits are assumed to be paid from the trust funds, even if the assets of the trust funds become exhausted at some point in the projection period.

Payroll Tax Revenue

The OASDI payroll tax rate for the employer and employee, each, is currently specified as 6.2 percent, resulting in a combined employer/employee tax rate of 12.4 percent. Income to the trust funds from payroll taxes is first computed by multiplying the combined payroll tax rate by the effective taxable payroll from the Economics Module. Because there is a time lag between incurring and collecting payroll taxes, an adjustment is made when computing the annual revenue from payroll taxes.

Benefit Outgo

A key component in projecting the amount of benefit payments for a given year is the projection of average benefits of worker beneficiaries who are newly awarded and those who are in current-payment status. The primary insurance amount (PIA) formula factors are as specified in current law. The two bend points for 2003 are indexed by the increase in the average wage supplied by the Economics Module.

The Cost Module uses a starting average benefit matrix for worker beneficiaries in current-payment status based on a one-percent sample of the Master Beneficiary Record (MBR) as of December 31, 2003. This starting matrix is read in once by the Cost Module in its initialization subroutine. For each sex, the starting matrix of retired-worker average benefits is broken out by *age of entitlement* to benefits (62 through 70+) and by *current age* (62 through 95+). The average benefits of retired workers who have converted from disability status are classified in a separate *age of entitlement* category. Similarly, for each sex, the starting matrix of disabled-worker average benefits is broken out by *benefit duration* (0 through 9+) and by *current age* (20 through 66).

The Cost Module updates this starting matrix each year of the projection. First, average benefits are computed for worker beneficiaries who are newly awarded during the year. To compute these amounts, the distributions of AIME levels by age and sex for OASI and DI are obtained from the Awards Module. The average benefit for each age and sex is computed by applying this distribution to the intervals of potential AIME dollars, and then converting to the average PIA. Benefit amounts are determined by applying adjustment factors necessitated by provisions in current law (e.g., actuarial reduction factors and delayed retirement credits). To complete the update of the matrix, two adjustments are made to the benefit levels of those who were awarded benefits in a previous year. The first is increasing benefit levels due to the COLA received from the Economics Module. The other adjustment is to reflect earnings after entitlement and mortality being higher for beneficiaries with lower benefits. Average benefits for worker beneficiaries are determined using the updated benefit matrix and the beneficiary matrix from the OASIB and DIB Modules.

Average benefits of auxiliary beneficiaries are computed by using the historical trend of the level of these benefits, as a percentage of either the average PIA or monthly benefit amount (MBA) of the primary worker beneficiary. The dollar amounts of these average benefits are computed by applying these factors to the appropriate average PIA or MBA projection. Computations are made by sex and type of auxiliary beneficiary.

Finally, the aggregate value of benefits scheduled to be paid is the sum of the products of average benefits and the numbers of beneficiaries by type of benefit. The numbers of beneficiaries by type of benefit are from the OASIB and DIB Modules.

Taxation of Benefits, Administrative Expenses, and Railroad Retirement Financial Interchange

Income from taxation of benefits in a given year is computed by applying a ratio to the projected benefit payments scheduled for that year. This ratio is the same as in the TR04II. The projection of administrative expenses and railroad retirement interchange uses the same methodology as in the TR04II.

Interest

The nominal yield on the combined OASDI Trust Funds is computed by multiplying the increase in the Consumer Price Index by the increase in the real interest rate assumed for the special public-debt obligations held by the trusts funds. Both of these values are allowed to vary and are obtained from the Assumptions Module.[30]

The amount of interest on trust fund assets at the beginning of the year is calculated by multiplying the nominal yield by the amount of these assets. Additionally, interest is calculated for amounts that enter and leave the trust funds during the year. Dollar amounts of tax income, taxation of benefits, scheduled benefits, railroad retirement interchange, and administrative expenses are exposed to, or from, the point in the year at which, on average, they are received or paid out. The amount of interest on the trust funds is then obtained by multiplying the net exposed amounts by the nominal yield for the appropriate fund.

[30] The simulated nominal yield on new issues is converted to a nominal yield on the combined OASDI Trust Funds in the Assumptions Module.

Year-by-Year Actuarial Measures

Once the various components of income and outgo have been computed for a given year, the Cost Module computes the annual cash-flow measures and the trust fund ratio for that year. The annual cash-flow measures include income rate, cost rate, and annual balance (all as a percentage of payroll).

Long-Range Actuarial Measures

The Cost Module computes various summary measures that are used to assess the financial status of the OASDI program during the projection period. These measures summarize, on a present-value basis, the projected annual income and outgo components of the combined OASDI Trust Funds over the 75-year period. The assumed nominal yield on trust fund assets is used to discount the annual income and outgo components. The summary measures include the summarized income rate, summarized cost rate, and actuarial balance, all as a percentage of taxable payroll and as a percentage of gross domestic product (GDP). An additional important summary measure is the 75-year open group unfunded obligation. All of these summary measures are stored for each simulation of the trust funds. Additional details concerning the calculations of these measures are given in appendix A.

The Cost Module also stores the years during the 75-year projection period that relate to certain trust fund events. For each simulation of the trust funds, the following special years are determined, if possible: (1) the first year trust funds are exhausted, (2) the first year trust funds are exhausted and remain exhausted, (3) the first year that outgo exceeds income excluding interest, and (4) the first year that outgo exceeds total income.

9. Summary Results

The Summary Results Module receives data from the Assumptions, Population, and Cost Modules for each of the stochastic simulations. It then computes the estimated probability distributions of two different types of data: annual data and summarized data.

Annual Data

The Summary Results Module produces the estimated distributions of projected data available on an annual, or year-by-year, basis. Such data include the annual variables that are projected in the Assumptions Module using standard time-series modeling. These variables are the total fertility rate, legal immigration, legal emigration, net other immigration, mortality changes by five-year age groups, unemployment rate, inflation rate, real interest rate, change in average wages, and disability incidence and recovery rates. In the Population Module, life expectancy data are computed and sent to the Summary Results Module for analysis. In addition, annual data projected in the Cost Module and sent to the Summary Results Module for analysis include trust fund ratio, income and cost rates, and covered workers per beneficiary. The Summary Results Module processes the data, most of which is stored in arrays dimensioned by valuation year and stochastic iteration. For each of the stochastic simulations, the data for a given year is sorted using Heapsort (Press, et al., 2001). The probability distributions of the year may then be computed. For each piece of data (e.g., the total fertility rate) the Summary Results Module computes the smoothed empirical estimates (Klugman, et al., 1998) of the 2.5th, 5th, 10th, 20th, 30th, 40th, 50th, 60th, 70th, 80th, 90th, 95th, and 97.5th percentiles. These percentiles are stored in new arrays and are written to disk, where they are accessible to post-processing steps used to create graphs and other printed output. The Summary Results Module also computes various 75-year and final 50-year averages (arithmetic or geometric, as appropriate) for each piece of annual data.

Summarized Data

Some data are only available as a summary measure for each of the stochastic simulations. Examples are actuarial balance, summarized income and cost rates, open group unfunded obligation, and the exhaustion year of the combined OASDI Trust Funds. The Summary Results Module processes these data, most of which are passed to the module run by run. At the end of the stochastic simulations, the module computes the estimated frequency distributions for each piece of data. It computes the mean, median, and bounds for 95-, 90-, 80-, 60-, 40-, and 20-percent confidence intervals.

IV. RESULTS

This chapter presents the OSM simulation results in three sections. The results presented in this chapter are based on sets of 5,000 simulations. The first section shows the results of the assumptions which include projected probability distributions for the equations presented in chapter II. In order to better illustrate how uncertainty is quantified, confidence interval bounds from the OSM results are often compared with the values assumed in the low cost (TR04I) and high cost (TR04III) alternatives from the 2004 Trustees Report. The second section contains selected actuarial estimates including annual trust fund ratios and balances, as well as summary actuarial measures (e.g., actuarial balances and summarized cost rates). The third section illustrates the sensitivity of the OSM to variations in the assumptions.

A. ASSUMPTIONS

The tables in the following subsections display results of the OSM for the equations presented in chapter II. The three rows of values shown in the tables are the level in the 75th projection year (2078), the average level over the entire 75-year projection period (2004-78), and the average over the final 50 years of the projection period (2029-78).[31] The corresponding columns for the three rows are the value from the TR04II, median value, values from the TR04I and TR04III, and bounds of the 95-, 90-, and 80-percent confidence intervals. For the mortality subsection, the increase in period life expectancy is shown rather than the 50-year and 75-year averages.

Unless otherwise noted, the average value is computed as an arithmetic average. When the geometric mean is specified, the values are transformed by adding one, averaged geometrically, and reverse transformed by subtracting one. This method is analogous to computing an average effective annual rate for compound interest (Kellison 1991).

The figures in the following subsections graph the values of the assumption variables throughout their entire historical and projection periods. For most variables, the expected future annual values from the equations presented in chapter II are shown along with the 95-percent confidence intervals for the annual values and the cumulative averages of each simulation. For the mortality subsection, the 95-percent confidence interval is shown for the annual values only. For the immigration subsection, values shown for legal emigration and net other immigration during the historical period are estimated since actual data are not available.

1. Fertility

Table IV.1 lists the results of the total fertility rate equation presented in chapter II. For the final 50-year average, the limits of the 80-percent confidence interval closely match the TR04I and the TR04III values (2.24 vs. 2.20 and 1.65 vs. 1.70, respectively).

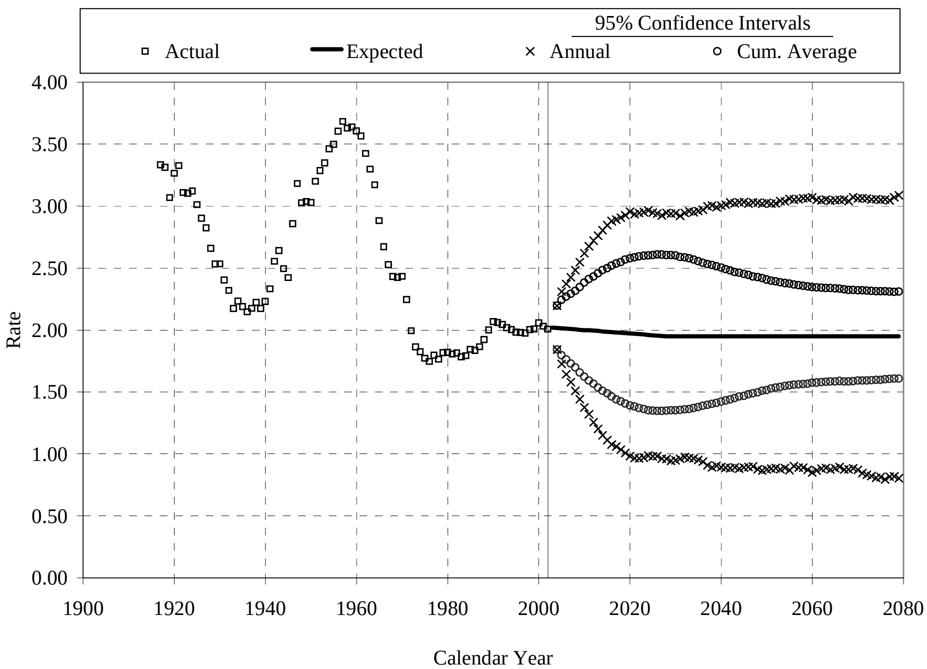
[31] A final 50-year average is presented because most of the variables in this section reach an assumed ultimate value prior to the end of the 25th projection year. Typically, this assumed ultimate value is a constant for the variable of interest. A notable exception is the mortality assumption in which death rates do not reach a constant level since they are derived from a multiple decrement model (where causes of death compete). For mortality, *increases* in life expectancies over the 75-year and final 50 years of the projection period, rather than *average levels*, are presented.

Table IV.1—Total Fertility Rate

	TR04II	Median	TR04I	TR04III	Confidence Intervals					
					95-percent		90-percent		80-percent	
Value in 2078	1.95	1.94	2.20	1.70	3.07	0.82	2.91	1.02	2.69	1.23
75-year average	1.96	1.96	2.17	1.75	2.31	1.61	2.25	1.66	2.18	1.73
Final 50-year average	1.95	1.95	2.20	1.70	2.39	1.50	2.32	1.57	2.24	1.65

Figure IV.1 displays the total fertility rate throughout the entire historical and projection periods. The annual values indicate that the standard deviation of the projected total fertility rate is about 0.5 child per woman.[32]

Figure IV.1—Total Fertility Rate, Calendar Years 1917-2078



2. Mortality

In order to simplify the presentation, mortality results are given here as period life expectancies at birth and age 65 for males and females, separately. Thus, the number of age-sex groups is reduced from 42 to just four.

[32] The width of the 95-percent confidence interval for the annual values is approximately two children per woman. For a normal distribution, this width represents about four standard deviations.

a. Period Life Expectancies at Birth

Tables IV.2 and IV.3 list the resulting period life expectancies at birth for males and females, respectively, from the mortality equations presented in chapter II. The increases shown are calculated as the difference between the values in 2078 and 2004 or the difference between the values in 2078 and 2029. The upper limit of the 90-percent confidence interval in 2078 is nearly equal to the TR04III value (85.8 vs. 85.9 for males, and 89.5 vs. 89.2 for females) while the lower limit is only slightly less than the TR04I value (77.5 vs. 78.0 for males, and 81.6 vs. 82.1 for females).

Table IV.2—Male Period Life Expectancies at Birth

					Confidence Intervals					
	TR04II	Median	TR04I	TR04III	95-percent		90-percent		80-percent	
Value in 2078	81.4	81.8	78.0	85.9	76.5	86.6	77.5	85.8	78.6	84.9
75-year increase	6.9	7.4	3.6	11.3	2.9	11.3	3.7	10.6	4.7	9.8
Final 50-year increase	4.1	4.4	2.2	6.8	2.8	6.4	3.1	6.0	3.4	5.5

Table IV.3—Female Period Life Expectancies at Birth

					Confidence Intervals					
	TR04II	Median	TR04I	TR04III	95-percent		90-percent		80-percent	
Value in 2078	85.2	85.4	82.1	89.2	80.9	90.4	81.6	89.5	82.4	88.6
75-year increase	5.6	5.9	2.6	9.6	2.1	10.2	2.7	9.5	3.4	8.7
Final 50-year increase	3.6	3.8	1.8	5.9	2.0	6.2	2.3	5.8	2.6	5.3

Figures IV.2 and IV.3 display the period life expectancies at birth for males and females, respectively, throughout the entire historical and projection periods.

Figure IV.2—Male Period Life Expectancies at Birth, Calendar Years 1900-2078

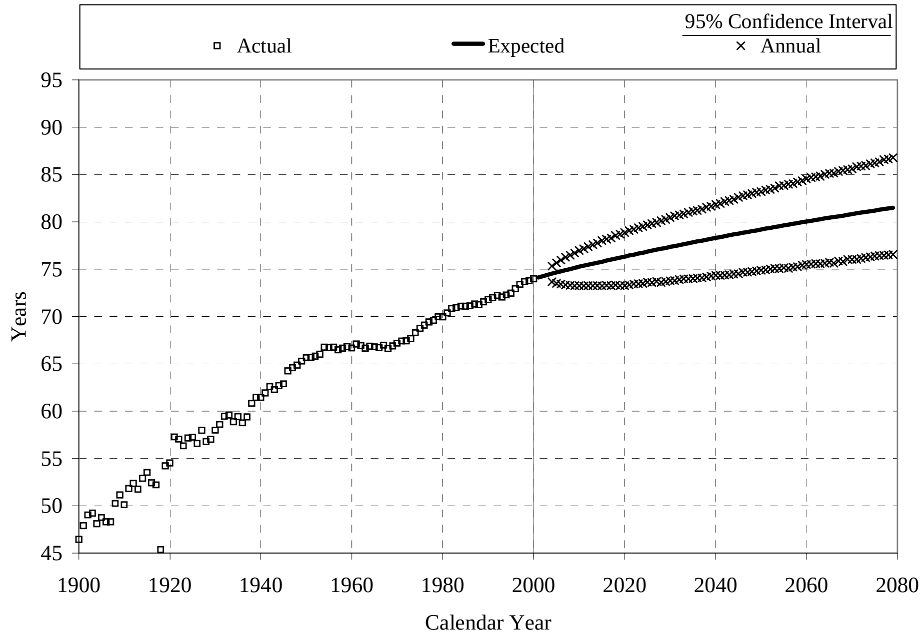
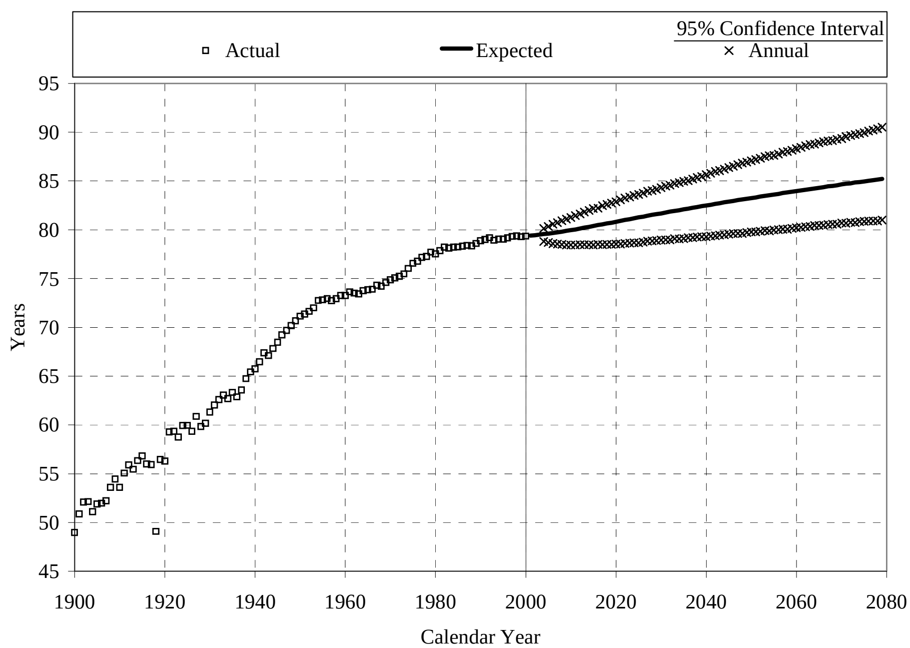


Figure IV.3—Female Period Life Expectancies at Birth, Calendar Years 1900-2078



b. Period Life Expectancies at Age 65

Tables IV.4 and IV.5 list the resulting period life expectancies at age 65 for males and females, respectively, from the mortality equations presented in chapter II. The values shown are analogous to those shown in the previous subsection. The upper limit of the 80-percent confidence interval in 2078 is close to that of the TR04III value (23.2 vs. 23.6 for males, and 26.3 vs. 26.1 for females) while the lower limit is slightly larger than the TR04I value (18.4 vs. 17.7 for males, and 20.7 vs. 20.3 for females).

Table IV.4—Male Period Life Expectancies at Age 65

	TR04II	Median	TR04I	TR04III	Confidence Intervals					
					95-percent		90-percent		80-percent	
Value in 2078	20.3	20.6	17.7	23.6	17.2	24.8	17.7	24.0	18.4	23.2
75-year increase	4.2	4.5	1.7	7.5	1.6	8.3	2.0	7.6	2.6	6.8
Final 50-year increase	2.7	2.9	1.1	4.8	1.4	5.1	1.6	4.6	1.9	4.2

Table IV.5—Female Period Life Expectancies at Age 65

	TR04II	Median	TR04I	TR04III	Confidence Intervals					
					95-percent		90-percent		80-percent	
Value in 2078	22.8	23.3	20.3	26.1	19.4	28.1	19.9	27.3	20.7	26.3
75-year increase	3.9	4.3	1.4	7.1	0.9	8.6	1.4	7.9	2.1	7.0
Final 50-year increase	2.6	2.9	1.1	4.6	1.2	5.3	1.4	5.0	1.8	4.4

Figures IV.4 and IV.5 display the period life expectancies at age 65 for males and females, respectively, throughout the entire historical and projection periods.

Figure IV.4—Male Period Life Expectancies at Age 65, Calendar Years 1900-2078

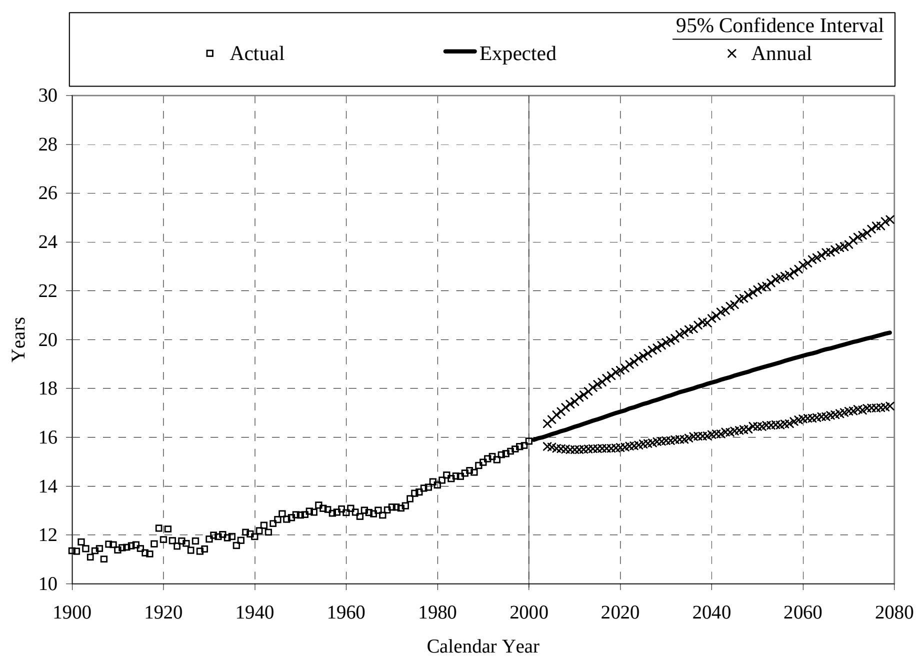
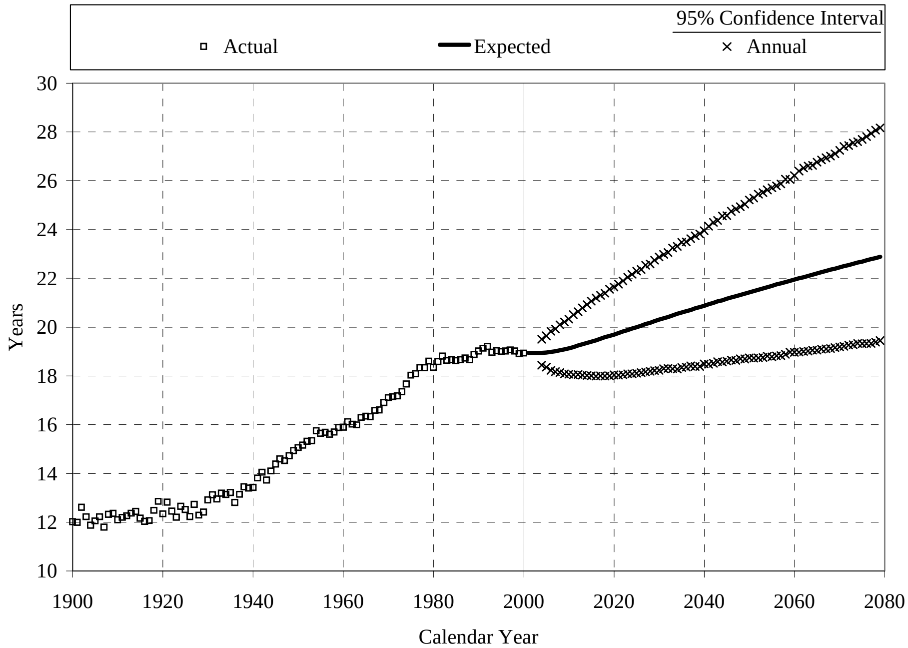


Figure IV.5—Female Period Life Expectancies at Age 65, Calendar Years 1900-2078



3. Immigration

a. Legal Immigration

Table IV.6 displays the results (in thousands) of the legal immigration equation presented in chapter II. For the final 50-year average, the bounds of the 80-percent confidence interval are close to the TR04I and the TR04III values (1,051 vs. 1,063 and 548 vs. 675, respectively).

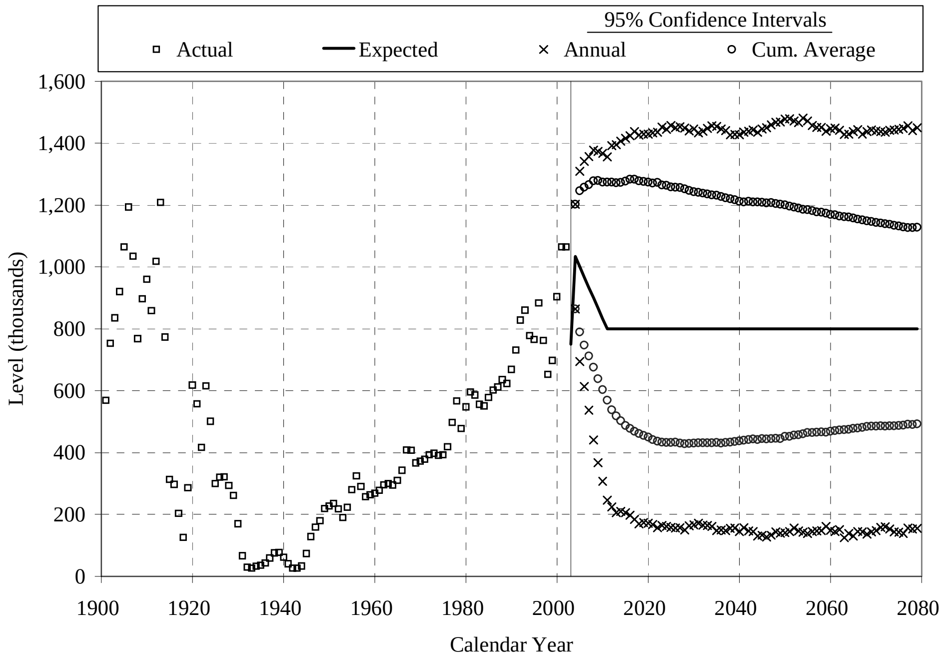
Table IV.6—Legal Immigration (thousands)

	TR04II	Median	TR04I	TR04III	Confidence Intervals					
					95-percent		90-percent		80-percent	
Value in 2078	800	797	1,063	675	1,440	153	1,349	252	1,232	371
75-year average	812	811	1,070	685	1,127	491	1,079	549	1,018	606
Final 50-year average	800	797	1,063	675	1,193	409	1,125	473	1,051	548

Figure IV.6 shows a graph of legal immigration throughout the entire historical and projection periods. The annual values indicate that the standard deviation of the projected legal immigration level is about 300,000.[33]

[33] The width of the 95-percent confidence interval for the annual values is approximately 1,200,000. For a normal distribution, this width represents about four standard deviations.

Figure IV.6—Legal Immigration, Calendar Years 1901-2078



b. Legal Emigration

Table IV.7 displays the results (in thousands) of the legal emigration equation presented in chapter II. The confidence intervals shown are not directly comparable with the values assumed for the TR04I and TR04III. This is because the Trustees' values are established by applying an assumed percentage to the level of legal immigration (20 percent for the TR04I and 30 percent for the TR04III). As a result, the level of emigration under the TR04I and the TR04III are both greater than that of the TR04II.

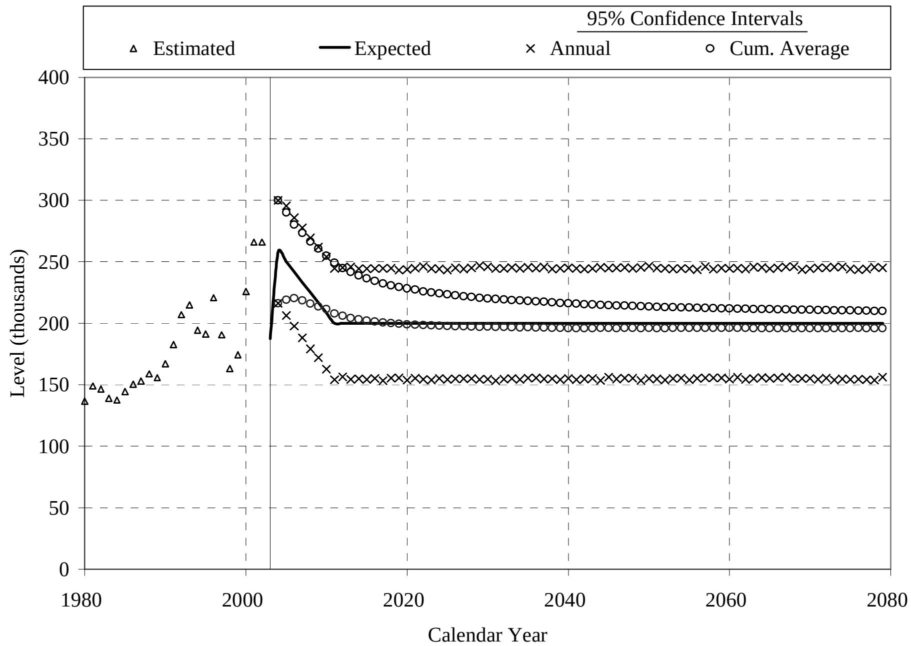
Table IV.7—Legal Emigration (thousands)

	TR04II	Median	TR04I	TR04III	Confidence Intervals					
					95-percent		90-percent		80-percent	
Value in 2078	200	200	213	203	154	246	162	238	170	230
75-year average	203	203	214	206	196	210	197	209	198	208
Final 50-year average	200	200	213	203	191	208	193	207	194	206

Figure IV.7 graphs the level of legal emigration throughout the entire historical and projection periods. The annual values indicate that the standard deviation of the projected legal emigration level is about 25,000.[34]

[34] The width of the 95-percent confidence interval for the annual values is approximately 100,000. For a normal distribution, this width represents about four standard deviations.

Figure IV.7—Legal Emigration, Calendar Years 1980-2078



c. Net Other Immigration

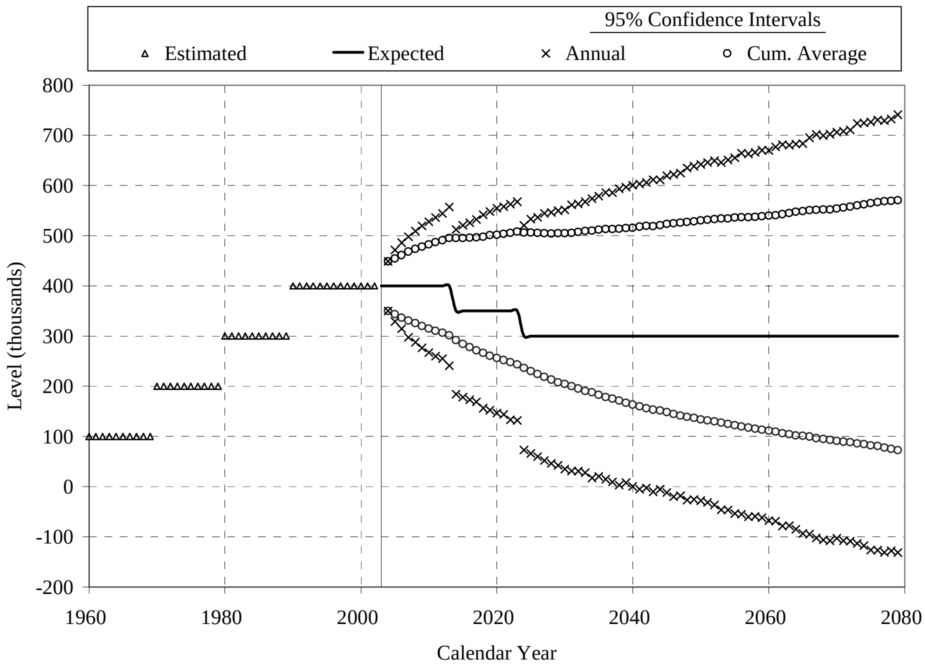
Table IV.8 displays the results (in thousands) of the net other immigration equation presented in chapter II. For the final 50-year average, the bounds of the 80-percent confidence interval are considerably wider than the interval defined by the TR04I and the TR04III values (505 vs. 450 and 86 vs. 200, respectively).

Table IV.8—Net Other Immigration (thousands)

	TR04II	Median	TR04I	TR04III	Confidence Intervals					
					95-percent		90-percent		80-percent	
Value in 2078	300	299	450	200	733	-128	663	-64	577	14
75-year average	320	320	470	207	570	75	526	110	481	152
Final 50-year average	300	302	450	200	622	-16	565	28	505	86

Figure IV.8 graphs the level of net other immigration throughout the entire historical and projection periods. Discontinuities are shown in the graph over the historical and the projected periods. The discontinuities shown over the historical years reflect that the estimated levels of net other immigration are determined as average annual levels for 10-year periods between census years. The discontinuities shown over the projected years are caused by the Trustees' assumption of a 50,000 decrease in the level of net other immigration in years 2014 and 2024. Ignoring the discontinuities, the graph of the projected values is quite typical of that of a random walk in that it produces a parabola. This is expected since the standard error of the forecast increases with the square root of the time variable.

Figure IV.8—Net Other Immigration, Calendar Years 1960-2078



4. Unemployment Rate

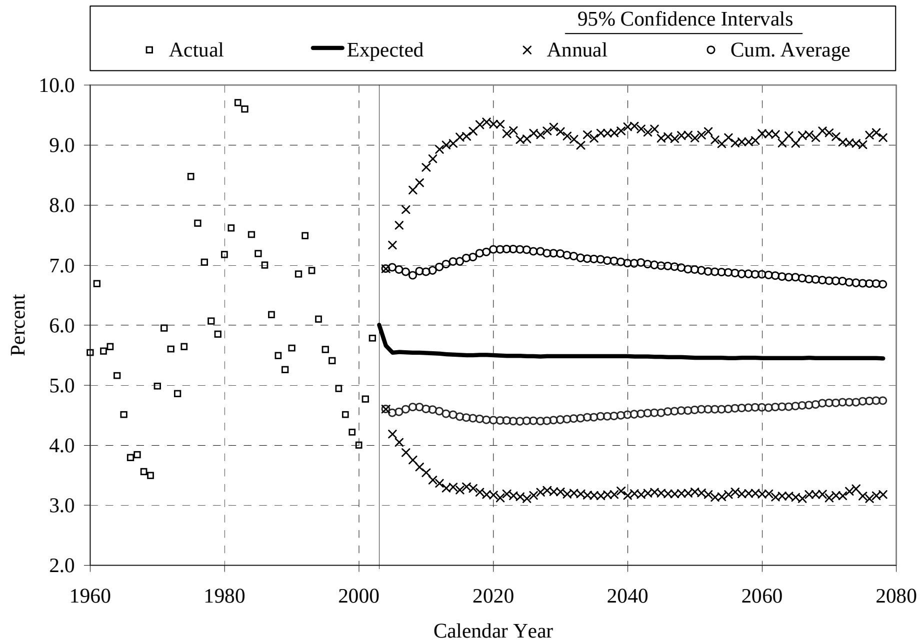
Table IV.9 shows the results of the unemployment rate equation presented in chapter II. The values presented in the table are expressed in percents. The lower and upper bounds of the 95-percent confidence interval for the 75-year cumulative average (4.74 and 6.68 percent, respectively) are close to the values assumed under the TR04I and the TR04III (4.61 and 6.40 percent, respectively).

Table IV.9—Unemployment Rate

	TR04II	Median	TR04I	TR04III	Confidence Intervals					
					95-percent		90-percent		80-percent	
Value in 2078	5.45	5.43	4.59	6.29	3.18	9.13	3.49	8.40	3.88	7.67
75-year average	5.48	5.63	4.61	6.40	4.74	6.68	4.86	6.49	5.02	6.31
Final 50-year average	5.47	5.59	4.58	6.34	4.53	6.92	4.67	6.70	4.87	6.45

Figure IV.9 shows a graph of the unemployment rate over the historical and projection periods. Due to the log-odds transformation of the unemployment rate variable, the confidence intervals are not symmetrical around the median value.

Figure IV.9—Unemployment Rate, Calendar Years 1960-2078



5. Inflation Rate

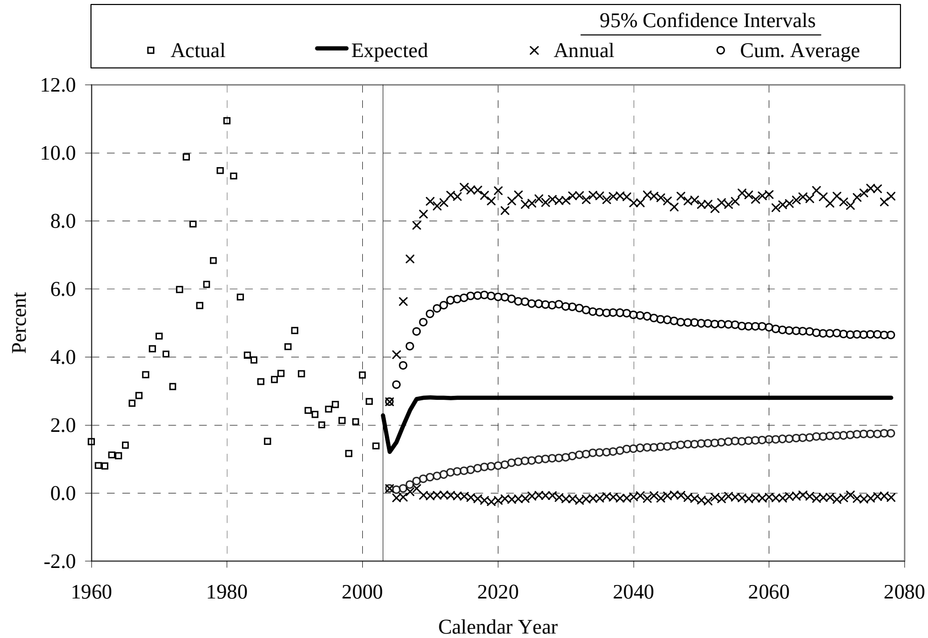
Table IV.10 displays the results of the CPI inflation rate equation presented in chapter II. The values presented in the table are growth rates expressed in percents, and the averages shown are annual compound averages. The lower bound of the 95-percent confidence interval for the 75-year cumulative average, at 1.75 percent, closely matches the 1.77 percent assumed under the TR04I, while the upper bound of 4.64 percent is much higher than the 3.80 percent assumed under the TR04III.

Table IV.10—Inflation Rate

	TR04II	Median	TR04I	TR04III	Confidence Intervals					
					95-percent		90-percent		80-percent	
Value in 2078	2.80	2.84	1.80	3.80	-0.13	8.73	0.23	7.66	0.65	6.37
75-year geometric average	2.75	3.03	1.77	3.80	1.75	4.64	1.93	4.36	2.14	4.05
Final 50-year geometric average	2.80	3.06	1.80	3.80	1.53	5.07	1.76	4.73	2.03	4.34

Figure IV.10 shows a graph of the inflation rate over the historical and projection periods. The asymmetry in the confidence intervals around the median values is due to the logarithmic transformation that was applied to the inflation rate variable.

Figure IV.10—Inflation Rate, Calendar Years 1960-2078



6. Real Interest Rate

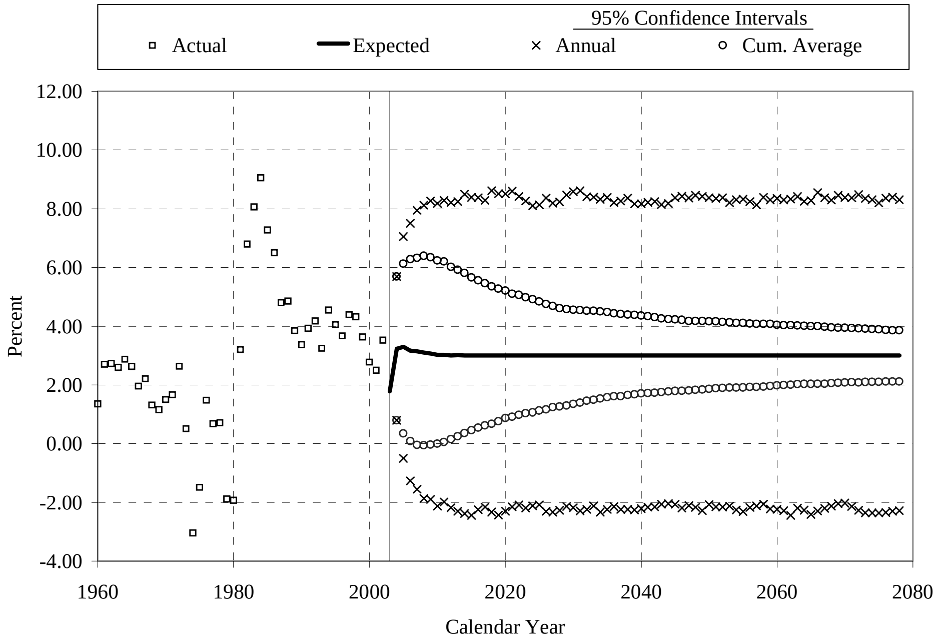
Table IV.11 shows the results of the real interest rate equation presented in chapter II. The values presented in the table are expressed in percents, and the averages shown are annual compound averages. For the 75-year cumulative average, the limits of the 90-percent confidence interval (3.73 percent and 2.26 percent) are close to the values assumed under the TR04I and the TR04III (3.69 percent and 2.24 percent, respectively).

Table IV.11—Real Interest Rate

	TR04II	Median	TR04I	TR04III	Confidence Intervals: 95-percent		90-percent		80-percent	
Value in 2078	3.00	2.96	3.70	2.20	8.31	-2.28	7.40	-1.49	6.38	-0.60
75-year geometric average	3.01	2.98	3.69	2.24	3.86	2.12	3.73	2.26	3.57	2.43
Final 50-year geometric average	3.00	2.98	3.70	2.20	4.14	1.85	3.95	2.03	3.73	2.24

Figure IV.11 shows a graph of the real interest rate on new special issue securities over the historical and projection periods.

Figure IV.11—Real Interest Rate, Calendar Years 1960-2078



7. Real Average Covered Wage

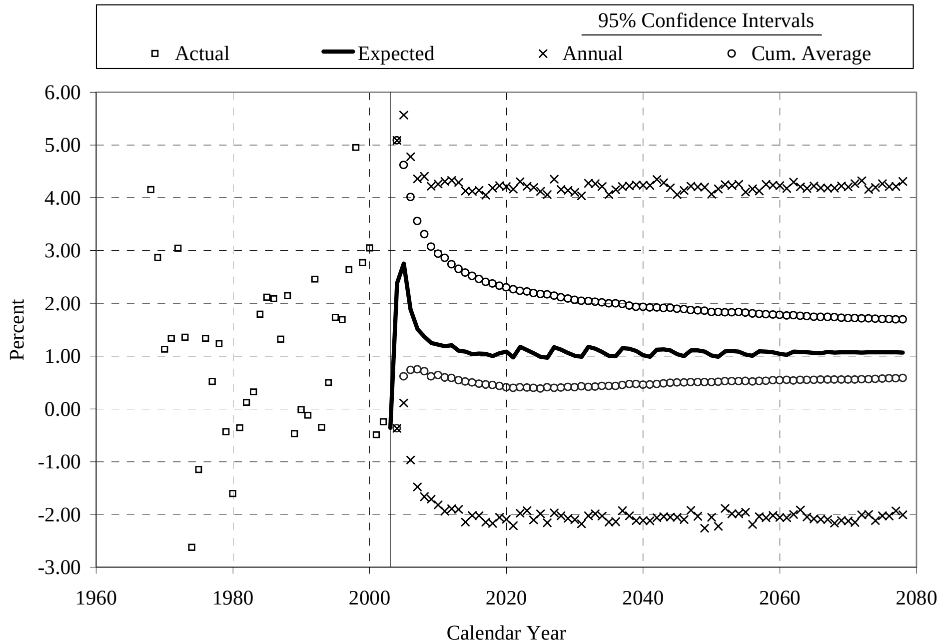
Table IV.12 displays the results of the real average covered wage equation presented in chapter II. The values presented in the table are growth rates expressed in percents, and the averages shown are annual compound averages. For the 75-year cumulative average, the limits of the 95-percent confidence interval (1.69 percent and 0.59 percent) closely match the values assumed under the TR04I and the TR04III (1.63 and 0.62 percent, respectively).

Table IV.12—Real Average Covered Wage

	TR04II	Median	TR04I	TR04III	Confidence Intervals					
					95-percent		90-percent		80-percent	
Value in 2078	1.07	1.05	1.57	0.57	4.31	-2.01	3.74	-1.55	3.12	-0.97
75-year geometric average	1.14	1.13	1.63	0.62	1.69	0.59	1.59	0.67	1.49	0.78
Final 50-year geometric average	1.07	1.07	1.57	0.57	1.75	0.38	1.62	0.48	1.51	0.61

Figure IV.12 shows a graph of the real average covered wage for the historical and projection periods.

Figure IV.12—Real Average Covered Wage, Calendar Years 1968-2078



8. Disability Incidence Rate

Tables IV.13 and IV.14 show the results of the age-adjusted disability incidence rate equations for males and females, respectively. The disability incidence rates shown are per thousand. For the 75-year cumulative average, values for males under the TR04I and the TR04III (4.95 and 7.27 per thousand, respectively) fall outside the limits of the 95-percent confidence interval (5.58 and 6.56 per thousand, respectively). Comparable values for females, under the TR04I and the TR04III (4.24 and 6.22 per thousand, respectively) also fall outside the limits of the 95-percent confidence interval (4.71 and 5.69 per thousand, respectively).

Table IV.13—Male Disability Incidence Rate

					Confidence Intervals					
	TR04II	Median	TR04I	TR04III	95-percent		90-percent		80-percent	
Value in 2078	6.25	6.25	5.00	7.50	4.13	8.42	4.49	8.07	4.89	7.63
75-year average	6.08	6.08	4.95	7.27	5.58	6.56	5.67	6.49	5.76	6.40
Final 50-year average	6.25	6.25	5.00	7.50	5.64	6.86	5.74	6.76	5.86	6.64

Table IV.14—Female Disability Incidence Rate

					Confidence Intervals					
	TR04II	Median	TR04I	TR04III	95-percent		90-percent		80-percent	
Value in 2078	5.27	5.29	4.22	6.33	3.24	7.34	3.58	7.01	3.96	6.64
75-year average	5.21	5.21	4.24	6.22	4.71	5.69	4.80	5.61	4.89	5.52
Final 50-year average	5.27	5.27	4.21	6.33	4.66	5.86	4.76	5.77	4.87	5.66

Figures IV.13 and IV.14 show graphs of male and female age-adjusted disability incidence rates, respectively, over the historical and projection periods.

Figure IV.13—Male Disability Incidence Rate, Calendar Years 1970-2078

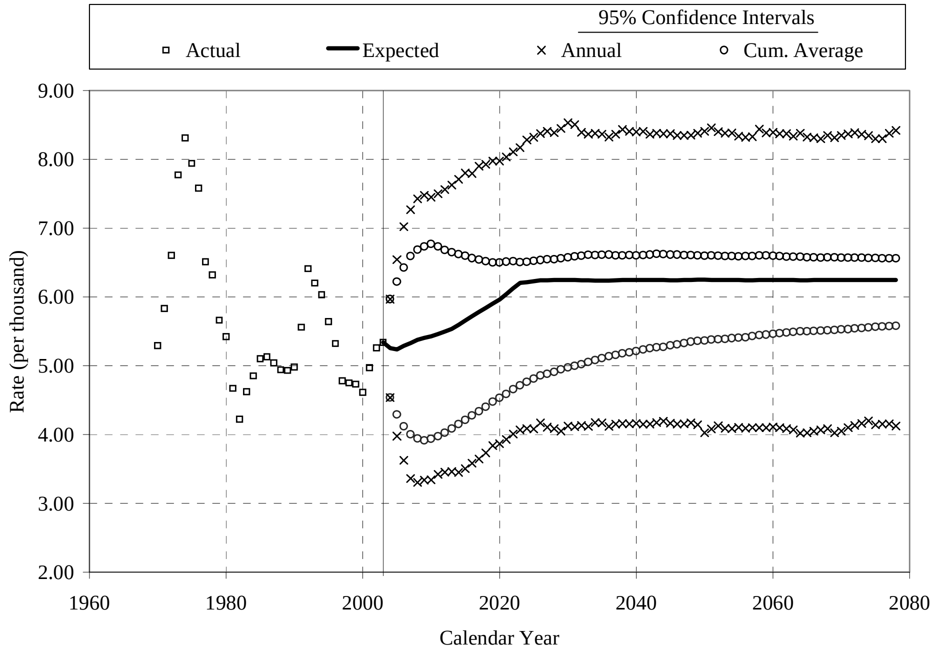
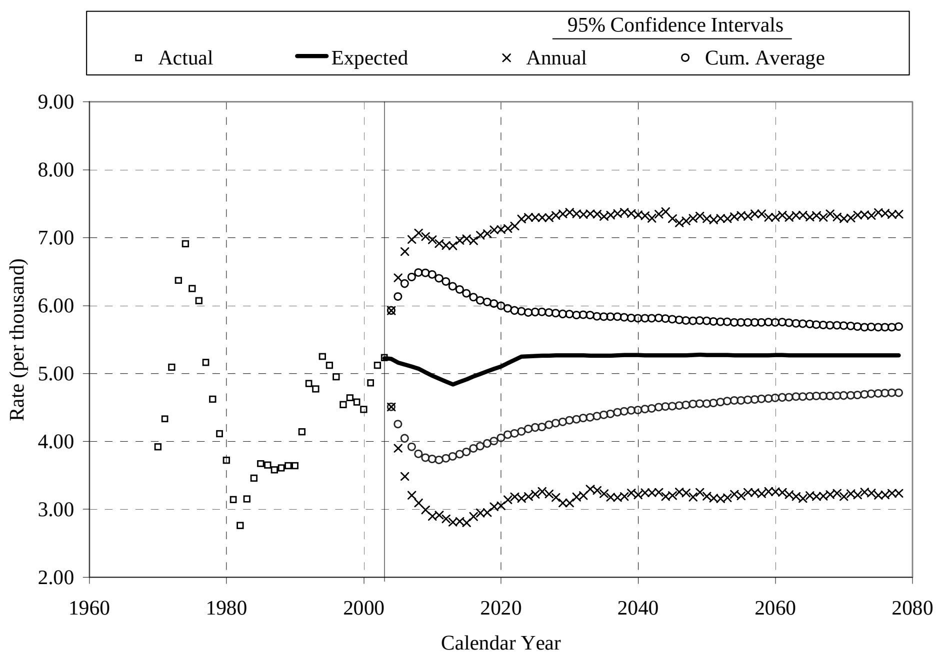


Figure IV.14—Female Disability Incidence Rate, Calendar Years 1970-2078



9. Disability Recovery Rate

Tables IV.15 and IV.16 show the results of the disability recovery rate equations for males and females, respectively. The disability recovery rates presented are per thousand. For the 75-year cumulative average, values for males under the TR04I and the TR04III (13.64 and 8.42 per thousand, respectively) fall outside the limits of the 95-percent confidence interval (12.37 and 10.59 per thousand). Comparable values for females assumed under the TR04I and the TR04III (12.33 and 7.74 per thousand, respectively) also fall outside the limits of the 95-percent confidence interval (11.21 and 9.54 per thousand).

Table IV.15—Male Disability Recovery Rate

					Confidence Intervals					
	TR04II	Median	TR04I	TR04III	95-percent		90-percent		80-percent	
Value in 2078	9.82	9.77	11.99	7.72	13.85	5.77	13.24	6.35	12.49	7.10
75-year average	11.46	11.47	13.64	8.42	12.37	10.59	12.23	10.73	12.06	10.89
Final 50-year average	9.89	9.89	12.03	7.77	10.98	8.82	10.82	8.99	10.62	9.19

Table IV.16—Female Disability Recovery Rate

					Confidence Intervals					
	TR04II	Median	TR04I	TR04III	95-percent		90-percent		80-percent	
Value in 2078	9.28	11.80	11.26	7.34	13.06	5.46	12.47	6.03	11.80	6.70
75-year average	10.37	10.38	12.33	7.74	11.21	9.54	11.08	9.69	10.92	9.84
Final 50-year average	9.32	9.32	11.27	7.38	10.33	8.31	10.17	8.48	9.99	8.67

Figures IV.15 and IV.16 show graphs of the age-adjusted disability recovery rates for males and females, respectively, over the historical and projection periods.

Figure IV.15—Male Disability Recovery Rate, Calendar Years 1986-2078

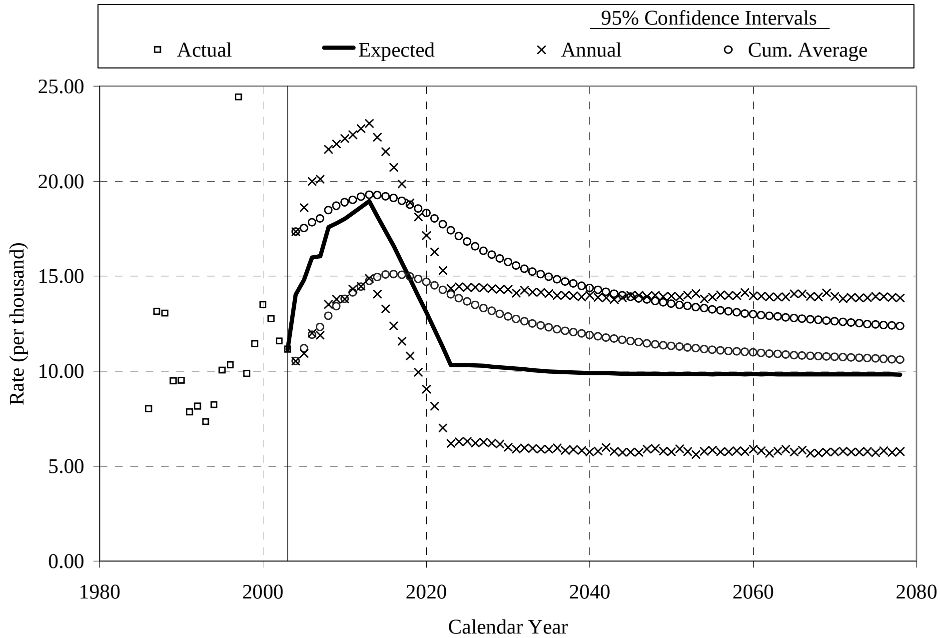
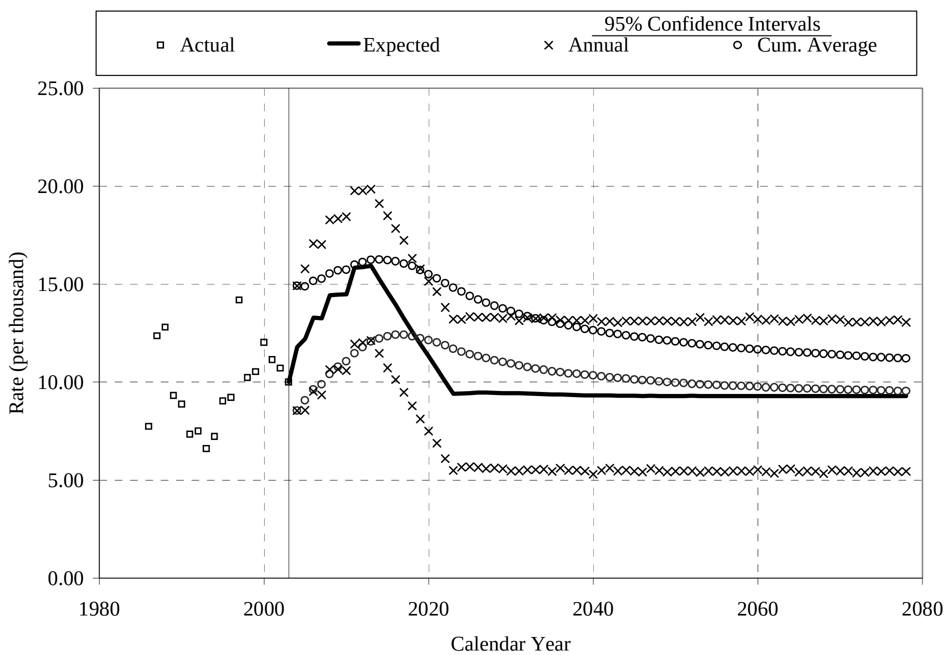


Figure IV.16—Female Disability Recovery Rate, Calendar Years 1986-2078



B. ACTUARIAL ESTIMATES

The tables in the following subsections contain estimates for the OASDI program. A row in the tables contains estimates of a particular measure. For each measure, the columns show results from both the OSM and the 2004 Trustees Report. The value from the TR04II and the median from the OSM are shown first since they both provide an estimate of the central case. The TR04I and TR04III values are shown next followed by the 95-, 90-, and 80-percent confidence intervals from the OSM. These pairs of values provide ranges of the uncertainty for each measure. Where necessary, the limits of the confidence intervals have been inverted to simplify the visual comparison of these to the TR04I and TR04III values.

1. Annual Measures

Table IV.17 shows selected measures of the OASDI program for the 75th projection year, 2078. The median estimates shown from the OSM are slightly more pessimistic (from the OASDI program perspective) than those from the TR04II. For each measure in the table, the upper bound of the 95-percent confidence interval (the second value listed under the 95-percent confidence interval) is more pessimistic than the TR04III value. Thus, the 95-percent confidence interval *includes* the values of these measures under the TR04III. In contrast (with the exception of the cost rate as a percent of GDP), the 95-percent confidence interval does *not include* the values of these measures under the TR04I.

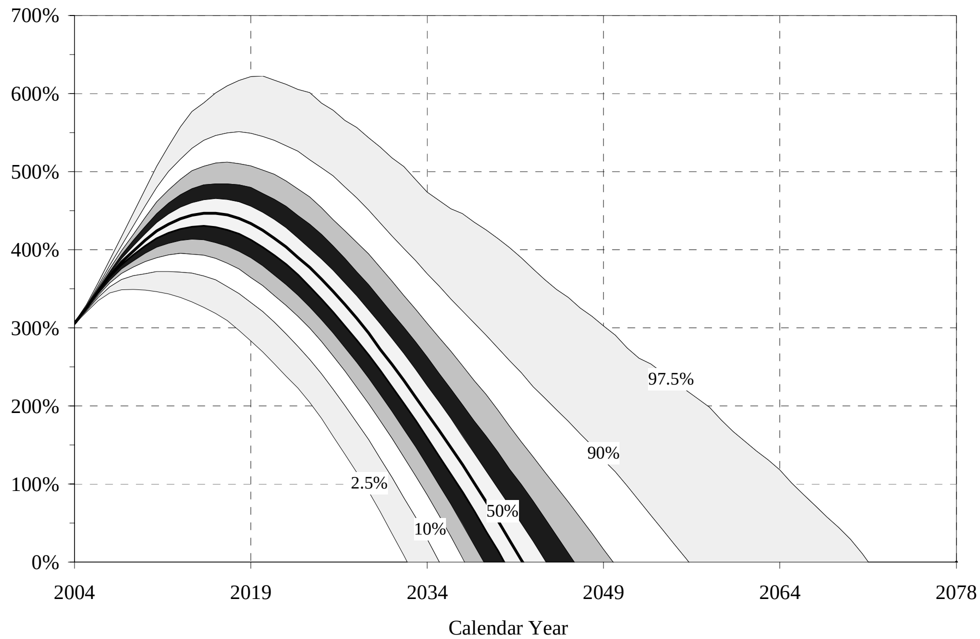
Table IV.17—Estimates of the OASDI Program, Calendar Year 2078

					Confidence Intervals					
	TR04II	Median	TR04I	TR04III	95-percent		90-percent		80-percent	
Cost Rate (% Payroll)	19.29	19.78	14.01	27.23	14.38	27.88	15.17	26.32	16.08	24.70
Cost Rate (% GDP)	6.62	6.78	5.20	8.61	4.95	9.54	5.23	9.00	5.52	8.46
Beneficiaries per 100 Workers	54	56	41	71	43	76	45	72	47	68

Figure IV.17 shows the estimated probability distribution of the annual trust fund ratio for the OASDI program from the OSM. Note that the smooth curves which result do not represent the path of any particular simulation. Instead, for each given year, the curved lines represent the distribution of trust fund ratios based on all stochastic simulation results for that year. The two extreme curves in this figure comprise a 95-percent confidence interval,[35] and deciles are shown for the range of the 10th through the 90th percentiles.

[35] Note that bounds of the 95-percent confidence interval are the 2.5th and 97.5th percentiles.

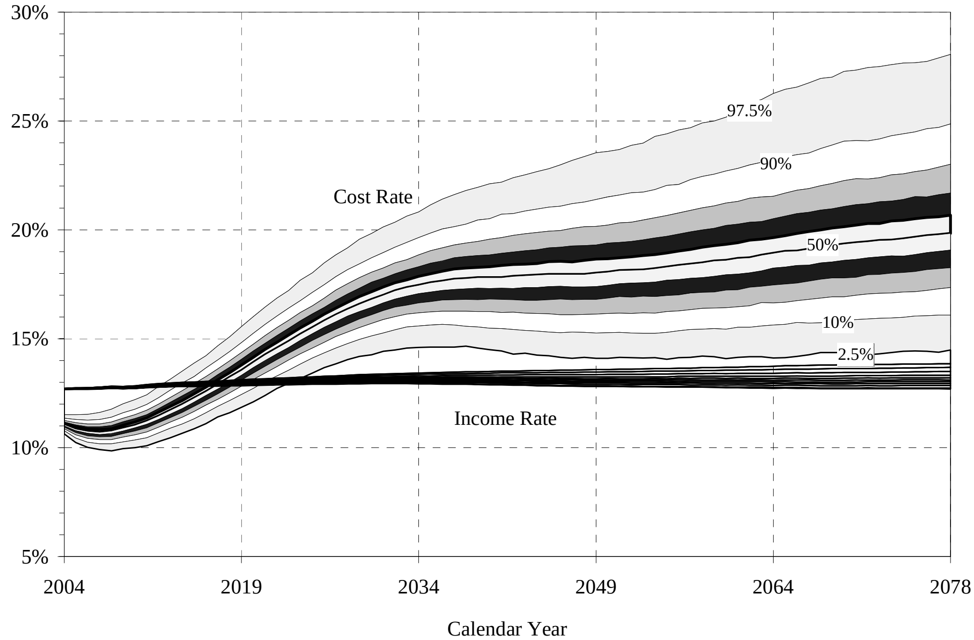
Figure IV.17—Annual Trust Fund Ratio, Calendar Years 2004-78



An estimate of the exhaustion year of the combined OASDI Trust Funds can be obtained from figure IV.17 by inspecting the *x*-intercept of a given curve. The *x*-intercept of the median curve indicates that according to the OSM, there is about the same chance of trust fund exhaustion prior to 2042 as there is after 2042. This exhaustion year is exactly the same as that indicated by the TR04II. Furthermore, the lower limit of the 95-percent confidence interval of the trust fund exhaustion year indicated by the OSM is only 1 year greater than that of the TR04III (2032 vs. 2031). However, the upper limit, as indicated by the OSM is more pessimistic than that of the TR04I (2071 vs. no exhaustion during the 75-year projection).

Figure IV.18 shows the estimated probability distribution of the annual cost and income rates for the OASDI program from the OSM. As before, the smooth curves which result do not represent the path of any particular simulation. Once again, the two extreme curves in this figure comprise a 95-percent confidence interval, and deciles are shown for the range of the 10^{th} through the 90^{th} percentiles. Note that for the annual income rate, the range of the 95-percent confidence interval is too narrow to resolve the decile values.

Figure IV.18—Annual Cost and Income Rates, Calendar Years 2004-78



Notice that the upper limit of the 95-percent confidence interval for the income rate never exceeds the lower limit of the 95-percent confidence interval for the cost rate after about 2025.

2. Summary Measures

Table IV.18 shows selected measures of the OASDI program for the 75-year projection period. The median estimates shown from the OSM are equal to or slightly more pessimistic than those from the TR04II. Estimates of the long-range actuarial balance, summarized cost rate, open group unfunded obligation, and exhaustion year measures under the TR04I and TR04III fall outside the bounds of the 95-percent confidence intervals from the OSM. Estimates of the summarized income rate and the first year cost exceeds tax income under the TR04I and TR04III fall inside or on the bounds of the 95-percent confidence intervals.

Table IV.18—Estimates of the OASDI Program, Calendar Years 2004-78

	TR04II	Median	TR04I	TR04III	Confidence Intervals: 95-percent		90-percent		80-percent	
Actuarial Balance	-1.89	-1.98	0.41	-4.96	-0.33	-4.02	-0.55	-3.63	-0.85	-3.25
Summarized Cost Rate	15.73	15.83	13.30	18.94	14.08	17.96	14.35	17.57	14.65	17.15
Summarized Income Rate	13.84	13.84	13.72	13.98	13.62	14.08	13.65	14.04	13.69	13.99
Open Group Unfunded Obligation (in trillions)	$3.7	$4.0	-$1.1	$10.3	$0.4	$9.2	$0.9	$8.1	$1.5	$7.1
First Year Cost Exceeds Tax Income	2018	2018	2022	2013	2023	2013	2022	2013	2021	2014
Exhaustion Year	2042	2042	1/	2031	2071	2032	2063	2033	2056	2035

1 There is no exhaustion year in the period 2004-78 for the TR04I.

Figure IV.19 shows a frequency distribution of the long-range actuarial balances estimated from the OSM. The width of each bar on the figure is 0.2 percent and the actuarial balance is expressed as a percentage of taxable payroll. It is interesting to note that only 34 of the 5,000 simulations resulted in a positive long-range actuarial balance. In other words, according to the OSM, there is a 99.3 percent probability that the OASDI program has a negative actuarial balance for the period 2004-78.

Figure IV.19—Frequency Distribution of the Long-Range Actuarial Balance, Calendar Years 2004-78

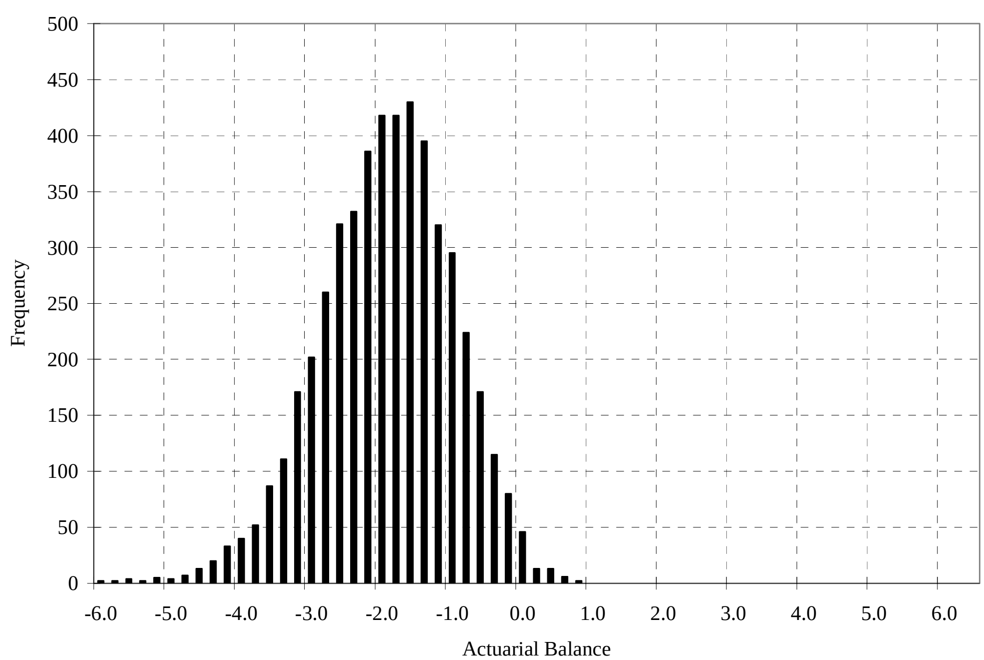
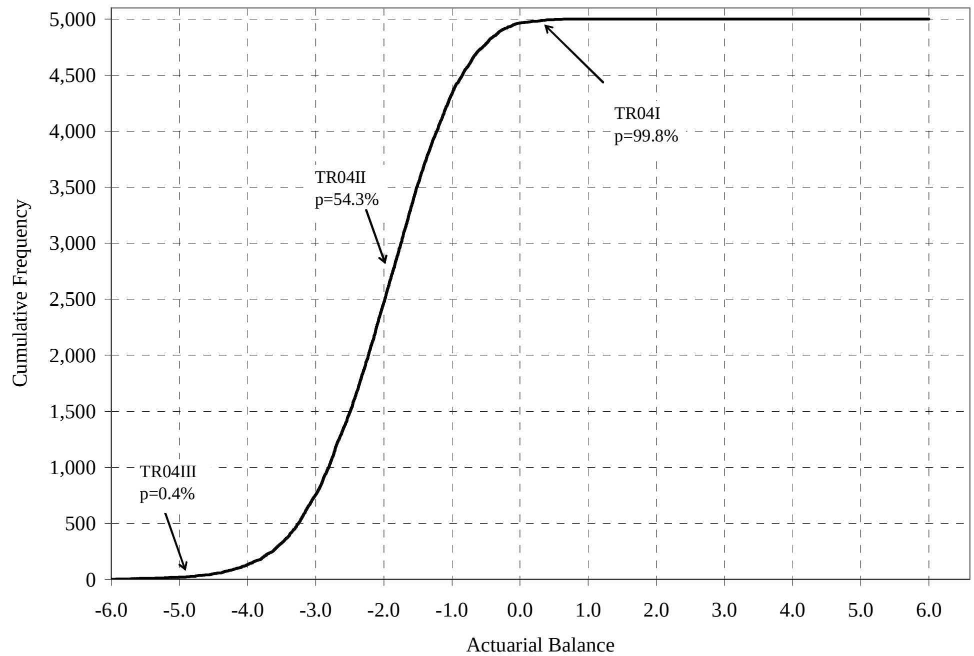


Figure IV.20 shows a cumulative frequency distribution of the long-range actuarial balances estimated from the OSM. The scale on the horizontal axis is the same as in figure IV.19. The TR04III, TR04II, and TR04I have each been identified on the figure and labeled with the probability (according to the OSM) that the actuarial balance is less than or equal to this level.

Figure IV.20—Cumulative Distribution of the Long-Range Actuarial Balance, Calendar Years 2004-78



C. SENSITIVITY ANALYSIS

Two types of sensitivity analyses have been performed. A stochastic sensitivity analysis consists of performing a set of simulations in which a single equation (or group of related equations) is modeled stochastically while all other equations are modeled as the TR04II. A deterministic sensitivity analysis consists of a single simulation in which the mean of an equation (or group of related equations) is replaced with the TR04I or TR04III value, while the remaining equations are left with their means equal to the TR04II values.

1. Stochastic Analysis

A stochastic sensitivity analysis is the process in which one, or some, of the demographic, economic, and disability variables are varied independently, while the other variables are set to their values under the TR04II. This allows a measurement of how much of the variance of the final distributions can be attributed to each variable or set of variables.

Table IV.19 presents the expected long-range actuarial balance, along with the standard deviation, for each of the stochastic sensitivity runs. Each of these sensitivity runs is based on 5,000 simulations of the stochastic model. The first sensitivity run presented in table IV.19 allows only the total fertility rate to vary. The next sensitivity run varies only the 42 mortality improvement rates (21 age groups for males and females). The third run, listed as immigration, allows only the three immigration assumptions (legal immigration, emigration, net other immigration) to vary. Grouping the four economic assumptions (unemployment, inflation, real interest, and real average wage growth rates) and allowing only these variables to vary produces the economic sensitivity run. The last sensitivity run varies only the disability incidence rates (male and female together). Varying the disability recovery rates resulted in a negligible change.

Table IV.19—Sensitivity to Varying Selected Assumptions, Calendar Years 2004-78

	Actuarial Balance		
Effect of Changing Individual Assumption:	Median	Average	Standard Deviation
Fertility	-1.92	-1.93	0.49
Mortality Improvement	-1.96	-1.97	0.56
Immigration	-1.89	-1.89	0.10
Economic	-1.90	-1.90	0.51
Disability Incidence	-1.89	-1.88	0.08
Effect of Changing All Assumptions Together	-1.98	-2.03	0.95

The long-range actuarial balance under the TR04II is -1.89 percent of taxable payroll.

2. Deterministic Analysis

A deterministic sensitivity analysis is the process in which the mean of one, or some, of the demographic, economic, and disability equations are set to equal the TR04I or the TR04III value, while the remaining variables are set to equal the TR04II value. In this case, the variance of all key input variables is set to zero and only one simulation is performed, since the zero variance ensures that all simulations produce the same results. This type of sensitivity analysis is similar to that done in the Trustees Report, and can therefore be used to compare the underlying nature of the OSM and the Trustees' model.

Table IV.20 compares the long-range actuarial balance results from the deterministic sensitivity run of the OSM to the results in the Trustees Report. The first row of each sensitivity grouping is from the OSM. The second row contains the actual sensitivity result from the Trustees Report. Technically speaking, the first sensitivity is not really a sensitivity analysis at all, but is included as a point of reference. The next three results (fertility, mortality, and immigration) are the demographic deterministic sensitivity runs. This is followed by three economic deterministic sensitivity runs (inflation, real interest, and real average wage). The final deterministic sensitivity run is disability incidence. Note that unemployment and disability recovery sensitivity runs are not included since no comparable sensitivity analysis is performed for the Trustees Report.

Table IV.20—Comparison of Stochastic and Deterministic Models, Calendar Years 2004-78

	Actuarial Balance		
Sensitivity:	Low Cost	Intermediate [1]	High Cost
None			
OSM	0.40	-1.89	-4.99
TR	0.41	-1.89	-4.96
Fertility			
OSM	-1.57	-1.89	-2.21
TR	-1.61	-1.89	-2.18
Mortality			
OSM	-1.17	-1.89	-2.76
TR	-1.29	-1.89	-2.59
Immigration			
OSM	-1.68	-1.89	-2.03
TR	-1.63	-1.89	-2.08
Inflation			
OSM	-2.10	-1.89	-1.63
TR	-2.11	-1.89	-1.67
Real Interest			
OSM	-1.39	-1.89	-2.47
TR	-1.38	-1.89	-2.48
Real Average Wage			
OSM	-1.32	-1.89	-2.50
TR	-1.35	-1.89	-2.42
Disability Incidence			
OSM	-1.57	-1.89	-2.22
TR	-1.61	-1.89	-2.17

[1] This column does not show a sensitivity analysis, but is shown for comparison purposes.

V. APPENDICES

A. PRINCIPAL MEASURES OF FINANCIAL STATUS

Three types of financial measures are useful in assessing the actuarial status of the combined OASDI Trust Funds under the financing approach specified in current law: (1) annual balances, income rates, and cost rates, (2) trust fund ratios, and (3) summary measures like actuarial balance and open group unfunded obligation. In assessing the financial condition of the program, particular attention should be paid to the level of the annual balance at the end of the long-range period, and the time at which the annual balance may change from positive to negative values. The ratio of beneficiaries to covered workers on a year-by-year basis (a program ratio rather than a financial ratio), provides a useful comparison to the cost of the OASDI program over the entire 75-year valuation period. Another important measure is the pattern of projected trust fund ratios. The trust fund ratio is the proportion of a year's projected outgo that can be paid with the funds available at the beginning of the year. Particular attention should be paid to the amount and year of maximum trust fund ratio, to the year of exhaustion of the funds, and to stability of the trust fund ratio in cases where the ratio remains positive at the end of the long-range period. The final measures discussed in this appendix summarize the total income and cost over valuation periods that extend through 75 years. These measures indicate whether projected income will be adequate for the period as a whole. The first such measure, actuarial balance, indicates the size of any shortfall as a percentage of the taxable payroll over the period. The second, open group unfunded obligation, indicates the size of any shortfall in present-value discounted dollars.

If the 75-year actuarial balance is zero or positive, then the trust fund ratio at the end of the period, by definition, will be at 100 percent (or greater) and financing for the program is considered to be adequate for the 75-year period as a whole. (Financial adequacy for each year is determined by whether the trust fund is zero or positive throughout the year.) Whether or not financial adequacy is stable in the sense that it is likely to continue for subsequent 75-year periods in succeeding reports is also important when considering the actuarial status of the program. One indication of this stability is the behavior of the trust fund ratio at the end of the projection period. If projected trust fund ratios for the last several years of the long-range period are positive and constant, or rising, then it is likely that subsequent Trustees Reports will also show projections of financial adequacy (assuming no changes in demographic and economic assumptions).

The remaining portion of this appendix discusses these basic measures of assessing the actuarial status of the combined OASDI Trust Funds. Values from the 2004 deterministic model are often given in order to aid in the discussion.

1. Trust Fund Operations

Starting with the dollar level of assets in a given year, the combined OASDI Trust Funds receive income from payroll taxes, income from taxation of benefits, and interest income. In turn, scheduled benefits, administrative expenses, and the net financial interchange to the Railroad Retirement Board, are all paid from the combined OASDI Trust Funds. Hence, the dollar value of assets at the end of a given year is numerically equal to the dollar value of assets at the beginning of the year, plus payroll taxes, plus income from taxation of benefits, plus interest income, less scheduled benefits, less administrative expenses, less net financial interchange to the Railroad Retirement Board.

2. Annual Income Rates, Cost Rates, and Balances

Basic to the consideration of the long-range actuarial status of the trust funds are the concepts of income rate and cost rate, each of which is expressed as a percentage of taxable payroll. The annual income rate is the ratio of income from revenues (payroll tax contributions and income from the taxation of benefits) to the OASDI taxable payroll for the year. The OASDI taxable payroll consists of the total earnings which are subject to OASDI taxes, with some relatively small adjustments. These adjustments include reflecting that individuals who work for more than one employer and have OASDI covered wages exceeding the contribution and benefit base during the year will be reimbursed for the payroll taxes they paid on wages in excess of the contribution and benefit base for that year. However, each employer pays, on behalf of each employee, payroll taxes on annual wages up to the contribution and benefit base, regardless of the amount of wages an employee receives from other employers. Because the taxable payroll reflects such adjustments as these, the annual income rate can be defined to be the sum of the OASDI combined employee-employer contribution rate (or the payroll-tax rate) scheduled in the law and the rate of income from taxation of benefits (which is, in turn, expressed as a percentage of taxable payroll). As such, it excludes net investment income. The annual cost rate is the ratio of the cost (or outgo, expenditures, or disbursements) of the program to the taxable payroll for the year. In this context, the outgo is defined to include benefit payments, administrative expenses, and net transfers from the trust funds to the Railroad Retirement program under the financial-interchange provisions. For any year, the income rate minus the cost rate is referred to as the annual balance for the year. Considering Social Security's cost as a percentage of the total U.S. economy, GDP provides an additional perspective.

The year-by-year relationship of the income and cost rates illustrates the expected pattern of cash flow for the OASDI program over the full 75-year period. The pattern of the OASDI program's estimated cost rate is generally much different than that of the income rate. The income rate generally increases only slightly during the next 75 years, from just under to just over 13 percent as income from taxation of benefits increases. There is only a very small difference in the income rate among the TR04I, TR04II, and TR04III. Under the TR04II, the OASDI cost rate is estimated to remain fairly stable and well below the income rate for the next several years, until about 2010. It then begins to increase rapidly and first exceeds the income rate for 2018, producing cash-flow deficits thereafter. The cost rate continues rising through about 2030 as the baby-boom generation reaches retirement age. By 2078, the projected continued reductions in death rates and relatively low birth rates will cause a significant upward shift in the average age of the population and will push the cost rate to over 19 percent of taxable payroll under the TR04II. Annual deficits generally increase throughout the remainder of the 75-year projection period, reaching nearly 6 percent of taxable payroll by 2078. The OASDI cost rates for the TR04I and TR04III follow generally similar patterns, but differ significantly in magnitude from those projected for the TR04II.

3. Comparison of Workers to Beneficiaries

The primary reason that the estimated OASDI cost rate generally increases rapidly after 2010 is that the number of beneficiaries is projected to increase more rapidly than the number of covered workers. This generally occurs because the relatively large number of persons born during the baby-boom generation will reach retirement age, and begin to receive benefits, while the relatively small number of persons born during the subsequent period of low fertility rates will comprise the labor force.

Based on the TR04II, the number of covered workers per OASDI beneficiary declines to 2.1 by 2031, and to 1.9 by 2062. Under the TR04I, for which high fertility rates and small reductions in death rates are assumed, this ratio declines to 2.3 by 2032, and then rises back to a level of 2.4 by 2044. Under the TR04III, for which low fertility rates and large reductions in death rates are assumed, the decline in this ratio is much greater, reaching 1.8 by 2037, and 1.4 by 2074.

The impact of the demographic shifts under the TR04I, TR04II, and TR04III on the OASDI cost rates is better understood by considering the projected number of beneficiaries per 100 workers. As compared to the 2003 level of 30 beneficiaries per 100 covered workers, this ratio is estimated to rise significantly by 2080 to 54, 41, and 71 under the TR04II, TR04I, and TR04III, respectively. The significance of these numbers can be seen by noting that the trend of the OASDI cost rates is essentially the same as the trend of the number of beneficiaries per 100 covered workers over the 75-year projection period. This emphasizes the extent to which the cost of the OASDI program as a percentage of taxable payroll is determined by the age distribution of the population. Because the cost rate is basically the product of the number of beneficiaries and their average benefit, divided by the product of the number of covered workers and their average taxable earnings (and because average benefits rise at about the same rate as average earnings), it is to be expected that the pattern of the annual cost rates is similar to that of the annual ratios of beneficiaries to workers.

4. Trust Fund Ratios

The trust fund ratio is a useful indicator of the adequacy of the financial resources of the Social Security program at any point in time. The combined OASDI Trust Fund ratio for a given year is, by definition, the ratio of the assets at the beginning of the year in the combined OASDI Trust Funds to the total outgo during that year, expressed as a percentage. In general, a yearly trust fund ratio of 100 percent or greater is desired. For any year in which the projected trust fund ratio is positive (i.e., the trust fund holds assets at the beginning of the year), but is not positive for the following year, the trust fund is projected to become exhausted during the year. Under present law, the combined OASDI Trust Funds do not currently have the authority to borrow. Therefore, exhaustion of the assets in either fund during a year would mean that there are no longer sufficient funds to cover the full amount of benefits scheduled under present law.

The trust fund ratio also serves an additional important purpose in assessing the actuarial status of the OASDI program. When the financing is adequate for the timely payment of full benefits throughout the long-range period, the stability of the trust fund ratio toward the end of the period indicates the likelihood that this projected adequacy will continue for subsequent Trustees

Reports. If the trust fund ratio toward the end of the period is level or increasing, then projected adequacy for the long-range period is likely to continue for subsequent reports.

Under the TR04II, the trust fund ratio for the combined OASDI Trust Funds rises from 306 percent at the beginning of 2004 to a peak of 448 percent at the beginning of 2015. Thereafter, the ratio declines, with the combined funds becoming exhausted in 2042. The trust fund ratio first declines in 2016, about 2 years before annual expenditures begin to exceed noninterest income. After 2015, the dollar amount of assets is projected to continue to rise through the beginning of 2028, because interest income more than offsets the shortfall in noninterest income. Beginning in 2018, the OASDI program is projected to experience increasingly large cash-flow shortfalls that will require the trust funds to redeem special public-debt obligations of the General Fund of the Treasury. The pattern of the trust fund ratios under the TR04III is similar to that under the TR04II, with a lower maximum value and an earlier exhaustion year (2031). While the pattern of the trust fund ratios under the TR04I seems similar to that under the TR04II, the trust fund ratios under the TR04I stabilize generally and remain over 480 percent for most of the second half of the 75-year long-range period.

5. Actuarial Balance, Summarized Rates, Open Group Unfunded Obligation

The deterministic model projects nominal dollar levels of trust fund assets over a 75-year period. In order to assess the financial status of the OASDI program quantitatively, in particular, to view the program as a whole at a specified point in time, the notion of present value, or time-value of money, is required. An essential ingredient in computing present values is the interest rate. Under the TR04II, the ultimate value of the annual nominal yield on the combined OASDI Trust Funds is 5.88 percent, achieved in 2018. The ultimate value of the annual nominal yield rate is 5.57 and 6.08 under the TR04I and TR04III respectively. In the short-range period, interest earned by these new issues, along with interest earned on the existing assets of the trust funds, determine implicitly the nominal yield earned by the combined OASDI Trust Funds in the short-range period 2004-13. In a five-year blending period, 2014-18, the annual yield rate on the combined OASDI Trust Funds is linearly interpolated between the yield rate in 2013 and the ultimate value achieved in 2018 to obtain the nominal yield on the combined funds in those years.

The 75-year long-range actuarial balance (henceforth referred to as the actuarial balance) is a measure, as a single number, of the program's financial status for the 75-year valuation period as a whole. It is essentially the difference between income and cost of the program expressed as a percentage of taxable payroll summarized over the valuation period. When the actuarial balance is negative, the magnitude of the negative actuarial balance, the actuarial deficit, can be interpreted as the percentage that would have to be added to the current law income rate in each of the next 75 years, or subtracted from the cost rate in each year, to bring the funds into actuarial balance. Under the TR04II, there is an actuarial deficit of 1.89 percent of taxable payroll for the combined OASDI Trust Funds. Under the TR04I, the actuarial balance is 0.41 percent of taxable payroll while under the TR04III, the actuarial deficit is 4.96 percent of taxable payroll.

Summarized values for the full 75-year period are useful in analyzing the long-range adequacy of financing for the program over the period as a whole under present law. The summarized income rate for the 75-year valuation period is equal to the present value of tax income, plus the beginning-of-year 2004 assets, expressed as a percentage of the present value of taxable payroll. Similarly, the summarized cost rate for the 75-year valuation period is equal to the present value of the cost for the 75-year period, plus the present value of the 76^{th} year's outgo (the so-called "target fund"), expressed as a percentage of the present value of taxable payroll. These summarized rates are useful for comparing the total cash flows of tax income and expenditures as an indicator of the degree to which accumulated assets and tax income during the period are sufficient to meet the outgo estimated for the long-range period. The OASDI long-range actuarial balance is defined to be the difference between the summarized 75-year income rate and the summarized 75-year cost rate.

The summarized income rate and summarized cost rate, as described above, are expressed as a percentage of the present value of taxable payroll. However, it is also useful to provide summary measures expressed in terms of the total economy of the United States. The income and cost of the OASDI program may also be expressed in terms of GDP. In this case, summary values for the OASDI program are expressed as a percentage of the present value of GDP (instead of payroll). The summarized long-range (75-year) balance as a percentage of GDP for the OASDI program is estimated to be -0.70 percent of GDP under the TR04II.

Conceptually, the open group unfunded obligation is the present value of the cash deficits observed over the 75-year period less the accumulated trust fund assets at the beginning of the period. In particular, it is the present value of the cost of the program, less the sum of the present value of tax income and the assets on hand at the beginning of the 75-year valuation period. Under the TR04II, the open group unfunded obligation was estimated to be \$3.7 trillion. It is -\$1.1 trillion and \$10.3 trillion under the TR04I and TR04III, respectively. A negative estimate of the open group unfunded obligation, as observed for the TR04I, indicates that over the 75-year period the OASDI program runs a net surplus (in present-value terms), i.e., the sum of accumulated trust fund assets and the present value of tax income is strictly greater than the present value of outgo over the 75-year period.

B. TIME-SERIES ANALYSIS

1. Background

Time-Series Modeling

Time-series analysis is a standard projection technique in econometric modeling. The equations used to project the assumptions are fit using these techniques. This appendix provides details about the models in general, presents statistical methods employed to test these models, and underscores the nuances inherent in the method of determining the equations. The reader may wish to consult Box, Jenkins and Reinsel (1994) or Hamilton (1994) for a standard presentation of this material. A more elementary reference is Pindyck and Rubinfeld (1998).

Stationary Time Series

A time series Y_t is *stationary* if neither the mean nor variance depends on time t:

- Y_t has a constant mean μ.
- For fixed k and all t, the covariance of Y_t and Y_{t-k} is γ_k, a constant depending on k.

In particular, the variance of Y_t is always equal to $\sigma^2 = \gamma_0$. A different concept is that of *strict stationarity.* This means that any k values of the time series have the same joint distribution as any other set of k values of the time series. If a time series Y_t is strictly stationary, then it is stationary. Henceforth, when we say that a time series Y_t is stationary we intend that it is strictly stationary. However, it is more straightforward to think of a time series as stationary in the simpler sense.

White Noise Process

Suppose that ε_t, the error term at time t, is normally distributed with mean zero and constant variance σ_ε^2. If ε_r and ε_s are uncorrelated for $r \neq s$, then the series ε_t is called a *white noise process*.

Random Walk

The simplest of all time-series models is the random walk. Here, if Y_t is the series to be estimated, then the random walk process is given by $Y_t = Y_{t-1} + \varepsilon_t$. The error term series, ε_t, is a white noise process. The forecast error variance increases as a linear function of the forecast lead time, l. In other words, the variance of the l-step-ahead forecast error is $l\sigma_\varepsilon^2$. The confidence intervals for these forecasts will therefore widen as l increases.

Moving Average (MA) Models

A time series is called a moving average model of order q, or simply an MA(q) process, if

$$Y_t = \mu + \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2} - \cdots - \theta_q\varepsilon_{t-q}.$$

In this equation, $\theta_1, \ldots, \theta_q$ are constant parameters and μ is the process mean. The error term series, ε_t, is a white noise process.

Autoregressive (AR) Models

A time series is called an autoregressive model of order *p*, or simply an AR(*p*) process, if

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \delta + \varepsilon_t.$$

In this equation, $\phi_1, \ldots, \phi_p$ are constant parameters and δ is a drift term. The naming of this model is apt because the coefficients of the time-series equation are obtained by regressing the equation on itself, more precisely, with its own *p* lagged values. The error term series, ε_t, is a white noise process. If this time series is stationary, then the mean μ of this process is computed to be

$$\mu = \frac{\delta}{1 - \phi_1 - \phi_2 - \cdots - \phi_p}.$$

Autoregressive Moving Average (ARMA) Models

As its name indicates, an autoregressive moving average model of order (*p*,*q*), or simply an ARMA(*p*,*q*) model, is a natural combination of an autoregressive and moving average model. The equation which specifies an ARMA(*p*,*q*) model takes the form

$$Y_t = \delta + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}.$$

Deviations Form of ARMA Model Equations

It is often more convenient to transform an ARMA model equation into deviations form using the equation

$$y_t = Y_t - \mu,$$

where μ is defined, as above, to be the mean of the process. The transformed model may be written as

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q},$$

and has a mean of zero.

Adding the original process mean to both sides of the equation produces

$$Y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}.$$

The lagged variables are left in deviations form, and the constant term, μ, on the right-hand side is the process mean.

Cholesky Decomposition

Suppose that $\vec{Z} = (Z_1, \ldots, Z_n)'$ is a vector of independently and identically distributed standard normal random variables. (If A is a matrix, then A' denotes its transpose.) Suppose we want to use these random variables to obtain a random vector $\vec{X} = (X_1, \ldots, X_n)'$ from a multivariate normal distribution with mean $\vec{\mu} = (\mu_1, \ldots, \mu_n)'$ and an $n \times n$ variance-covariance matrix $\mathbf{V} = (\sigma_{ij})$ [with $\sigma_{ij} = Cov(X_i, X_j)$ for $i, j = 1, \ldots, n$]. Since **V** is positive definite and symmetric, a standard result in linear algebra yields a lower triangular matrix, **L**, such that $\mathbf{V} = \mathbf{LL}'$. The random vector $X = \vec{\mu} + \mathbf{L}\vec{Z}$ is then a random vector with the desired properties. The decomposition $\mathbf{V} = \mathbf{LL}'$ is called a Cholesky decomposition; see Atkinson (1989) for more details. In what follows we call the matrix **L** a Cholesky matrix.

For our applications, a Cholesky decomposition is used to convert a random vector, $\vec{\varepsilon}$, of independent standard normal variates to a multivariate normal distribution with mean $\vec{\mu} = \vec{0}$ (the zero vector) and a variance-covariance matrix **V** obtained by using historical data. If **L** is a lower triangular Cholesky matrix associated with **V** then the vector $\mathbf{L}\vec{\varepsilon}$ has the required multivariate normal distribution.

Vector Autoregressive Models

Vector autoregressive models allow the joint modeling of time-series processes. For the sake of simplicity, suppose that three variables $y_{1,t}$, $y_{2,t}$, and $y_{3,t}$ depend on time *t*. Data may indicate that these variables may be related to each other's past values. The simplest such case is when the relationship is limited to the time-1 lag, i.e., when $y_{1,t}$, $y_{2,t}$, and $y_{3,t}$ may be modeled in terms of $y_{1,t-1}$, $y_{2,t-1}$, and $y_{3,t-1}$. In this case, a three-variable VAR(1) model takes the form

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{bmatrix} = B \cdot \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ y_{3,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}$$

for some 3×3 matrix *B*.

Alternatively stated, if $\vec{y}_t = (y_{1,t}, y_{2,t}, y_{3,t})'$ and $\vec{\varepsilon}_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t})'$ then the model takes the form $\vec{y}_t = B\vec{y}_{t-1} + \vec{\varepsilon}_t$ for some 3×3 matrix *B*.

The *k*-variable VAR(*p*) model, with p lags, naturally extends from this. If $\vec{y}_t = (y_{1,t}, y_{2,t}, \ldots, y_{k,t})'$ and $\vec{\varepsilon}_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, \ldots, \varepsilon_{k,t})'$, then the *k*-variable VAR(*p*) model takes the form $\vec{y}_t = B_1\vec{y}_{t-1} + B_2\vec{y}_{t-2} + \cdots + B_p\vec{y}_{t-p} + \vec{\varepsilon}$ for $k \times k$ matrices $B_1, \ldots, B_p$.

2. Methods

Modified Autoregressive Moving Average (ARMA) Models

The equations used to model the assumptions are autoregressive moving average models with the additional requirement that the mean of the variable Y_t is always equal to its value under the TR04II. The value of the variable under the TR04II is written Y_t^{TR}. Thus, the mean of Y_t varies, in general, in the early years of the long-range period. The resulting model is called a *modified autoregressive moving average model*. In general, the equation in a modified autoregressive moving average model takes the form

$$Y_t = \delta_t + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}.$$

Writing this equation in "deviations form," with $y_t = Y_t - \mu_t$, gives

$$Y_t = \mu_t + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}.$$

Since $\mu_t = Y_t^{TR}$, we obtain the expression

$$Y_t = Y_t^{TR} + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}.$$

Equations in Chapter II are all presented in this form, with the letters *Y* and *y* replaced with a more suggestive naming.

C. MONTE CARLO SIMULATION

A standard reference for this appendix is Knuth (1997). In choosing a random number generator, it is essential to make the following considerations:

- the generator should have the correct statistical properties
- the stream should be easy to reproduce
- the cycle length should be long
- computational efficiency and storage needs should be met
- the generator should be portable across platforms
- it should be easy to generate separate streams

Given a seed I_0 and a judicious choice of a and m, the linear congruence $I_{j+1} = aI_j \pmod{m}$ will generate a sequence of numbers. The values $a = 16{,}807$ and $m = 2^{31} - 1$ have the property that, for any nonzero initial seed, a large number of variables may be generated before cycling occurs. Upon division by m, a sequence of variates U_j of a uniform distribution on the interval (0,1) is generated. This linear congruential generator with these values of a and m is the so-called "Minimal Standard" generator of Park and Miller and appears in Press et al. (2001). As explained there, the value of 0 must never be allowed as the initial seed. Also, since a and m are relatively prime, the value of 0 will never occur provided that the initial seed is nonzero. This choice of a and m gives a cycle of maximum possible length $2^{31} - 2$. The Minimal Standard generator can be modified so as to remove low-order serial correlations inherent in the random numbers produced by that method. The method is adapted from Press et al. (2001) and uses the Park-Miller algorithm along with a Bays-Durham shuffle. This is the process we used to generate uniform (0,1) variates. There are other methods which will further eliminate such serial correlations. We chose this particular method to balance the need to remove such serial correlations with the expense of run-time.

To generate the standard normal (mean 0 and variance 1) variates used to generate the random error terms for our equations, one may transform uniform variates using the Box-Muller method: generate two independent uniform (0,1) variates, U_1 and U_2, and compute $X = \sqrt{-2\ln U_1}\cos(2\pi U_1)$ and $Y = \sqrt{-2\ln U_1}\sin(2\pi U_2)$. In order to avoid the use of trigonometric functions, we instead used the Polar method. This, as a preliminary step, transforms the two uniform (0,1) variates U_1 and U_2 to uniform (–1,1) variates V_1 and V_2 (using the transformation $V = 2U + 1$). If $S = V_1^2 + V_2^2$ then $X = \sqrt{\dfrac{-2\ln S}{S}}$ and $Y = \sqrt{\dfrac{-2\ln S}{S}}$ will be two independent random draws from a standard normal distribution, provided that S is less than one. (If not, then we repeat this process until it is.) Although this method produces two standard normal variates, we always used the first one (X) and discarded the second one (Y). Each equation for each assumption is separately seeded to have its own random number stream.

D. EQUATION PARAMETER ESTIMATES AND REGRESSION STATISTICS

This appendix contains the parameter values and standard errors for each numbered equation described in chapter II. The parameter values include, where applicable, the estimated autoregressive (AR) and moving average (MA) coefficients. In addition, the assumed annual projected values of each variable from the 2004 Trustees Report alternative II (e.g., F_t^{TR}) are presented in a table at the end of this appendix.

The standard errors of each regression were used to generate random variation in each of the variables. The random variation in one variable is usually generated independently of the random variation in another variable. However, the mortality improvement variables, economic variable VAR, and disability variables were each generated in such a way that the random error terms were correlated. For each of these, a Cholesky decomposition was performed on a residual variance-covariance matrix. The correlated random error terms were then obtained by multiplying the lower triangular Cholesky matrix by a vector of standard normal random errors.

Total Fertility Rate

$$F_t = F_t^{TR} + \phi_1 f_{t-1} + \phi_2 f_{t-2} + \phi_3 f_{t-3} + \phi_4 f_{t-4} + \varepsilon_t - \theta_1 \varepsilon_{t-1}, \qquad 0.5 \le F_t \le 3.4 \tag{1}$$

Note: The total fertility rate was bounded on this interval because the computer model became unstable if the simulated total fertility rate was less than 0.5. The upper bound was set at 3.4 to preserve the symmetry around the ultimate TR04II value of 1.95.

AR Parameters	MA Parameters	Process mean over historical period
$\phi_1 =$ 1.991694	$\theta_1 = 0.665162$	$\mu = 2.469526$
$\phi_2 =$ -1.508138		
$\phi_3 =$ 0.914735		
$\phi_4 =$ -0.419396		

Standard Error
$\hat{\sigma}_\varepsilon = 0.090936$

Rate of Decrease in Central Death Rate

$$MR_{k,t} = MR_{k,t}^{TR} + \phi_k mr_{k,t-1} + \varepsilon_{k,t} \quad (2)$$

Group (k)	Sex	Age	AR Parameter (ϕ_k)	Process mean over historical period
1	M	Under 1	-0.351201	2.869243
2	F		-0.372752	2.842529
3	M	1 to 4	-0.416565	3.501578
4	F		-0.427767	3.565887
5	M	5 to 9	-0.227505	2.949994
6	F		-0.271437	2.997825
7	M	10 to 14	-0.234871	2.155480
8	F		-0.182700	2.320293
9	M	15 to 19	-0.193210	0.906328
10	F		-0.168664	1.871810
11	M	20 to 24	-0.175331	0.822418
12	F		-0.200078	1.914533
13	M	25 to 29	-0.213296	0.689647
14	F		-0.185487	1.657118
15	M	30 to 34	-0.214036	0.882346
16	F		-0.221462	1.788244
17	M	35 to 39	-0.213548	1.141628
18	F		-0.237834	1.777003
19	M	40 to 44	-0.137733	1.173295
20	F		-0.223119	1.681488
21	M	45 to 49	-0.153830	1.058397
22	F		-0.253837	1.501890
23	M	50 to 54	-0.082579	0.992394
24	F		-0.250153	1.418907
25	M	55 to 59	-0.025914	0.891541
26	F		-0.221067	1.234432
27	M	60 to 64	-0.107071	0.783572
28	F		-0.340946	1.133483
29	M	65 to 69	-0.217861	0.634373
30	F		-0.219608	0.975814
31	M	70 to 74	-0.280665	0.610547
32	F		-0.269514	0.978308
33	M	75 to 79	-0.256160	0.552180
34	F		-0.280206	0.894627
35	M	80 to 84	-0.331191	0.540486
36	F		-0.295512	0.816556
37	M	85 to 89	-0.241108	0.361083
38	F		-0.185227	0.580038
39	M	90 to 94	-0.242183	0.274489
40	F		-0.207037	0.467668
41	M	95 and Up	-0.158566	0.139841
42	F		-0.184255	0.322523

Cholesky Decomposition of Residual Variance-Covariance Matrix

$$\mathbf{V} = \mathbf{LL}' \qquad \mathbf{L} = [\mathbf{A} \mid \mathbf{B} \mid \mathbf{C}] \qquad (\mathbf{L} \text{ is a } 42 \times 42 \text{ matrix})$$

$$
\mathbf{A} = \begin{bmatrix}
4.62 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4.58 & 1.07 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4.50 & 1.57 & 6.41 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4.82 & 2.12 & 6.61 & 2.12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3.45 & 1.26 & 4.79 & 1.53 & 3.63 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3.82 & 0.85 & 6.13 & 1.90 & 3.14 & 3.30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3.48 & 0.70 & 5.14 & 2.19 & 3.50 & 1.33 & 3.41 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4.53 & 1.17 & 6.53 & 2.51 & 4.34 & 2.71 & 2.22 & 3.50 & 0 & 0 & 0 & 0 & 0 & 0 \\
5.06 & 1.36 & 8.82 & 3.80 & 4.25 & 1.69 & 3.76 & 1.85 & 4.24 & 0 & 0 & 0 & 0 & 0 \\
4.19 & 1.04 & 7.47 & 2.90 & 3.63 & 2.02 & 3.12 & 2.56 & 1.90 & 2.92 & 0 & 0 & 0 & 0 \\
4.46 & 2.21 & 9.39 & 3.80 & 4.60 & 1.71 & 3.99 & 1.57 & 4.46 & -0.84 & 3.49 & 0 & 0 & 0 \\
5.07 & 1.73 & 9.02 & 4.46 & 4.51 & 2.66 & 4.05 & 2.27 & 3.36 & 2.56 & 0.08 & 3.12 & 0 & 0 \\
5.75 & 2.49 & 11.22 & 5.42 & 5.59 & 2.70 & 4.82 & 2.51 & 5.68 & 0.76 & 1.61 & 1.75 & 3.13 & 0 \\
5.73 & 2.47 & 10.87 & 5.51 & 5.31 & 3.18 & 4.58 & 3.06 & 4.38 & 2.75 & -0.26 & 3.59 & 1.61 & 2.25 \\
5.54 & 2.37 & 9.69 & 4.50 & 4.59 & 2.22 & 4.86 & 2.42 & 4.51 & 0.99 & 0.89 & 1.51 & 2.42 & 0.56 \\
4.61 & 1.93 & 8.22 & 4.26 & 3.99 & 2.32 & 3.70 & 2.16 & 3.17 & 1.72 & 0.08 & 2.13 & 0.87 & 1.45 \\
4.06 & 1.26 & 5.96 & 2.36 & 2.74 & 0.88 & 3.18 & 1.38 & 2.35 & 0.74 & 0.72 & 0.10 & 1.50 & 0.10 \\
3.05 & 0.96 & 4.76 & 2.28 & 2.24 & 1.13 & 2.41 & 1.20 & 1.34 & 1.31 & 0.49 & 0.72 & 0.17 & 0.70 \\
2.80 & 0.73 & 3.13 & 0.53 & 1.06 & 0.13 & 1.78 & 0.82 & 1.01 & 0.59 & 0.88 & -1.08 & 0.39 & -0.19 \\
2.21 & 0.31 & 2.67 & 0.79 & 1.03 & 0.61 & 1.19 & 0.97 & 0.63 & 0.96 & 0.79 & 0.02 & -0.30 & 0.04 \\
2.28 & 0.11 & 1.71 & -0.11 & 0.53 & -0.02 & 1.17 & 0.48 & 0.47 & 0.46 & 0.68 & -0.91 & 0.15 & -0.13 \\
1.77 & -0.06 & 1.91 & 0.04 & 0.57 & 0.46 & 0.93 & 0.38 & 0.15 & 0.70 & 0.30 & -0.06 & -0.25 & 0.16 \\
1.62 & -0.15 & 0.93 & -0.37 & 0.00 & -0.41 & 0.59 & 0.14 & 0.17 & 0.45 & 0.77 & -0.88 & -0.09 & 0.11 \\
1.23 & 0.16 & 1.57 & -0.09 & 0.41 & 0.11 & 0.70 & 0.16 & -0.10 & 0.68 & 0.44 & -0.40 & -0.23 & 0.38 \\
1.20 & -0.41 & 0.54 & -0.55 & 0.15 & -0.16 & 0.44 & 0.01 & 0.09 & 0.48 & 0.87 & -1.28 & -0.23 & 0.15 \\
1.02 & -0.12 & 1.02 & -0.27 & 0.01 & 0.20 & 0.55 & -0.10 & -0.40 & 0.69 & 0.68 & -0.86 & -0.42 & 0.24 \\
0.97 & -0.12 & 0.38 & -0.61 & -0.06 & 0.01 & 0.72 & 0.17 & -0.26 & 0.26 & 0.46 & -1.11 & -0.13 & 0.27 \\
0.77 & -0.07 & 0.71 & -0.38 & -0.25 & 0.21 & 0.62 & 0.19 & -0.67 & 0.54 & 0.37 & -0.45 & -0.29 & 0.45 \\
0.99 & -0.28 & 0.47 & -0.41 & -0.36 & 0.03 & 0.40 & -0.02 & -0.13 & 0.37 & 0.51 & -1.04 & -0.10 & 0.26 \\
0.74 & -0.11 & 0.66 & -0.45 & -0.48 & 0.18 & 0.51 & 0.19 & -0.62 & 0.59 & 0.44 & -0.59 & -0.35 & 0.44 \\
0.57 & -0.24 & 0.31 & -0.72 & -0.22 & 0.10 & 0.39 & 0.11 & -0.39 & 0.53 & 0.68 & -1.01 & -0.39 & 0.08 \\
0.61 & -0.12 & 0.31 & -0.56 & -0.41 & 0.28 & 0.59 & 0.28 & -0.83 & 0.72 & 0.67 & -0.70 & -0.70 & 0.48 \\
0.76 & -0.57 & 0.41 & -0.67 & -0.54 & -0.12 & 0.25 & -0.09 & -0.74 & 0.40 & 0.55 & -0.86 & -0.40 & 0.43 \\
0.72 & -0.46 & 0.27 & -0.58 & -0.75 & 0.07 & 0.41 & -0.13 & -0.98 & 0.42 & 0.53 & -0.72 & -0.56 & 0.62 \\
0.61 & -0.41 & 0.24 & -0.76 & -0.63 & -0.04 & 0.39 & -0.08 & -0.75 & 0.46 & 0.68 & -1.34 & -0.79 & 0.29 \\
0.66 & -0.22 & 0.13 & -0.67 & -0.80 & 0.08 & 0.40 & -0.26 & -0.91 & 0.53 & 0.78 & -1.11 & -0.97 & 0.58 \\
0.35 & -0.63 & -0.01 & -0.76 & -0.69 & 0.36 & 0.54 & 0.08 & -1.00 & 0.77 & 0.50 & -0.79 & -0.90 & 0.64 \\
0.50 & -0.58 & -0.19 & -0.75 & -0.63 & 0.37 & 0.56 & -0.04 & -1.04 & 0.68 & 0.52 & -0.95 & -1.04 & 0.85 \\
0.42 & -0.54 & -0.07 & -0.86 & -0.71 & 0.59 & 0.50 & 0.27 & -1.02 & 0.69 & 0.49 & -0.81 & -0.93 & 0.65 \\
0.53 & -0.51 & -0.13 & -1.00 & -0.69 & 0.57 & 0.64 & -0.07 & -1.11 & 0.78 & 0.56 & -0.92 & -1.09 & 0.80 \\
0.51 & -0.29 & -0.16 & -0.72 & -0.86 & 1.13 & 0.26 & 0.40 & -0.79 & 0.52 & 0.25 & -1.00 & -1.04 & 0.76 \\
0.60 & -0.36 & -0.13 & -0.87 & -0.84 & 0.91 & 0.50 & 0.14 & -1.07 & 0.62 & 0.32 & -1.14 & -1.06 & 0.87
\end{bmatrix}
$$

$$
\mathbf{B} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2.30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.08 & 1.78 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.66 & -0.16 & 2.39 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.48 & 0.72 & 1.46 & 1.69 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.48 & -0.11 & 1.96 & 0.44 & 1.96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.45 & 0.56 & 0.99 & 0.59 & 1.04 & 1.48 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.76 & -0.09 & 1.55 & 0.49 & 1.14 & 0.23 & 1.51 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.20 & 0.44 & 0.87 & 0.45 & 0.48 & 0.22 & 0.68 & 1.13 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.57 & -0.33 & 1.53 & 0.51 & 1.29 & 0.10 & 0.63 & -0.03 & 1.54 & 0 & 0 & 0 & 0 & 0 \\
0.25 & -0.05 & 0.86 & 0.61 & 0.43 & 0.64 & 0.17 & 0.39 & 0.98 & 1.18 & 0 & 0 & 0 & 0 \\
0.67 & 0.05 & 0.99 & 0.47 & 1.25 & 0.10 & 0.84 & 0.26 & 0.90 & -0.04 & 1.14 & 0 & 0 & 0 \\
0.39 & 0.31 & 0.70 & 0.91 & 0.55 & 0.33 & 0.53 & 0.44 & 0.39 & 0.23 & 1.06 & 1.05 & 0 & 0 \\
0.62 & -0.27 & 1.01 & 0.43 & 0.75 & -0.03 & 1.11 & 0.29 & 1.19 & 0.03 & 0.08 & -0.11 & 1.21 & 0 \\
0.36 & 0.04 & 0.76 & 0.62 & 0.25 & 0.29 & 0.74 & 0.35 & 0.67 & 0.52 & 0.15 & 0.24 & 0.64 & 0.92 \\
0.33 & -0.08 & 1.08 & 0.57 & 1.15 & 0.22 & 0.62 & 0.21 & 0.96 & -0.02 & 0.55 & 0.16 & 0.60 & -0.06 \\
0.09 & 0.52 & 0.78 & 0.68 & 0.56 & 0.31 & 0.39 & 0.14 & 0.60 & 0.16 & 0.65 & 0.50 & 0.26 & 0.26 \\
0.54 & -0.06 & 0.89 & 0.48 & 0.75 & 0.28 & 0.79 & 0.22 & 1.11 & 0.15 & 0.50 & -0.10 & 0.97 & 0.10 \\
0.44 & 0.30 & 0.76 & 0.61 & 0.36 & 0.37 & 0.63 & 0.29 & 0.84 & 0.50 & 0.61 & 0.07 & 0.59 & 0.38 \\
-0.05 & 0.25 & 1.00 & 0.77 & 0.51 & 0.15 & 0.67 & 0.23 & 1.09 & 0.12 & 0.89 & 0.06 & 0.59 & 0.41 \\
-0.12 & 0.64 & 1.03 & 1.11 & 0.47 & 0.32 & 0.62 & 0.22 & 0.97 & 0.31 & 0.88 & 0.37 & 0.48 & 0.54 \\
0.07 & 0.21 & 0.90 & 0.77 & 0.59 & 0.54 & 0.62 & 0.23 & 1.24 & 0.48 & 0.69 & -0.07 & 0.76 & 0.52 \\
-0.09 & 0.47 & 0.93 & 0.90 & 0.62 & 0.52 & 0.54 & 0.11 & 1.32 & 0.51 & 0.76 & 0.06 & 0.71 & 0.77 \\
-0.15 & 0.38 & 1.34 & 0.87 & 0.16 & 0.18 & 0.64 & 0.41 & 1.28 & 0.14 & 0.87 & 0.00 & 1.10 & 0.64 \\
-0.49 & 0.52 & 1.38 & 1.20 & 0.21 & 0.07 & 0.88 & 0.13 & 1.34 & 0.39 & 0.83 & -0.12 & 1.02 & 0.83 \\
-0.22 & 0.48 & 1.50 & 0.91 & 0.18 & 0.14 & 0.51 & 0.30 & 1.15 & 0.22 & 0.91 & 0.09 & 1.13 & 0.69 \\
-0.57 & 0.52 & 1.45 & 1.01 & 0.14 & 0.06 & 0.77 & 0.11 & 1.30 & 0.39 & 0.83 & -0.09 & 1.04 & 0.90 \\
-0.12 & 0.41 & 1.98 & 1.11 & 0.05 & 0.12 & 0.24 & 0.20 & 0.82 & 0.12 & 1.29 & 0.30 & 1.13 & 0.68 \\
-0.31 & 0.47 & 1.72 & 1.05 & 0.01 & 0.09 & 0.48 & -0.08 & 1.11 & 0.41 & 1.07 & 0.06 & 1.04 & 1.08
\end{bmatrix}
$$

$$
\mathbf{C} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.06 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.69 & 1.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.25 & -0.01 & 1.07 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.08 & 0.63 & 0.77 & 0.70 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.38 & 0.33 & 0.08 & -0.12 & 1.09 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.30 & 0.89 & 0.02 & 0.43 & 0.78 & 0.75 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.67 & 0.23 & 0.58 & 0.22 & 0.41 & 0.05 & 1.18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.38 & 0.47 & 0.71 & 0.63 & 0.36 & 0.35 & 0.83 & 0.81 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.62 & 0.44 & 0.29 & 0.21 & 0.72 & 0.30 & 0.55 & 0.45 & 1.42 & 0 & 0 & 0 & 0 & 0 \\
0.69 & 1.05 & 0.33 & 0.46 & 0.80 & 0.34 & 0.61 & 0.75 & 0.96 & 0.83 & 0 & 0 & 0 & 0 \\
0.69 & 0.37 & 0.20 & 0.28 & 0.66 & 0.31 & 0.63 & 0.61 & 1.48 & -0.03 & 0.54 & 0 & 0 & 0 \\
0.71 & 1.03 & 0.23 & 0.50 & 0.77 & 0.36 & 0.69 & 0.76 & 0.94 & 0.85 & 0.23 & 0.34 & 0 & 0 \\
0.85 & 0.41 & 0.04 & 0.58 & 0.44 & 0.35 & 0.71 & 0.89 & 1.53 & -0.24 & 0.94 & 0.23 & 1.23 & 0 \\
0.84 & 1.08 & 0.10 & 0.63 & 0.68 & 0.35 & 0.77 & 0.95 & 1.03 & 0.80 & 0.67 & 0.31 & 0.57 & 0.41
\end{bmatrix}
$$

Legal Immigration

$$IM_t = IM_t^{TR} + \phi_1 im_{t-1} + \phi_2 im_{t-2} + \phi_3 im_{t-3} + \phi_4 im_{t-4} + \varepsilon_t - \theta_1 \varepsilon_{t-1}, \qquad 0 \le IM_t \le 2 \times IM_t^{TR} \tag{3}$$

Note: Because legal immigration is a non-negative quantity, its lower bound was set at zero. The upper limit was chosen to preserve the symmetry around the TR04II value.

AR Parameters	MA Parameters	Process mean over historical period
$\phi_1 = 1.079587$	$\theta_1 = -0.487496$	$\mu = 460{,}545$
$\phi_2 = -0.538171$		
$\phi_3 = 0.689674$		
$\phi_4 = -0.309780$		

Standard Error
$\hat{\sigma}_\varepsilon = 84{,}770$

Legal Emigration

$$EM_t = EM_t^{TR} + \phi_1 em_{t-1} + \phi_2 em_{t-2} + \phi_3 em_{t-3} + \phi_4 em_{t-4} + \varepsilon_t - \theta_1 \varepsilon_{t-1}, \qquad 0 \le EM_t \le 2 \times EM_t^{TR} \tag{4}$$

Note: Because legal emigration is a non-negative quantity, its lower bound was set at zero. The upper limit was chosen to preserve the symmetry around the TR04II value.

AR Parameters	MA Parameters
$\phi_1 = 0.269897$	$\theta_1 = -0.121874$
$\phi_2 = -0.134543$	
$\phi_3 = 0.172419$	
$\phi_4 = -0.077445$	

Standard Error
$\hat{\sigma}_\varepsilon = 21{,}192$

Net Other Immigration

$$\Delta O_t = \Delta O_t^{TR} + \varepsilon_t \qquad\qquad O_{2003} = O_{2003}^{TR} = 400{,}000 \tag{5}$$

Standard Error
$\hat{\sigma}_\varepsilon = 25{,}371$

Economic Variable VAR

Unemployment Rate (expressed as log-odds ratio)

$$U_t = U_t^{TR} + a_{111} u_{t-1} + a_{112} u_{t-2} + a_{121} i_{t-1} + a_{122} i_{t-2} + a_{131} r_{t-1} + a_{132} r_{t-2} + \varepsilon_{1t} \tag{6}$$

AR Parameters		Process mean over historical period
$a_{111} = 0.960146$	$a_{112} = -0.304386$	$\mu = -2.796475$
$a_{121} = 0.400633$	$a_{122} = -0.083907$	
$a_{131} = 0.746191$	$a_{132} = 0.610372$	

Inflation Rate (transformed into logs with lower bound of -3.0%)

$$I_t = I_t^{TR} + a_{211} u_{t-1} + a_{212} u_{t-2} + a_{221} i_{t-1} + a_{222} i_{t-2} + a_{231} r_{t-1} + a_{232} r_{t-2} + \varepsilon_{2t} \tag{7}$$

AR Parameters		Process mean over historical period
$a_{211} = -0.769000$	$a_{212} = 0.721055$	$\mu = -2.719855$
$a_{221} = 0.595616$	$a_{222} = 0.301637$	
$a_{231} = -4.846046$	$a_{232} = 1.803246$	

Real Interest Rate

$$R_t = R_t^{TR} + a_{311}u_{t-1} + a_{312}u_{t-2} + a_{321}i_{t-1} + a_{322}i_{t-2} + a_{331}r_{t-1} + a_{332}r_{t-2} + \varepsilon_{3t} \quad (8)$$

Note: The real interest rate was bounded below such that the nominal interest rate would be non-negative.

AR Parameters		Process mean over historical period
a_{311} = 0.060540	a_{312} = -0.048520	μ = 0.030517
a_{321} = 0.028107	a_{322} = -0.027562	
a_{331} = 1.234137	a_{332} = -0.319613	

Matrix Notation

$$\begin{bmatrix} U_t \\ I_t \\ R_t \end{bmatrix} = \begin{bmatrix} U_t^{TR} \\ I_t^{TR} \\ R_t^{TR} \end{bmatrix} + \mathbf{A_1} \begin{bmatrix} u_{t-1} \\ i_{t-1} \\ r_{t-1} \end{bmatrix} + \mathbf{A_2} \begin{bmatrix} u_{t-2} \\ i_{t-2} \\ r_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}$$

$$\mathbf{A_1} = \begin{bmatrix} a_{111} & a_{121} & a_{131} \\ a_{211} & a_{221} & a_{231} \\ a_{311} & a_{321} & a_{331} \end{bmatrix} \qquad \mathbf{A_2} = \begin{bmatrix} a_{112} & a_{122} & a_{132} \\ a_{212} & a_{222} & a_{232} \\ a_{312} & a_{322} & a_{332} \end{bmatrix}$$

Cholesky Decomposition of Residual Variance-Covariance Matrix

$$\mathbf{V} = \mathbf{LL}' \qquad \mathbf{L} = \begin{bmatrix} 0.113117 & 0 & 0 \\ -0.050445 & 0.139394 & 0 \\ 0.000952 & -0.007164 & 0.009837 \end{bmatrix}$$

Real Average Wage (percent change)

$$W_t = W_t^{TR} + \beta_2 u_t + \beta_3 u_{t-1} + \varepsilon_t \quad (9)$$

AR Parameters	Process mean over historical period
β_2 = -0.061391	μ = 0.012822
β_3 = 0.035378	

Standard Error

$\hat{\sigma}_\varepsilon$ = 0.011806

Disability Incidence - Male

$$DIM_t = DIM_t^{TR} + \phi_1 dim_{t-1} + \phi_2 dim_{t-2} + \varepsilon_t \tag{10}$$

AR Parameters	Process mean over historical period
$\phi_1 =$ 1.465965	μ =5.559148
$\phi_2 =$ -0.634930	

Disability Incidence - Female

$$DIF_t = DIF_t^{TR} + \phi_1 dif_{t-1} + \phi_2 dif_{t-2} + \varepsilon_t \tag{11}$$

AR Parameters	Process mean over historical period
$\phi_1 =$ 1.447164	μ =4.53556
$\phi_2 =$ -0.618748	

Cholesky Decomposition of Residual Variance-Covariance Matrix

$$\mathbf{V} = \mathbf{LL}' \qquad \mathbf{L} = \begin{bmatrix} 0.369409 & 0 \\ 0.310160 & 0.198227 \end{bmatrix}$$

Disability Recovery - Male

$$DRM_t = DRM_t^{TR} + \phi_1 drm_{t-1} + \varepsilon_t \tag{12}$$

AR Parameters	Process mean over historical period
$\phi_1 =$ 0.577874	μ =10.51046

Disability Recovery - Female

$$DRF_t = DRF_t^{TR} + \phi_1 drf_{t-1} + \varepsilon_t \tag{13}$$

AR Parameters	Process mean over historical period
$\phi_1 =$ 0.569980	μ =9.590448

Cholesky Decomposition of Residual Variance-Covariance Matrix

$$\mathbf{V} = \mathbf{LL}' \qquad \mathbf{L} = \begin{bmatrix} 1.697850 & 0 \\ 1.538223 & 0.470810 \end{bmatrix}$$

Equation Parameters from the 2004 Trustees Report Assumptions

t	F_t^{TR}	$MR_{1,t}^{TR}$	$MR_{2,t}^{TR}$	$MR_{3,t}^{TR}$	$MR_{4,t}^{TR}$	$MR_{5,t}^{TR}$	$MR_{6,t}^{TR}$	$MR_{7,t}^{TR}$	$MR_{8,t}^{TR}$
2004	2.0167	2.9097	2.7474	3.1458	2.9534	3.0533	2.5483	1.8772	1.4612
2005	2.0139	2.6733	2.5448	2.7958	2.6463	2.7069	2.3225	1.7531	1.4460
2006	2.0112	2.4845	2.3829	2.5161	2.4008	2.4350	2.1433	1.6580	1.4344
2007	2.0084	2.3338	2.2535	2.2923	2.2041	2.2203	2.0006	1.5840	1.4253
2008	2.0056	2.2133	2.1500	2.1128	2.0464	2.0501	1.8865	1.5260	1.4178
2009	2.0028	2.1170	2.0672	1.9688	1.9197	1.9145	1.7949	1.4800	1.4114
2010	2.0000	2.0399	2.0009	1.8530	1.8178	1.8062	1.7213	1.4432	1.4058
2011	1.9973	1.9781	1.9478	1.7598	1.7356	1.7194	1.6619	1.4134	1.4007
2012	1.9945	1.9287	1.9051	1.6846	1.6693	1.6495	1.6137	1.3892	1.3960
2013	1.9917	1.8889	1.8709	1.6239	1.6156	1.5931	1.5746	1.3692	1.3917
2014	1.9889	1.8571	1.8434	1.5746	1.5721	1.5474	1.5427	1.3528	1.3875
2015	1.9861	1.8314	1.8213	1.5347	1.5366	1.5103	1.5165	1.3390	1.3836
2016	1.9833	1.8107	1.8034	1.5021	1.5076	1.4800	1.4950	1.3273	1.3798
2017	1.9806	1.7940	1.7890	1.4754	1.4839	1.4552	1.4772	1.3174	1.3762
2018	1.9778	1.7805	1.7773	1.4535	1.4643	1.4347	1.4623	1.3089	1.3727
2019	1.9750	1.7695	1.7677	1.4354	1.4480	1.4177	1.4498	1.3015	1.3693
2020	1.9723	1.7605	1.7599	1.4203	1.4344	1.4035	1.4392	1.2950	1.3660
2021	1.9695	1.7532	1.7535	1.4077	1.4229	1.3916	1.4301	1.2893	1.3627
2022	1.9667	1.7471	1.7483	1.3970	1.4132	1.3815	1.4223	1.2841	1.3596
2023	1.9639	1.7421	1.7439	1.3879	1.4049	1.3728	1.4155	1.2794	1.3564
2024	1.9611	1.7379	1.7402	1.3801	1.3976	1.3653	1.4094	1.2751	1.3534
2025	1.9584	1.7344	1.7371	1.3733	1.3912	1.3587	1.4041	1.2712	1.3503
2026	1.9556	1.7314	1.7344	1.3673	1.3856	1.3529	1.3992	1.2675	1.3473
2027	1.9528	1.7288	1.7321	1.3619	1.3805	1.3477	1.3947	1.2639	1.3444
2028	1.9500	1.7212	1.7255	1.3486	1.3684	1.3348	1.3849	1.2576	1.3408
2029	1.9500	1.7203	1.7247	1.3458	1.3656	1.3321	1.3822	1.2549	1.3380
2030	1.9500	1.7193	1.7238	1.3431	1.3629	1.3294	1.3794	1.2523	1.3352
2031	1.9500	1.7184	1.7229	1.3404	1.3601	1.3266	1.3766	1.2497	1.3324
2032	1.9500	1.7175	1.7220	1.3377	1.3573	1.3239	1.3738	1.2471	1.3296
2033	1.9500	1.7165	1.7212	1.3350	1.3546	1.3212	1.3711	1.2445	1.3268
2034	1.9500	1.7156	1.7203	1.3323	1.3518	1.3185	1.3683	1.2419	1.3240
2035	1.9500	1.7146	1.7194	1.3296	1.3491	1.3158	1.3656	1.2394	1.3213
2036	1.9500	1.7137	1.7185	1.3270	1.3464	1.3131	1.3628	1.2368	1.3185
2037	1.9500	1.7127	1.7176	1.3243	1.3436	1.3105	1.3601	1.2343	1.3157
2038	1.9500	1.7117	1.7167	1.3216	1.3409	1.3078	1.3573	1.2317	1.3130
2039	1.9500	1.7108	1.7158	1.3190	1.3382	1.3051	1.3546	1.2292	1.3103
2040	1.9500	1.7098	1.7149	1.3163	1.3355	1.3025	1.3519	1.2267	1.3075
2041	1.9500	1.7088	1.7139	1.3137	1.3328	1.2998	1.3491	1.2242	1.3048
2042	1.9500	1.7078	1.7130	1.3110	1.3301	1.2972	1.3464	1.2217	1.3021
2043	1.9500	1.7068	1.7121	1.3084	1.3274	1.2946	1.3437	1.2193	1.2994
2044	1.9500	1.7058	1.7111	1.3058	1.3247	1.2920	1.3410	1.2168	1.2967
2045	1.9500	1.7048	1.7102	1.3032	1.3220	1.2893	1.3383	1.2144	1.2940
2046	1.9500	1.7038	1.7092	1.3006	1.3194	1.2868	1.3357	1.2120	1.2913
2047	1.9500	1.7027	1.7083	1.2980	1.3167	1.2841	1.3330	1.2095	1.2887
2048	1.9500	1.7017	1.7073	1.2954	1.3141	1.2816	1.3303	1.2071	1.2860
2049	1.9500	1.7007	1.7063	1.2929	1.3114	1.2790	1.3276	1.2047	1.2834
2050	1.9500	1.6996	1.7054	1.2903	1.3088	1.2764	1.3250	1.2024	1.2807
2051	1.9500	1.6986	1.7044	1.2877	1.3062	1.2739	1.3223	1.2000	1.2781
2052	1.9500	1.6975	1.7034	1.2852	1.3035	1.2713	1.3197	1.1977	1.2755
2053	1.9500	1.6965	1.7024	1.2826	1.3009	1.2688	1.3170	1.1953	1.2729
2054	1.9500	1.6954	1.7014	1.2801	1.2983	1.2663	1.3144	1.1930	1.2703
2055	1.9500	1.6943	1.7004	1.2776	1.2957	1.2637	1.3118	1.1907	1.2677
2056	1.9500	1.6932	1.6994	1.2751	1.2931	1.2612	1.3091	1.1884	1.2651
2057	1.9500	1.6921	1.6984	1.2726	1.2905	1.2587	1.3065	1.1861	1.2625
2058	1.9500	1.6910	1.6973	1.2701	1.2880	1.2563	1.3039	1.1838	1.2600
2059	1.9500	1.6899	1.6963	1.2676	1.2854	1.2538	1.3013	1.1816	1.2574
2060	1.9500	1.6888	1.6953	1.2651	1.2828	1.2513	1.2987	1.1793	1.2549
2061	1.9500	1.6877	1.6942	1.2627	1.2803	1.2488	1.2962	1.1771	1.2524
2062	1.9500	1.6866	1.6932	1.2602	1.2777	1.2464	1.2936	1.1749	1.2498
2063	1.9500	1.6855	1.6921	1.2578	1.2752	1.2440	1.2910	1.1727	1.2473
2064	1.9500	1.6843	1.6910	1.2553	1.2727	1.2415	1.2885	1.1704	1.2448
2065	1.9500	1.6832	1.6900	1.2529	1.2702	1.2391	1.2859	1.1683	1.2423
2066	1.9500	1.6820	1.6889	1.2505	1.2677	1.2367	1.2834	1.1661	1.2399
2067	1.9500	1.6809	1.6878	1.2481	1.2652	1.2343	1.2809	1.1639	1.2374
2068	1.9500	1.6797	1.6867	1.2457	1.2627	1.2319	1.2784	1.1618	1.2349
2069	1.9500	1.6785	1.6856	1.2433	1.2602	1.2295	1.2758	1.1597	1.2325
2070	1.9500	1.6773	1.6845	1.2409	1.2578	1.2272	1.2733	1.1575	1.2301
2071	1.9500	1.6762	1.6834	1.2385	1.2553	1.2248	1.2708	1.1554	1.2276
2072	1.9500	1.6750	1.6823	1.2362	1.2528	1.2225	1.2683	1.1533	1.2252
2073	1.9500	1.6738	1.6812	1.2338	1.2504	1.2201	1.2659	1.1513	1.2228
2074	1.9500	1.6726	1.6801	1.2315	1.2480	1.2178	1.2634	1.1492	1.2204
2075	1.9500	1.6713	1.6789	1.2291	1.2455	1.2155	1.2609	1.1471	1.2180
2076	1.9500	1.6701	1.6778	1.2268	1.2431	1.2132	1.2585	1.1451	1.2157
2077	1.9500	1.6689	1.6766	1.2245	1.2407	1.2109	1.2561	1.1431	1.2133
2078	1.9500	1.6676	1.6755	1.2222	1.2383	1.2086	1.2536	1.1411	1.2110
2079	1.9500	1.6664	1.6743	1.2199	1.2360	1.2063	1.2512	1.1390	1.2086

Equation Parameters from the 2004 Trustees Report Assumptions

t	$MR_{9,t}^{TR}$	$MR_{10,t}^{TR}$	$MR_{11,t}^{TR}$	$MR_{12,t}^{TR}$	$MR_{13,t}^{TR}$	$MR_{14,t}^{TR}$	$MR_{15,t}^{TR}$	$MR_{16,t}^{TR}$	$MR_{17,t}^{TR}$
2004	1.2537	1.0197	1.4051	1.1711	1.5643	0.7808	0.5897	0.0972	0.1171
2005	1.1661	0.9759	1.2872	1.0909	1.4114	0.7787	0.6290	0.2320	0.2625
2006	1.0973	0.9413	1.1944	1.0295	1.2938	0.7799	0.6686	0.3430	0.3855
2007	1.0429	0.9139	1.1212	0.9820	1.2027	0.7827	0.7054	0.4335	0.4883
2008	0.9999	0.8920	1.0631	0.9449	1.1316	0.7858	0.7380	0.5070	0.5731
2009	0.9656	0.8744	1.0169	0.9156	1.0757	0.7887	0.7660	0.5661	0.6425
2010	0.9382	0.8602	0.9800	0.8923	1.0316	0.7910	0.7895	0.6135	0.6989
2011	0.9162	0.8486	0.9504	0.8736	0.9965	0.7928	0.8088	0.6513	0.7443
2012	0.8986	0.8391	0.9267	0.8585	0.9685	0.7940	0.8244	0.6812	0.7807
2013	0.8843	0.8313	0.9076	0.8462	0.9460	0.7947	0.8369	0.7048	0.8097
2014	0.8728	0.8248	0.8922	0.8361	0.9278	0.7949	0.8468	0.7232	0.8326
2015	0.8634	0.8193	0.8797	0.8277	0.9131	0.7947	0.8545	0.7376	0.8507
2016	0.8557	0.8148	0.8695	0.8207	0.9011	0.7941	0.8603	0.7486	0.8647
2017	0.8495	0.8109	0.8612	0.8148	0.8912	0.7933	0.8647	0.7570	0.8755
2018	0.8443	0.8076	0.8544	0.8098	0.8831	0.7922	0.8679	0.7633	0.8837
2019	0.8400	0.8047	0.8489	0.8055	0.8764	0.7910	0.8700	0.7679	0.8898
2020	0.8365	0.8022	0.8442	0.8017	0.8708	0.7896	0.8715	0.7712	0.8943
2021	0.8335	0.8000	0.8404	0.7984	0.8660	0.7882	0.8723	0.7733	0.8974
2022	0.8309	0.7980	0.8371	0.7955	0.8620	0.7867	0.8726	0.7746	0.8994
2023	0.8288	0.7962	0.8344	0.7929	0.8585	0.7851	0.8725	0.7753	0.9006
2024	0.8269	0.7946	0.8320	0.7905	0.8555	0.7834	0.8720	0.7754	0.9011
2025	0.8253	0.7931	0.8300	0.7883	0.8528	0.7818	0.8714	0.7750	0.9010
2026	0.8239	0.7918	0.8283	0.7862	0.8505	0.7801	0.8705	0.7744	0.9005
2027	0.8226	0.7905	0.8267	0.7843	0.8484	0.7784	0.8695	0.7734	0.8997
2028	0.8195	0.7881	0.8227	0.7809	0.8435	0.7770	0.8705	0.7759	0.9034
2029	0.8188	0.7872	0.8220	0.7796	0.8423	0.7752	0.8688	0.7738	0.9011
2030	0.8182	0.7863	0.8213	0.7782	0.8412	0.7735	0.8672	0.7718	0.8989
2031	0.8176	0.7854	0.8206	0.7768	0.8401	0.7718	0.8655	0.7699	0.8967
2032	0.8170	0.7844	0.8199	0.7755	0.8390	0.7701	0.8639	0.7679	0.8945
2033	0.8164	0.7835	0.8192	0.7742	0.8379	0.7684	0.8623	0.7660	0.8924
2034	0.8158	0.7827	0.8185	0.7729	0.8368	0.7667	0.8608	0.7641	0.8903
2035	0.8151	0.7818	0.8179	0.7716	0.8358	0.7651	0.8592	0.7622	0.8882
2036	0.8146	0.7809	0.8172	0.7703	0.8347	0.7635	0.8577	0.7603	0.8861
2037	0.8140	0.7800	0.8166	0.7690	0.8337	0.7619	0.8562	0.7585	0.8840
2038	0.8134	0.7792	0.8160	0.7678	0.8326	0.7603	0.8547	0.7567	0.8820
2039	0.8129	0.7784	0.8153	0.7665	0.8316	0.7587	0.8532	0.7549	0.8800
2040	0.8123	0.7775	0.8147	0.7653	0.8306	0.7572	0.8517	0.7531	0.8780
2041	0.8118	0.7767	0.8141	0.7641	0.8296	0.7556	0.8503	0.7513	0.8761
2042	0.8112	0.7759	0.8135	0.7628	0.8286	0.7541	0.8488	0.7496	0.8741
2043	0.8107	0.7751	0.8129	0.7617	0.8277	0.7526	0.8474	0.7479	0.8722
2044	0.8101	0.7743	0.8123	0.7605	0.8267	0.7511	0.8460	0.7462	0.8703
2045	0.8096	0.7735	0.8117	0.7593	0.8258	0.7496	0.8446	0.7445	0.8684
2046	0.8091	0.7727	0.8111	0.7582	0.8248	0.7482	0.8432	0.7428	0.8665
2047	0.8086	0.7719	0.8106	0.7570	0.8239	0.7468	0.8419	0.7412	0.8647
2048	0.8081	0.7712	0.8100	0.7559	0.8230	0.7453	0.8406	0.7395	0.8629
2049	0.8076	0.7704	0.8094	0.7548	0.8221	0.7439	0.8392	0.7379	0.8611
2050	0.8071	0.7697	0.8089	0.7537	0.8212	0.7425	0.8379	0.7363	0.8593
2051	0.8066	0.7689	0.8083	0.7526	0.8204	0.7412	0.8366	0.7348	0.8576
2052	0.8061	0.7682	0.8078	0.7515	0.8195	0.7398	0.8354	0.7332	0.8558
2053	0.8056	0.7675	0.8073	0.7505	0.8186	0.7385	0.8341	0.7317	0.8541
2054	0.8052	0.7668	0.8068	0.7494	0.8178	0.7371	0.8329	0.7302	0.8524
2055	0.8047	0.7660	0.8062	0.7484	0.8170	0.7358	0.8316	0.7287	0.8507
2056	0.8043	0.7653	0.8057	0.7473	0.8161	0.7346	0.8304	0.7272	0.8491
2057	0.8038	0.7646	0.8052	0.7463	0.8153	0.7332	0.8292	0.7257	0.8474
2058	0.8034	0.7640	0.8047	0.7453	0.8145	0.7320	0.8280	0.7243	0.8458
2059	0.8029	0.7633	0.8042	0.7443	0.8137	0.7307	0.8268	0.7228	0.8442
2060	0.8025	0.7626	0.8037	0.7433	0.8129	0.7295	0.8257	0.7214	0.8426
2061	0.8020	0.7619	0.8032	0.7423	0.8121	0.7283	0.8245	0.7200	0.8410
2062	0.8016	0.7613	0.8028	0.7414	0.8114	0.7271	0.8234	0.7186	0.8395
2063	0.8012	0.7606	0.8023	0.7404	0.8106	0.7258	0.8223	0.7173	0.8379
2064	0.8008	0.7600	0.8018	0.7395	0.8098	0.7247	0.8211	0.7159	0.8364
2065	0.8004	0.7593	0.8014	0.7385	0.8091	0.7235	0.8201	0.7146	0.8349
2066	0.7999	0.7587	0.8009	0.7376	0.8084	0.7224	0.8190	0.7133	0.8334
2067	0.7995	0.7581	0.8005	0.7367	0.8076	0.7212	0.8179	0.7119	0.8320
2068	0.7991	0.7575	0.8000	0.7358	0.8069	0.7201	0.8168	0.7106	0.8305
2069	0.7987	0.7568	0.7996	0.7349	0.8062	0.7190	0.8158	0.7094	0.8291
2070	0.7984	0.7562	0.7991	0.7340	0.8055	0.7178	0.8147	0.7081	0.8277
2071	0.7980	0.7556	0.7987	0.7331	0.8048	0.7167	0.8137	0.7069	0.8263
2072	0.7976	0.7550	0.7983	0.7323	0.8041	0.7156	0.8127	0.7056	0.8249
2073	0.7972	0.7545	0.7979	0.7314	0.8034	0.7146	0.8117	0.7044	0.8235
2074	0.7968	0.7539	0.7974	0.7305	0.8028	0.7135	0.8107	0.7032	0.8222
2075	0.7965	0.7533	0.7970	0.7297	0.8021	0.7125	0.8097	0.7020	0.8208
2076	0.7961	0.7527	0.7966	0.7289	0.8014	0.7114	0.8088	0.7008	0.8195
2077	0.7958	0.7521	0.7962	0.7281	0.8008	0.7104	0.8078	0.6997	0.8182
2078	0.7954	0.7516	0.7958	0.7272	0.8001	0.7094	0.8069	0.6985	0.8169
2079	0.7950	0.7510	0.7954	0.7264	0.7995	0.7084	0.8059	0.6974	0.8156

Equation Parameters from the 2004 Trustees Report Assumptions

t	$MR_{18,t}{}^{TR}$	$MR_{19,t}{}^{TR}$	$MR_{20,t}{}^{TR}$	$MR_{21,t}{}^{TR}$	$MR_{22,t}{}^{TR}$	$MR_{23,t}{}^{TR}$	$MR_{24,t}{}^{TR}$	$MR_{25,t}{}^{TR}$	$MR_{26,t}{}^{TR}$
2004	-0.0165	0.1965	0.5096	0.9120	1.1420	1.7302	1.3338	1.7585	0.9555
2005	0.1443	0.3353	0.5679	0.9229	1.0793	1.5762	1.2363	1.5985	0.9336
2006	0.2764	0.4528	0.6169	0.9365	1.0300	1.4559	1.1587	1.4721	0.9166
2007	0.3840	0.5509	0.6574	0.9503	0.9909	1.3614	1.0966	1.3718	0.9032
2008	0.4710	0.6318	0.6904	0.9631	0.9596	1.2867	1.0465	1.2918	0.8923
2009	0.5410	0.6979	0.7169	0.9740	0.9342	1.2272	1.0060	1.2278	0.8830
2010	0.5971	0.7515	0.7380	0.9831	0.9134	1.1797	0.9729	1.1762	0.8751
2011	0.6417	0.7946	0.7546	0.9902	0.8963	1.1413	0.9458	1.1345	0.8681
2012	0.6771	0.8290	0.7674	0.9955	0.8820	1.1102	0.9234	1.1006	0.8617
2013	0.7050	0.8563	0.7772	0.9993	0.8699	1.0848	0.9047	1.0728	0.8558
2014	0.7268	0.8778	0.7845	1.0017	0.8596	1.0639	0.8890	1.0498	0.8503
2015	0.7438	0.8945	0.7898	1.0029	0.8507	1.0465	0.8756	1.0307	0.8451
2016	0.7569	0.9074	0.7935	1.0032	0.8429	1.0318	0.8642	1.0146	0.8401
2017	0.7669	0.9171	0.7958	1.0027	0.8360	1.0195	0.8542	1.0010	0.8352
2018	0.7743	0.9243	0.7972	1.0015	0.8298	1.0088	0.8455	0.9893	0.8305
2019	0.7798	0.9295	0.7977	0.9997	0.8243	0.9996	0.8378	0.9791	0.8259
2020	0.7836	0.9331	0.7975	0.9976	0.8191	0.9915	0.8309	0.9702	0.8215
2021	0.7862	0.9354	0.7968	0.9951	0.8144	0.9843	0.8246	0.9622	0.8171
2022	0.7878	0.9366	0.7957	0.9923	0.8100	0.9778	0.8188	0.9551	0.8127
2023	0.7886	0.9370	0.7943	0.9893	0.8058	0.9719	0.8135	0.9486	0.8085
2024	0.7887	0.9368	0.7926	0.9861	0.8018	0.9664	0.8085	0.9425	0.8043
2025	0.7884	0.9360	0.7907	0.9828	0.7980	0.9613	0.8038	0.9369	0.8001
2026	0.7876	0.9347	0.7887	0.9794	0.7944	0.9565	0.7993	0.9316	0.7960
2027	0.7865	0.9332	0.7865	0.9759	0.7908	0.9520	0.7950	0.9266	0.7920
2028	0.7894	0.9362	0.7861	0.9740	0.7863	0.9449	0.7888	0.9189	0.7879
2029	0.7870	0.9332	0.7835	0.9701	0.7832	0.9412	0.7852	0.9148	0.7839
2030	0.7847	0.9303	0.7808	0.9663	0.7801	0.9376	0.7817	0.9108	0.7800
2031	0.7824	0.9275	0.7782	0.9625	0.7771	0.9340	0.7782	0.9069	0.7762
2032	0.7801	0.9246	0.7756	0.9587	0.7741	0.9304	0.7747	0.9029	0.7723
2033	0.7778	0.9218	0.7730	0.9550	0.7711	0.9269	0.7713	0.8990	0.7685
2034	0.7756	0.9190	0.7705	0.9513	0.7682	0.9233	0.7679	0.8951	0.7648
2035	0.7734	0.9162	0.7680	0.9476	0.7653	0.9198	0.7645	0.8912	0.7610
2036	0.7711	0.9135	0.7655	0.9440	0.7624	0.9163	0.7612	0.8873	0.7574
2037	0.7690	0.9108	0.7630	0.9404	0.7596	0.9128	0.7578	0.8835	0.7537
2038	0.7668	0.9081	0.7606	0.9368	0.7568	0.9093	0.7546	0.8797	0.7501
2039	0.7647	0.9054	0.7582	0.9333	0.7540	0.9059	0.7513	0.8759	0.7465
2040	0.7626	0.9028	0.7558	0.9298	0.7513	0.9024	0.7481	0.8721	0.7430
2041	0.7605	0.9002	0.7535	0.9263	0.7485	0.8990	0.7449	0.8684	0.7395
2042	0.7585	0.8976	0.7511	0.9229	0.7459	0.8956	0.7418	0.8646	0.7360
2043	0.7564	0.8951	0.7489	0.9195	0.7432	0.8923	0.7387	0.8610	0.7326
2044	0.7544	0.8925	0.7466	0.9162	0.7406	0.8889	0.7356	0.8573	0.7292
2045	0.7524	0.8900	0.7443	0.9128	0.7380	0.8856	0.7325	0.8536	0.7258
2046	0.7505	0.8876	0.7421	0.9095	0.7354	0.8823	0.7295	0.8500	0.7225
2047	0.7485	0.8851	0.7399	0.9063	0.7329	0.8790	0.7265	0.8464	0.7192
2048	0.7466	0.8827	0.7377	0.9030	0.7304	0.8757	0.7235	0.8428	0.7159
2049	0.7447	0.8803	0.7356	0.8998	0.7279	0.8725	0.7206	0.8392	0.7127
2050	0.7428	0.8779	0.7335	0.8967	0.7254	0.8692	0.7177	0.8357	0.7095
2051	0.7410	0.8756	0.7314	0.8935	0.7230	0.8660	0.7148	0.8322	0.7063
2052	0.7391	0.8733	0.7293	0.8904	0.7206	0.8628	0.7120	0.8287	0.7032
2053	0.7373	0.8710	0.7273	0.8874	0.7183	0.8596	0.7092	0.8253	0.7002
2054	0.7355	0.8687	0.7252	0.8843	0.7159	0.8565	0.7064	0.8218	0.6971
2055	0.7338	0.8665	0.7232	0.8813	0.7136	0.8534	0.7036	0.8184	0.6941
2056	0.7320	0.8643	0.7213	0.8783	0.7113	0.8503	0.7009	0.8150	0.6911
2057	0.7303	0.8621	0.7193	0.8754	0.7091	0.8472	0.6982	0.8116	0.6881
2058	0.7285	0.8599	0.7174	0.8724	0.7068	0.8441	0.6955	0.8083	0.6852
2059	0.7269	0.8578	0.7155	0.8696	0.7046	0.8410	0.6929	0.8050	0.6823
2060	0.7252	0.8556	0.7136	0.8667	0.7025	0.8380	0.6903	0.8017	0.6795
2061	0.7235	0.8535	0.7117	0.8639	0.7003	0.8350	0.6877	0.7984	0.6766
2062	0.7219	0.8515	0.7099	0.8611	0.6982	0.8320	0.6852	0.7952	0.6738
2063	0.7203	0.8494	0.7080	0.8583	0.6961	0.8291	0.6826	0.7920	0.6711
2064	0.7187	0.8474	0.7062	0.8556	0.6940	0.8261	0.6801	0.7887	0.6683
2065	0.7171	0.8453	0.7045	0.8528	0.6920	0.8232	0.6777	0.7856	0.6656
2066	0.7155	0.8434	0.7027	0.8502	0.6899	0.8203	0.6752	0.7824	0.6629
2067	0.7140	0.8414	0.7009	0.8475	0.6879	0.8174	0.6728	0.7793	0.6603
2068	0.7125	0.8395	0.6992	0.8449	0.6860	0.8145	0.6704	0.7762	0.6577
2069	0.7110	0.8375	0.6975	0.8423	0.6840	0.8116	0.6680	0.7731	0.6551
2070	0.7095	0.8356	0.6958	0.8397	0.6821	0.8088	0.6657	0.7700	0.6525
2071	0.7080	0.8337	0.6942	0.8372	0.6802	0.8060	0.6633	0.7670	0.6500
2072	0.7065	0.8319	0.6925	0.8347	0.6783	0.8032	0.6610	0.7640	0.6475
2073	0.7051	0.8300	0.6909	0.8322	0.6764	0.8004	0.6588	0.7610	0.6450
2074	0.7037	0.8282	0.6893	0.8297	0.6746	0.7977	0.6565	0.7580	0.6426
2075	0.7023	0.8264	0.6877	0.8273	0.6728	0.7950	0.6543	0.7551	0.6402
2076	0.7009	0.8246	0.6862	0.8249	0.6710	0.7922	0.6521	0.7522	0.6378
2077	0.6995	0.8229	0.6846	0.8225	0.6692	0.7895	0.6499	0.7493	0.6355
2078	0.6982	0.8211	0.6831	0.8201	0.6674	0.7869	0.6478	0.7464	0.6331
2079	0.6968	0.8194	0.6816	0.8178	0.6657	0.7842	0.6457	0.7436	0.6308

Equation Parameters from the 2004 Trustees Report Assumptions

t	$MR_{27,t}{}^{TR}$	$MR_{28,t}{}^{TR}$	$MR_{29,t}{}^{TR}$	$MR_{30,t}{}^{TR}$	$MR_{31,t}{}^{TR}$	$MR_{32,t}{}^{TR}$	$MR_{33,t}{}^{TR}$	$MR_{34,t}{}^{TR}$	$MR_{35,t}{}^{TR}$
2004	1.5425	0.6333	1.0988	0.1406	1.1434	0.1198	0.9292	0.0663	0.4925
2005	1.4227	0.6763	1.0477	0.2716	1.0847	0.2540	0.9204	0.2200	0.5784
2006	1.3283	0.7123	1.0088	0.3790	1.0397	0.3654	0.9150	0.3476	0.6485
2007	1.2536	0.7417	0.9789	0.4664	1.0047	0.4568	0.9115	0.4523	0.7050
2008	1.1940	0.7654	0.9553	0.5370	0.9772	0.5312	0.9088	0.5376	0.7502
2009	1.1463	0.7842	0.9366	0.5936	0.9551	0.5912	0.9065	0.6065	0.7861
2010	1.1077	0.7989	0.9212	0.6386	0.9371	0.6393	0.9042	0.6616	0.8142
2011	1.0764	0.8100	0.9085	0.6742	0.9222	0.6774	0.9017	0.7054	0.8360
2012	1.0508	0.8182	0.8978	0.7020	0.9097	0.7073	0.8990	0.7399	0.8526
2013	1.0296	0.8241	0.8885	0.7236	0.8990	0.7307	0.8960	0.7667	0.8650
2014	1.0119	0.8280	0.8803	0.7401	0.8896	0.7485	0.8927	0.7873	0.8740
2015	0.9970	0.8302	0.8730	0.7524	0.8812	0.7620	0.8892	0.8028	0.8802
2016	0.9843	0.8312	0.8663	0.7615	0.8738	0.7718	0.8854	0.8142	0.8840
2017	0.9733	0.8312	0.8602	0.7678	0.8669	0.7788	0.8815	0.8223	0.8862
2018	0.9637	0.8303	0.8544	0.7720	0.8605	0.7835	0.8773	0.8277	0.8868
2019	0.9552	0.8287	0.8490	0.7745	0.8545	0.7862	0.8731	0.8310	0.8863
2020	0.9476	0.8266	0.8438	0.7756	0.8488	0.7875	0.8686	0.8324	0.8848
2021	0.9407	0.8240	0.8388	0.7756	0.8433	0.7875	0.8642	0.8325	0.8825
2022	0.9343	0.8211	0.8340	0.7747	0.8381	0.7866	0.8596	0.8315	0.8797
2023	0.9284	0.8180	0.8293	0.7731	0.8330	0.7849	0.8550	0.8296	0.8763
2024	0.9229	0.8146	0.8247	0.7709	0.8281	0.7826	0.8503	0.8270	0.8726
2025	0.9176	0.8111	0.8202	0.7683	0.8232	0.7798	0.8456	0.8238	0.8686
2026	0.9126	0.8074	0.8157	0.7654	0.8185	0.7766	0.8409	0.8201	0.8644
2027	0.9078	0.8037	0.8114	0.7622	0.8138	0.7731	0.8361	0.8161	0.8599
2028	0.9012	0.8018	0.8069	0.7631	0.8089	0.7743	0.8323	0.8176	0.8585
2029	0.8970	0.7976	0.8027	0.7586	0.8045	0.7695	0.8274	0.8120	0.8532
2030	0.8929	0.7934	0.7985	0.7543	0.8001	0.7647	0.8226	0.8065	0.8480
2031	0.8888	0.7893	0.7944	0.7500	0.7957	0.7600	0.8178	0.8011	0.8428
2032	0.8848	0.7852	0.7903	0.7457	0.7913	0.7554	0.8130	0.7957	0.8376
2033	0.8807	0.7811	0.7862	0.7414	0.7870	0.7507	0.8083	0.7903	0.8325
2034	0.8767	0.7771	0.7822	0.7373	0.7827	0.7461	0.8035	0.7850	0.8273
2035	0.8727	0.7732	0.7782	0.7331	0.7785	0.7416	0.7989	0.7798	0.8223
2036	0.8687	0.7692	0.7743	0.7290	0.7743	0.7371	0.7943	0.7746	0.8173
2037	0.8648	0.7654	0.7703	0.7249	0.7701	0.7327	0.7897	0.7694	0.8123
2038	0.8609	0.7615	0.7664	0.7209	0.7659	0.7283	0.7851	0.7643	0.8073
2039	0.8569	0.7577	0.7625	0.7169	0.7618	0.7239	0.7806	0.7592	0.8024
2040	0.8531	0.7539	0.7587	0.7130	0.7577	0.7196	0.7761	0.7542	0.7975
2041	0.8492	0.7502	0.7549	0.7091	0.7537	0.7154	0.7717	0.7493	0.7927
2042	0.8454	0.7465	0.7511	0.7052	0.7497	0.7111	0.7672	0.7444	0.7878
2043	0.8416	0.7428	0.7473	0.7014	0.7457	0.7070	0.7629	0.7395	0.7831
2044	0.8378	0.7392	0.7436	0.6977	0.7417	0.7028	0.7585	0.7347	0.7783
2045	0.8341	0.7356	0.7399	0.6939	0.7378	0.6987	0.7542	0.7299	0.7737
2046	0.8304	0.7321	0.7362	0.6902	0.7339	0.6947	0.7499	0.7252	0.7690
2047	0.8267	0.7285	0.7326	0.6866	0.7301	0.6907	0.7457	0.7205	0.7644
2048	0.8230	0.7251	0.7290	0.6830	0.7263	0.6867	0.7415	0.7159	0.7598
2049	0.8194	0.7216	0.7255	0.6794	0.7225	0.6828	0.7373	0.7113	0.7552
2050	0.8158	0.7182	0.7219	0.6758	0.7187	0.6790	0.7332	0.7068	0.7507
2051	0.8122	0.7149	0.7184	0.6723	0.7150	0.6751	0.7291	0.7023	0.7462
2052	0.8086	0.7115	0.7149	0.6689	0.7113	0.6713	0.7250	0.6979	0.7418
2053	0.8051	0.7082	0.7114	0.6654	0.7076	0.6676	0.7210	0.6935	0.7374
2054	0.8015	0.7050	0.7080	0.6621	0.7040	0.6639	0.7170	0.6892	0.7330
2055	0.7981	0.7018	0.7046	0.6587	0.7004	0.6602	0.7131	0.6849	0.7287
2056	0.7946	0.6985	0.7013	0.6554	0.6969	0.6566	0.7091	0.6807	0.7244
2057	0.7911	0.6954	0.6979	0.6521	0.6933	0.6530	0.7053	0.6764	0.7201
2058	0.7877	0.6923	0.6946	0.6489	0.6898	0.6495	0.7014	0.6723	0.7159
2059	0.7844	0.6892	0.6914	0.6457	0.6864	0.6459	0.6976	0.6682	0.7117
2060	0.7810	0.6861	0.6881	0.6425	0.6830	0.6425	0.6938	0.6641	0.7075
2061	0.7777	0.6831	0.6849	0.6394	0.6796	0.6391	0.6900	0.6601	0.7034
2062	0.7744	0.6802	0.6817	0.6363	0.6762	0.6357	0.6863	0.6561	0.6993
2063	0.7711	0.6772	0.6785	0.6332	0.6728	0.6323	0.6827	0.6522	0.6953
2064	0.7678	0.6743	0.6754	0.6302	0.6695	0.6290	0.6790	0.6483	0.6913
2065	0.7646	0.6714	0.6723	0.6272	0.6662	0.6257	0.6754	0.6445	0.6873
2066	0.7614	0.6686	0.6692	0.6242	0.6630	0.6225	0.6718	0.6407	0.6834
2067	0.7582	0.6657	0.6662	0.6213	0.6598	0.6193	0.6683	0.6369	0.6795
2068	0.7550	0.6629	0.6632	0.6184	0.6566	0.6161	0.6647	0.6332	0.6756
2069	0.7519	0.6602	0.6602	0.6155	0.6534	0.6130	0.6613	0.6295	0.6718
2070	0.7488	0.6574	0.6572	0.6127	0.6503	0.6099	0.6578	0.6259	0.6680
2071	0.7457	0.6548	0.6543	0.6099	0.6472	0.6069	0.6544	0.6223	0.6642
2072	0.7427	0.6521	0.6513	0.6072	0.6441	0.6039	0.6510	0.6187	0.6605
2073	0.7396	0.6495	0.6485	0.6044	0.6411	0.6009	0.6477	0.6152	0.6568
2074	0.7366	0.6469	0.6456	0.6017	0.6381	0.5979	0.6443	0.6118	0.6531
2075	0.7336	0.6443	0.6428	0.5991	0.6351	0.5950	0.6410	0.6083	0.6495
2076	0.7307	0.6418	0.6400	0.5964	0.6322	0.5921	0.6378	0.6049	0.6459
2077	0.7278	0.6392	0.6372	0.5938	0.6292	0.5893	0.6346	0.6016	0.6424
2078	0.7249	0.6368	0.6345	0.5913	0.6263	0.5865	0.6314	0.5983	0.6389
2079	0.7220	0.6343	0.6317	0.5887	0.6235	0.5837	0.6282	0.5950	0.6354

Equation Parameters from the 2004 Trustees Report Assumptions

t	$MR_{36,t}^{TR}$	$MR_{37,t}^{TR}$	$MR_{38,t}^{TR}$	$MR_{39,t}^{TR}$	$MR_{40,t}^{TR}$	$MR_{41,t}^{TR}$	$MR_{42,t}^{TR}$	IM_t^{TR}	EM_t^{TR}
2004	-0.0041	0.0426	-0.0753	-0.4091	-0.4196	-1.4868	-1.3316	1,033,333	258,333
2005	0.1766	0.1738	0.0691	-0.1827	-0.2000	-1.0437	-0.9210	1,000,000	250,000
2006	0.3255	0.2810	0.1901	0.0006	-0.0190	-0.6872	-0.5883	966,667	241,667
2007	0.4473	0.3681	0.2905	0.1485	0.1293	-0.4008	-0.3196	933,333	233,333
2008	0.5460	0.4385	0.3728	0.2675	0.2500	-0.1711	-0.1031	900,000	225,000
2009	0.6255	0.4952	0.4400	0.3629	0.3478	0.0130	0.0709	866,667	216,667
2010	0.6891	0.5405	0.4944	0.4393	0.4267	0.1602	0.2106	833,333	208,333
2011	0.7395	0.5767	0.5382	0.5002	0.4900	0.2778	0.3223	800,000	200,000
2012	0.7792	0.6053	0.5732	0.5486	0.5407	0.3715	0.4116	800,000	200,000
2013	0.8100	0.6278	0.6011	0.5869	0.5810	0.4461	0.4828	800,000	200,000
2014	0.8337	0.6453	0.6230	0.6170	0.6129	0.5052	0.5393	800,000	200,000
2015	0.8516	0.6589	0.6401	0.6406	0.6380	0.5521	0.5840	800,000	200,000
2016	0.8648	0.6692	0.6533	0.6590	0.6575	0.5890	0.6193	800,000	200,000
2017	0.8742	0.6770	0.6633	0.6731	0.6726	0.6179	0.6469	800,000	200,000
2018	0.8804	0.6826	0.6708	0.6838	0.6841	0.6405	0.6685	800,000	200,000
2019	0.8843	0.6865	0.6762	0.6918	0.6928	0.6579	0.6851	800,000	200,000
2020	0.8861	0.6891	0.6800	0.6976	0.6990	0.6712	0.6978	800,000	200,000
2021	0.8863	0.6906	0.6823	0.7016	0.7034	0.6813	0.7073	800,000	200,000
2022	0.8852	0.6912	0.6836	0.7042	0.7063	0.6887	0.7142	800,000	200,000
2023	0.8831	0.6911	0.6841	0.7056	0.7080	0.6940	0.7192	800,000	200,000
2024	0.8802	0.6904	0.6838	0.7062	0.7087	0.6976	0.7224	800,000	200,000
2025	0.8767	0.6893	0.6829	0.7060	0.7086	0.6999	0.7244	800,000	200,000
2026	0.8726	0.6878	0.6816	0.7053	0.7078	0.7010	0.7254	800,000	200,000
2027	0.8681	0.6861	0.6799	0.7040	0.7066	0.7014	0.7255	800,000	200,000
2028	0.8698	0.6882	0.6833	0.7089	0.7121	0.7126	0.7362	800,000	200,000
2029	0.8635	0.6853	0.6801	0.7057	0.7088	0.7093	0.7329	800,000	200,000
2030	0.8573	0.6823	0.6769	0.7026	0.7055	0.7062	0.7296	800,000	200,000
2031	0.8511	0.6793	0.6737	0.6995	0.7022	0.7030	0.7262	800,000	200,000
2032	0.8450	0.6764	0.6705	0.6964	0.6989	0.6998	0.7229	800,000	200,000
2033	0.8390	0.6735	0.6673	0.6933	0.6956	0.6966	0.7196	800,000	200,000
2034	0.8329	0.6706	0.6642	0.6903	0.6923	0.6935	0.7163	800,000	200,000
2035	0.8269	0.6677	0.6611	0.6872	0.6891	0.6903	0.7131	800,000	200,000
2036	0.8210	0.6648	0.6579	0.6842	0.6858	0.6872	0.7098	800,000	200,000
2037	0.8152	0.6619	0.6548	0.6811	0.6826	0.6841	0.7065	800,000	200,000
2038	0.8093	0.6591	0.6517	0.6781	0.6794	0.6810	0.7033	800,000	200,000
2039	0.8036	0.6562	0.6486	0.6751	0.6762	0.6779	0.7000	800,000	200,000
2040	0.7979	0.6534	0.6456	0.6721	0.6729	0.6748	0.6968	800,000	200,000
2041	0.7922	0.6506	0.6425	0.6691	0.6697	0.6718	0.6936	800,000	200,000
2042	0.7866	0.6477	0.6395	0.6661	0.6666	0.6687	0.6904	800,000	200,000
2043	0.7810	0.6450	0.6364	0.6632	0.6634	0.6657	0.6871	800,000	200,000
2044	0.7755	0.6422	0.6334	0.6602	0.6603	0.6627	0.6839	800,000	200,000
2045	0.7700	0.6394	0.6304	0.6573	0.6571	0.6596	0.6808	800,000	200,000
2046	0.7646	0.6367	0.6274	0.6544	0.6540	0.6566	0.6776	800,000	200,000
2047	0.7593	0.6339	0.6245	0.6515	0.6509	0.6536	0.6744	800,000	200,000
2048	0.7539	0.6312	0.6215	0.6486	0.6478	0.6507	0.6713	800,000	200,000
2049	0.7487	0.6285	0.6185	0.6457	0.6447	0.6477	0.6681	800,000	200,000
2050	0.7435	0.6258	0.6156	0.6428	0.6416	0.6447	0.6650	800,000	200,000
2051	0.7383	0.6231	0.6127	0.6400	0.6385	0.6418	0.6619	800,000	200,000
2052	0.7332	0.6204	0.6098	0.6371	0.6355	0.6389	0.6588	800,000	200,000
2053	0.7281	0.6178	0.6069	0.6343	0.6324	0.6360	0.6557	800,000	200,000
2054	0.7231	0.6151	0.6040	0.6315	0.6294	0.6331	0.6526	800,000	200,000
2055	0.7182	0.6125	0.6011	0.6287	0.6264	0.6302	0.6495	800,000	200,000
2056	0.7133	0.6099	0.5983	0.6259	0.6234	0.6273	0.6464	800,000	200,000
2057	0.7084	0.6073	0.5954	0.6231	0.6204	0.6244	0.6434	800,000	200,000
2058	0.7036	0.6047	0.5926	0.6204	0.6174	0.6216	0.6403	800,000	200,000
2059	0.6988	0.6021	0.5898	0.6176	0.6145	0.6187	0.6373	800,000	200,000
2060	0.6941	0.5996	0.5870	0.6149	0.6115	0.6159	0.6343	800,000	200,000
2061	0.6895	0.5970	0.5842	0.6122	0.6086	0.6131	0.6313	800,000	200,000
2062	0.6848	0.5945	0.5815	0.6095	0.6057	0.6103	0.6283	800,000	200,000
2063	0.6803	0.5920	0.5787	0.6068	0.6028	0.6075	0.6253	800,000	200,000
2064	0.6757	0.5895	0.5760	0.6041	0.5999	0.6048	0.6223	800,000	200,000
2065	0.6713	0.5870	0.5733	0.6015	0.5970	0.6020	0.6194	800,000	200,000
2066	0.6668	0.5845	0.5705	0.5988	0.5941	0.5993	0.6164	800,000	200,000
2067	0.6625	0.5820	0.5679	0.5962	0.5913	0.5965	0.6135	800,000	200,000
2068	0.6581	0.5796	0.5652	0.5935	0.5884	0.5938	0.6106	800,000	200,000
2069	0.6538	0.5772	0.5625	0.5909	0.5856	0.5911	0.6077	800,000	200,000
2070	0.6496	0.5747	0.5599	0.5884	0.5828	0.5884	0.6048	800,000	200,000
2071	0.6454	0.5723	0.5572	0.5858	0.5800	0.5858	0.6019	800,000	200,000
2072	0.6413	0.5699	0.5546	0.5832	0.5772	0.5831	0.5990	800,000	200,000
2073	0.6372	0.5675	0.5520	0.5806	0.5745	0.5805	0.5962	800,000	200,000
2074	0.6331	0.5652	0.5494	0.5781	0.5717	0.5778	0.5933	800,000	200,000
2075	0.6291	0.5628	0.5468	0.5756	0.5690	0.5752	0.5905	800,000	200,000
2076	0.6252	0.5605	0.5443	0.5731	0.5662	0.5726	0.5877	800,000	200,000
2077	0.6212	0.5581	0.5417	0.5706	0.5635	0.5700	0.5849	800,000	200,000
2078	0.6174	0.5558	0.5392	0.5681	0.5608	0.5674	0.5821	800,000	200,000
2079	0.6135	0.5535	0.5367	0.5656	0.5582	0.5649	0.5793	800,000	200,000

Equation Parameters from the 2004 Trustees Report Assumptions

t	ΔO_t^{TR}	U_t^{TR}	I_t^{TR}	R_t^{TR}	W_t^{TR}	DIM_t^{TR}	DIF_t^{TR}	DRM_t^{TR}	DRF_t^{TR}
2004	0	-2.8137	-3.1669	0.0323	0.0238	5.2575	5.2156	14.0153	11.7905
2005	0	-2.8354	-3.1018	0.0329	0.0275	5.2389	5.1614	14.7945	12.2154
2006	0	-2.8334	-3.0007	0.0317	0.0189	5.2879	5.1323	15.9836	13.2971
2007	0	-2.8345	-2.9096	0.0314	0.0151	5.3314	5.1024	16.0449	13.2619
2008	0	-2.8361	-2.8531	0.0310	0.0137	5.3753	5.0688	17.5710	14.4340
2009	0	-2.8361	-2.8478	0.0307	0.0125	5.4026	5.0187	17.7771	14.4573
2010	0	-2.8369	-2.8454	0.0303	0.0122	5.4279	4.9677	18.0307	14.4862
2011	0	-2.8382	-2.8477	0.0302	0.0119	5.4606	4.9230	18.3203	15.8297
2012	0	-2.8391	-2.8468	0.0300	0.0121	5.4968	4.8817	18.6180	15.8735
2013	0	-2.8401	-2.8483	0.0301	0.0110	5.5336	4.8394	18.9424	15.9427
2014	-50,000	-2.8415	-2.8473	0.0300	0.0109	5.5953	4.8770	18.1460	15.2665
2015	0	-2.8430	-2.8473	0.0300	0.0103	5.6561	4.9157	17.3567	14.6057
2016	0	-2.8439	-2.8473	0.0300	0.0105	5.7185	4.9563	16.5604	13.9452
2017	0	-2.8436	-2.8473	0.0300	0.0104	5.7826	4.9957	15.7101	13.2610
2018	0	-2.8423	-2.8473	0.0300	0.0100	5.8435	5.0321	14.8327	12.5886
2019	0	-2.8431	-2.8473	0.0300	0.0105	5.9027	5.0689	13.9549	11.9400
2020	0	-2.8438	-2.8473	0.0300	0.0108	5.9631	5.1053	13.0818	11.3106
2021	0	-2.8449	-2.8473	0.0300	0.0098	6.0423	5.1523	12.1935	10.6925
2022	0	-2.8460	-2.8473	0.0300	0.0117	6.1223	5.2007	11.2658	10.0499
2023	0	-2.8460	-2.8473	0.0300	0.0111	6.2016	5.2480	10.3280	9.4046
2024	-50,000	-2.8461	-2.8473	0.0300	0.0105	6.2155	5.2538	10.3248	9.4176
2025	0	-2.8463	-2.8473	0.0300	0.0099	6.2282	5.2581	10.3208	9.4420
2026	0	-2.8469	-2.8473	0.0300	0.0097	6.2436	5.2644	10.3084	9.4631
2027	0	-2.8476	-2.8473	0.0300	0.0117	6.2436	5.2648	10.2754	9.4658
2028	0	-2.8472	-2.8473	0.0300	0.0112	6.2453	5.2674	10.2361	9.4581
2029	0	-2.8469	-2.8473	0.0300	0.0106	6.2473	5.2701	10.1971	9.4462
2030	0	-2.8468	-2.8473	0.0300	0.0100	6.2472	5.2707	10.1689	9.4495
2031	0	-2.8468	-2.8473	0.0300	0.0099	6.2462	5.2695	10.1362	9.4432
2032	0	-2.8470	-2.8473	0.0300	0.0117	6.2441	5.2682	10.0944	9.4271
2033	0	-2.8468	-2.8473	0.0300	0.0114	6.2411	5.2652	10.0497	9.4033
2034	0	-2.8467	-2.8473	0.0300	0.0108	6.2385	5.2637	10.0070	9.3877
2035	0	-2.8466	-2.8473	0.0300	0.0101	6.2379	5.2631	9.9843	9.3740
2036	0	-2.8466	-2.8473	0.0300	0.0100	6.2393	5.2639	9.9690	9.3749
2037	0	-2.8467	-2.8473	0.0300	0.0115	6.2435	5.2678	9.9495	9.3648
2038	0	-2.8468	-2.8473	0.0300	0.0114	6.2461	5.2711	9.9275	9.3471
2039	0	-2.8470	-2.8473	0.0300	0.0110	6.2479	5.2732	9.9120	9.3334
2040	0	-2.8473	-2.8473	0.0300	0.0102	6.2483	5.2729	9.9027	9.3281
2041	0	-2.8476	-2.8473	0.0300	0.0099	6.2477	5.2707	9.8982	9.3270
2042	0	-2.8480	-2.8473	0.0300	0.0112	6.2478	5.2703	9.8964	9.3294
2043	0	-2.8484	-2.8473	0.0300	0.0113	6.2469	5.2700	9.8798	9.3134
2044	0	-2.8488	-2.8473	0.0300	0.0111	6.2457	5.2700	9.8694	9.3020
2045	0	-2.8492	-2.8473	0.0300	0.0103	6.2444	5.2702	9.8600	9.2999
2046	0	-2.8497	-2.8473	0.0300	0.0100	6.2436	5.2692	9.8610	9.2958
2047	0	-2.8503	-2.8473	0.0300	0.0111	6.2464	5.2706	9.8674	9.3086
2048	0	-2.8507	-2.8473	0.0300	0.0111	6.2483	5.2723	9.8601	9.2960
2049	0	-2.8510	-2.8473	0.0300	0.0109	6.2500	5.2738	9.8542	9.2902
2050	0	-2.8514	-2.8473	0.0300	0.0101	6.2501	5.2734	9.8488	9.2901
2051	0	-2.8517	-2.8473	0.0300	0.0099	6.2485	5.2717	9.8531	9.2927
2052	0	-2.8520	-2.8473	0.0300	0.0109	6.2496	5.2727	9.8573	9.3020
2053	0	-2.8522	-2.8473	0.0300	0.0110	6.2487	5.2713	9.8520	9.2962
2054	0	-2.8524	-2.8473	0.0300	0.0109	6.2475	5.2705	9.8446	9.2910
2055	0	-2.8525	-2.8473	0.0300	0.0103	6.2455	5.2688	9.8361	9.2859
2056	0	-2.8524	-2.8473	0.0300	0.0100	6.2418	5.2663	9.8389	9.2848
2057	0	-2.8524	-2.8473	0.0300	0.0109	6.2440	5.2679	9.8431	9.2950
2058	0	-2.8524	-2.8473	0.0300	0.0109	6.2455	5.2689	9.8399	9.2920
2059	0	-2.8524	-2.8473	0.0300	0.0107	6.2475	5.2705	9.8363	9.2940
2060	0	-2.8524	-2.8473	0.0300	0.0104	6.2488	5.2722	9.8381	9.2888
2061	0	-2.8525	-2.8473	0.0300	0.0102	6.2479	5.2724	9.8345	9.2894
2062	0	-2.8525	-2.8473	0.0300	0.0109	6.2479	5.2707	9.8385	9.2932
2063	0	-2.8525	-2.8473	0.0300	0.0108	6.2458	5.2684	9.8339	9.2893
2064	0	-2.8525	-2.8473	0.0300	0.0107	6.2438	5.2674	9.8291	9.2872
2065	0	-2.8525	-2.8473	0.0300	0.0106	6.2432	5.2676	9.8275	9.2850
2066	0	-2.8524	-2.8473	0.0300	0.0106	6.2461	5.2696	9.8265	9.2831
2067	0	-2.8524	-2.8473	0.0300	0.0108	6.2458	5.2695	9.8294	9.2862
2068	0	-2.8524	-2.8473	0.0300	0.0107	6.2460	5.2701	9.8294	9.2854
2069	0	-2.8525	-2.8473	0.0300	0.0107	6.2462	5.2705	9.8278	9.2856
2070	0	-2.8526	-2.8473	0.0300	0.0107	6.2465	5.2706	9.8248	9.2845
2071	0	-2.8528	-2.8473	0.0300	0.0107	6.2467	5.2704	9.8257	9.2818
2072	0	-2.8529	-2.8473	0.0300	0.0107	6.2468	5.2705	9.8253	9.2813
2073	0	-2.8530	-2.8473	0.0300	0.0107	6.2469	5.2707	9.8241	9.2838
2074	0	-2.8531	-2.8473	0.0300	0.0107	6.2470	5.2706	9.8218	9.2816
2075	0	-2.8532	-2.8473	0.0300	0.0107	6.2471	5.2707	9.8246	9.2820
2076	0	-2.8534	-2.8473	0.0300	0.0107	6.2471	5.2705	9.8233	9.2802
2077	0	-2.8535	-2.8473	0.0300	0.0107	6.2472	5.2705	9.8214	9.2803
2078	0	-2.8536	-2.8473	0.0300	0.0107	6.2472	5.2704	9.8190	9.2842
2079	0	-2.8537	-2.8473	0.0300	0.0107	6.2474	5.2705	9.8203	9.2839

E. GLOSSARY

Actuarial balance. The difference between the summarized income rate and the summarized cost rate over a given time period.

Administrative expenses. Expenses incurred by the Social Security Administration and the Department of the Treasury in administering the OASDI program and the provisions of the Internal Revenue Code relating to the collection of contributions. Such administrative expenses are paid from the OASI and DI Trust Funds.

Age-adjusted rates. The crude rate that would occur in the standard population if it were to experience the same rates by age as in the selected year.

Alternatives I, II, or III. See "Assumptions."

Annual maximum taxable level. See "Contribution and benefit base."

Assets. Treasury notes and bonds, other securities guaranteed by the Federal Government, certain Federally sponsored agency obligations, and cash, held by the trust funds for investment purposes.

Assumptions. Values relating to future trends in certain key factors which affect the balance in the trust funds. Demographic assumptions include fertility, mortality, net immigration, marriage, and divorce. Economic assumptions include unemployment rates, average earnings, inflation, interest rates, and productivity. Program-specific assumptions include retirement patterns, and disability incidence and termination rates. Three sets of demographic, economic, and program-specific assumptions are presented in the 2004 Trustees Report:

- Alternative II (intermediate assumptions): represents the Trustees' best estimates of likely future demographic, economic, and program-specific conditions.
- Alternative I (low cost assumptions): assumes relatively rapid economic growth, low inflation, and favorable (from the standpoint of program financing) demographic conditions.
- Alternative III (high cost assumptions): assumes relatively slow economic growth, high inflation, and unfavorable (from the standpoint of program financing) demographic conditions.

Automatic cost-of-living benefit increase. The annual increase in benefits, effective for December, reflecting the increase in the cost of living. The benefit increase equals the percentage increase in the Consumer Price Index for Urban Wage Earners and Clerical Workers (CPI-W) measured from the average over July, August, and September of the preceding year to the average for the same 3 months in the current year. If the increase is less than one-tenth of 1 percent, when rounded, there is no automatic increase for the current year; the increase for the next year would reflect the net increase in the CPI over a 2-year period.

Average indexed monthly earnings (AIME). The amount of earnings used in determining the primary insurance amount (PIA) for most workers who attain age 62, become disabled, or die after 1978. A worker's actual past earnings are adjusted by changes in the average wage index, in order to bring them up to their approximately equivalent value at the time of retirement or other eligibility for benefits.

Average wage index. The average amount of total wages for each year after 1950, including wages in noncovered employment and wages in covered employment in excess of the OASDI contribution and benefit base. (See Title 20, chapter III, section 404.211(c) of the Code of Federal Regulations for a more precise definition.) These average wage amounts are used to index the taxable earnings of most workers first becoming eligible for benefits in 1979 or later, and for automatic adjustments in the contribution and benefit base, bend points, earnings test exempt amounts, and other wage-indexed amounts.

Award. An administrative determination that an individual is entitled to receive a specified type of OASDI benefit. Awards can represent not only new entrants to the benefit rolls but also persons already on the rolls who become entitled to a different type of benefit. Awards usually result in the immediate payment of benefits, although payments may be deferred or withheld depending on the individual's particular circumstances.

Bend points. The dollar amounts defining the AIME or PIA brackets in the benefit formulas. Each bracket is an AIME or PIA interval throughout which each dollar generates a uniform benefit increment.

Central death rate. The annual number of deaths for a particular group divided by the estimated population of that group at midyear.

Consumer Price Index (CPI). An official measure of inflation in consumer prices. In this study, all references to the CPI relate to the Consumer Price Index for Urban Wage Earners and Clerical Workers (CPI-W).

Contribution and benefit base. Annual dollar amount above which earnings in employment covered under the OASDI program are neither taxable nor creditable for benefit computation purposes. (Also referred to as maximum contribution and benefit base, annual creditable maximum, taxable maximum, and maximum taxable.)

Contributions. The amount based on a percentage of earnings, up to an annual maximum, that must be paid by:

- employers and employees on wages from employment under the Federal Insurance Contributions Act,
- the self-employed on net earnings from self-employment under the Self-Employment Contributions Act, and
- States on the wages of State and local government employees covered under the Social Security Act through voluntary agreements under section 218 of the Act.

Generally, employers withhold contributions from wages, add an equal amount of contributions, and pay both on a current basis. Also referred to as taxes, and as tax contributions.

Cost-of-living adjustment (COLA). See “Automatic cost-of-living benefit increase.”

Cost rate. The cost rate for a year is the ratio of the cost of the program to the taxable payroll for the year. In this context, the cost is defined to include scheduled benefit payments, special monthly payments to certain uninsured persons who have three or more quarters of coverage (and whose payments are therefore not reimbursable from the General Fund of the Treasury), administrative expenses, net transfers from the trust funds to the Railroad Retirement program under the financial-interchange provisions, and payments for vocational rehabilitation services for disabled beneficiaries; it excludes special monthly payments to certain uninsured persons whose payments are reimbursable from the General Fund of the Treasury (as described above), and transfers under the interfund borrowing provisions.

Covered earnings. Earnings in employment or self-employment covered by the OASDI program.

Covered employment. All employment for which earnings are creditable for Social Security purposes. Almost all employment is covered under the program. Some exceptions are:

- State and local government employees whose employer has not elected to be covered under Social Security and who are participating in an employer-provided pension plan.
- Current Federal civilian workers hired before 1984 who have not elected to be covered.
- Self-employed workers earning less than $400 in a calendar year.

Covered worker. A person who has earnings creditable for Social Security purposes on the basis of services for wages in covered employment and/or on the basis of income from covered self-employment.

Covered worker rate. The ratio of OASDI covered workers to the Social Security area population.

Current-payment status. The status of beneficiaries presently scheduled to be paid regular monthly benefits; includes those scheduled to be paid less than a full month’s benefits, but excludes those who receive retroactive benefits that are not scheduled to be paid regularly.

Currently insured status. See “Insured status.”

Deterministic model. A model with specified assumptions for and relationships among variables. Under such model, any specified set of assumptions fully determines a single outcome directly reflecting the specifications.

Disability incidence rate. The proportion of the exposed population at the beginning of that year who become newly entitled to disability benefits during the year.

Disability Insurance (DI). See “Trust fund.”

Disability insured status. See “Insured status.”

Disability recovery rate. The proportion of disabled-worker beneficiaries whose disability benefits terminate as a result of the individual's recovery from disability during the year.

Disbursements. See "Outgo."

Earnings. Unless otherwise qualified, all wages from employment and net earnings from self-employment, whether or not taxable or covered.

Emigration. See "Legal emigration."

Entitlement. The conditions upon which an individual may collect a Social Security cash benefit. There are many definitions, depending on the type of benefit.

Retired-worker benefit—An individual is entitled to a retirement insurance benefit if that individual:

(a) is at least 62 for a full month and is fully insured; and

(b) has filed an application for retirement benefits, or is entitled to disabled-worker benefits in the month before the month of attainment of the normal retirement age.

If the individual meets the requirements in (a) before the month the individual filed the application, that individual may be entitled to retirement insurance benefits back to the first month in which the individual met those requirements, subject to certain limitations.

Disabled-worker benefit—An individual is entitled to a disability insurance benefit if that individual:

(a) is disabled according to Social Security's definition of disability,

(b) has filed an application for disabled-worker's benefits,

(c) has disability insured status,

(d) has completed a five-month waiting period (unless exempt), and

(e) has not attained the normal retirement age.

For other definitions, please refer to the Social Security Handbook, available for download at www.socialsecurity.gov/OP_Home/handbook/download.html.

Exhaustion year. See "Year of exhaustion."

Financial interchange (Railroad). Provisions of the Railroad Retirement Act providing for transfers between the trust funds and the Social Security Equivalent Benefit Account of the Railroad Retirement program in order to place each trust fund in the same position as it would have been in if railroad employment had always been covered by Social Security.

Fully insured status. See "Insured status."

Gross Domestic Product (GDP). The total dollar value of all goods and services produced by labor and property located in the United States, regardless of who supplies the labor or property.

High cost assumptions. See “Assumptions.”

Immigration. See “Legal immigration.” See “Total immigration.” See “Net other immigration.”

Income rate. Ratio of income from tax revenues on a liability basis (payroll-tax contributions and income from the taxation of scheduled benefits) to the OASDI taxable payroll for the year.

Inflation rate. The annual growth rate in the Consumer Price Index for Urban Wage Earners and Clerical Workers (CPI-W).

Insured status. There are three types of insured status which can be acquired by a worker under the OASDI program. Each of these statuses is determined by the number and recency of quarters of coverage (QCs) earned.

- **Fully insured status** is acquired by any worker whose total number of QCs is greater than or equal to the number of years elapsed after the year of attainment of age 21 (with a minimum of six QCs required). Once a worker has accumulated 40 QCs, the worker remains permanently fully insured.
- **Disability insured status** is acquired by any fully insured worker over age 30 who has accumulated 20 QCs during the 40-quarter period ending with the current quarter, any fully insured worker aged 24-30 who has accumulated QCs during one-half of the quarters elapsed after the quarter of attainment of age 21 up to and including the current quarter, and any fully insured worker under age 24 who has accumulated six QCs during the 12-quarter period ending with the current quarter.
- **Currently insured status** is acquired by any worker who has accumulated six QCs during the 13-quarter period ending with the current quarter.

Periods of disability are excluded from the above described QC requirements for insured status (but do not reduce the minimum of six QCs).

Interest. A payment in exchange for the use of money during a specified period.

Interest rate. Amount of interest payment per unit of principal. Interest rates on new public-debt obligations issuable to Federal trust funds are determined monthly. (See “Special public-debt obligation.”) Such rates are set equal to the average market yield on all outstanding marketable U.S. securities not due or not callable until after four years from the date of the determination. The effective interest rate for a trust fund is the ratio of the interest earned by the fund over a given period of time to the average level of assets held by the fund during the period. The effective rate of interest thus represents a measure of the overall average interest earnings on the fund’s portfolio assets.

Intermediate assumptions. See “Assumptions.”

Labor force participation rate. The number of persons who are employed or actively seeking work as a percent of the civilian noninstitutional population.

Legal emigration. The number of persons who lawfully leave the United States, and are no longer considered to be a part of the Social Security program.

Legal immigration. The number of persons lawfully admitted for permanent residence into the United States.

Life expectancy (period). The average number of years of life remaining if a group of people at a given age were to experience the mortality rates for that year over the course of their remaining lives. This quantity is computed for each year of each simulation, and is done so

- At age 65, and
- At birth.

Long range. The next 75 years. Long-range actuarial estimates are made for this period because it is approximately the maximum remaining lifetime of current Social Security participants.

Low cost assumptions. See "Assumptions."

Master Beneficiary Record (MBR). A complete record of beneficiaries which contains such items as social security number, beneficiary identification code, date of birth, date of entitlement, etc. The *x*% MBR represents a sampling of *x*% of these records from the collection of all records.

Module. In Fortran, a nonexecutable program unit that contains type definitions, object declarations, procedure definitions (module procedures), external procedure interfaces, user-defined generic names, and user-defined operators and assignments. Any such definitions not specified to be private to the module containing them are available to be shared with those programs that use the module. Thus, modules provide a convenient sharing and encapsulation mechanism for data, types, procedures, and procedure interfaces. The OSM contains the following modules:

- Assumptions: Projects some of the key demographic and economic assumptions using time-series models.
- Population: Projects the Social Security area population for each year based on levels of fertility, mortality, and immigration generated in the Assumptions Module.
- Economics: Projects U.S. economy-wide employment and wages, OASDI covered employment and wages, taxable payroll, the average wage index and the COLA.
- Insured: Projects the percentage of the population that will be fully insured and disability insured.
- Disability Insurance Beneficiaries (DIB): Projects the annual values for the disabled-worker beneficiaries in current-payment status.

- Old-Age and Survivors Insurance Beneficiaries (OASIB): Projects the number of retired-worker beneficiaries and all aggregate dependent beneficiaries.
- Awards: Projects levels of benefits, in terms of Average Indexed Monthly Earnings (AIME), for those beneficiaries newly awarded.
- Cost: Computes the year-by-year progress of the combined OASDI Trust Funds and produces summary measures used to assess the long-range financial status of the OASDI program.
- Summary Results: Computes the estimated probability distributions of annual and summarized data received from the Assumptions, Population, and Cost Modules.

Monthly benefit amount (MBA). The actual cash benefit scheduled to be paid to an entitled individual.

Net other immigration. The annual flow of persons into the United States minus the annual flow of persons out of the United States who do not meet the definition of legal immigration or legal emigration. Thus, net other immigration includes unauthorized persons and those not seeking permanent residence.

Nominal yield (on special public-debt obligation). The effective annual interest rate, based on the semiannual compounding of interest. See “Interest rate.”

Normal retirement age (NRA). The age at which a person may first become entitled to unreduced retirement benefits. For persons reaching age 62 before 2000, the normal retirement age is 65. It will increase gradually to 67 for persons reaching that age in 2027 or later, beginning with an increase to 65 years and 2 months for persons reaching age 65 in 2003.

OCACT Stochastic Model (OSM). The particular stochastic model developed by the Office of the Chief Actuary. See “Stochastic model.”

Old-Age and Survivors Insurance (OASI). See “Trust fund.”

Open group unfunded obligation. This measure is determined as of the valuation date over a specified time period (such as over the long-range 75-year period). It is computed as the difference between:

(a) The present value of the future cost of the program between the valuation date and the end of the specified time period, and

(b) The sum of the assets in the trust fund as of the valuation date and the present value of the future scheduled tax income of the program between the valuation date and the end of the specified time period.

Outgo. Actual expenditures (outgo) made or expected to be made under current law, including benefits paid or payable, special monthly payments to certain uninsured persons who have three or more quarters of coverage (and whose payments are therefore not reimbursable from the General Fund of the Treasury), administrative expenses, net transfers from the trust funds to the Railroad Retirement program under the financial-interchange provisions, and payments for vocational rehabilitation services for disabled beneficiaries; it excludes special monthly payments to certain uninsured persons whose payments are reimbursable from the General Fund of the Treasury (as described above), and transfers under the interfund borrowing provisions.

Payroll taxes. A tax levied on the gross earned income of workers.

Period life expectancy. See "Life expectancy (period)."

Population in the Social Security area. The population comprised of (i) residents of the 50 States and the District of Columbia (adjusted for net census undercount); (ii) civilian residents of Puerto Rico, the Virgin Islands, Guam, American Samoa and the Northern Mariana Islands; (iii) Federal civilian employees and persons in the U.S. Armed Forces abroad and their dependents; (iv) crew members of U.S. merchant vessels; and (v) all other U.S. citizens abroad.

Present value. The equivalent value, at a given time, of a future stream of payments (either income or cost). The present value of a future stream of payments may be thought of as the lump-sum amount that, if invested today, together with interest earnings would be just enough to meet each of the payments as they fell due. Present values are widely used in calculations involving financial transactions over long periods of time to account for the time value of money (interest). For the purpose of present-value calculations for this study, values are discounted by the effective yield on trust fund assets.

Primary insurance amount (PIA). The monthly amount payable to a retired worker who begins to receive benefits at normal retirement age or (generally) to a disabled worker. This amount, which is related to the worker's average monthly wage or average indexed monthly earnings, is also the amount used as a base for computing all types of benefits payable on the basis of one individual's earnings record.

Primary-insurance-amount formula. The mathematical formula relating the PIA to the AIME for workers who attain age 62, become disabled, or die after 1978. The PIA is equal to the sum of 90 percent of AIME up to the first bend point plus 32 percent of AIME up to the second bend point, plus 15 percent of AIME in excess of the second bend point. Automatic benefit increases are applied beginning with the year of eligibility.

Quarters of coverage. Basic unit of measurement for determining insured status. In 2004, a worker receives one quarter of coverage (up to a total of four) for each $900 of annual covered earnings. The amount of earnings required for a quarter of coverage is subject to annual automatic increases in proportion to increases in the Average Wage Index.

Railroad retirement. A Federal insurance program, somewhat similar to Social Security, designed for workers in the railroad industry. The provisions of the Railroad Retirement Act provide for a system of coordination and financial interchange between the Railroad Retirement program and the Social Security program.

Real average covered wage. The ratio of the average nominal OASDI covered wage to the adjusted inflation rate.

Real interest rate. The annual (compounded) nominal yield divided by the inflation rate.

Self-employment. Operation of a trade or business by an individual or by a partnership in which an individual is a member.

Social Security area population. See “Population in the Social Security area.”

Special public-debt obligation. Securities of the United States Government issued exclusively to the OASI, DI, and other Federal trust funds. Section 201(d) of the Social Security Act provides that the public-debt obligations issued for purchase by the OASI and DI Trust Funds shall have maturities fixed with due regard for the needs of the funds. The usual practice has been to spread the holdings of special issues, as of each June 30, so that the amounts maturing in each of the next 15 years are approximately equal. Special public-debt obligations are redeemable at par value at any time and carry interest rates determined by law. (See “Interest rate.”)

Stochastic model. A model used for projecting a probability distribution of potential outcomes. Such models allow for random variation in one or more variables through time. The random variation is generally based on fluctuations observed in historical data for a selected period. Distributions of potential outcomes are derived from a large number of simulations, each of which reflects random variation in the variable(s).

Summarized cost rate. The ratio of the present value of cost to the present value of the taxable payroll for the years in a given period, expressed as a percentage. This percentage can be used as a measure of the relative level of cost during the period in question. For purposes of evaluating the financial adequacy of the program, the summarized cost rate is adjusted to include the cost of reaching and maintaining a target trust fund level. Because a trust fund level of about one year’s cost is considered to be an adequate reserve for unforeseen contingencies, the targeted trust fund ratio used in determining summarized cost rates is 100 percent of annual cost. Accordingly the adjusted summarized cost rate is equal to the ratio of (a) the sum of the present value of the cost during the period plus the present value of the increase needed to attain the targeted ending trust fund level, to (b) the present value of the taxable payroll during the projection period.

Summarized income rate. The ratio of the present value of scheduled tax income to the present value of taxable payroll for the years in a given period, expressed as a percentage. This percentage can be used as a measure of the relative level of income during the period in question. For the purposes of evaluating the financial adequacy of the program, the summarized income rate is adjusted to include assets on hand at the beginning of the period. Accordingly, the adjusted summarized income rate equals the ratio of (a) the sum of the trust fund balance at the beginning of the period plus the present value of the total income from taxes during the period, to (b) the present value of the taxable payroll for the years in the period.

Taxable earnings. Wages and/or self-employment income, in employment covered by the OASDI and/or HI programs, that is under the applicable annual maximum taxable limit. For 1994 and later, no maximum taxable limit applies to the HI program.

Taxable payroll. A weighted average of taxable wages and taxable self-employment income. When multiplied by the combined employee-employer tax rate, it yields the total amount of taxes incurred by employees, employers, and the self-employed for work during the period.

Taxable self-employment income. The maximum amount of net earnings from self-employment by an earner which, when added to any taxable wages, does not exceed the contribution and benefit base.

Taxable wages. See “Taxable earnings.”

Taxation of benefits. During 1984-93, up to one-half of an individual’s or a couple’s OASDI benefits was potentially subject to Federal income taxation under certain circumstances. The revenue derived from this provision was allocated to the OASI and DI Trust Funds on the basis of the income taxes paid on the benefits from each fund. Beginning in 1994, the maximum portion of OASDI benefits potentially subject to taxation was increased to 85 percent. The additional revenue derived from taxation of benefits in excess of one-half, up to 85 percent, is allocated to the HI Trust Fund.

Taxes. See “Contributions.”

Total fertility rate. The average number of children who would be born to a hypothetical woman in her lifetime if she were to experience the birth rates by age observed in, or assumed for, a specified calendar year, and if she were to survive the entire childbearing period.

Total immigration. Legal immigration minus legal emigration plus net other immigration.

TR04I, TR04II, TR04III. See “Assumptions.”

Trust fund. Separate accounts in the United States Treasury into which are deposited the taxes received under the Federal Insurance Contributions Act and the Self-Employment Contributions Act, contributions resulting from coverage of State and local government employees; any sums received under the financial interchange with the railroad retirement account; voluntary hospital and medical insurance premiums; and transfers of Federal general revenues. Funds not withdrawn for current monthly or service benefits, the financial interchange, and administrative expenses are invested in interest-bearing Federal securities, as required by law; the interest earned is also deposited in the trust funds.

- **Old-Age and Survivors Insurance (OASI).** The trust fund used for paying monthly benefits to retired-worker (old-age) beneficiaries and their spouses and children, and to survivors of deceased insured workers.

- **Disability Insurance (DI).** The trust fund used for paying monthly benefits to disabled-worker beneficiaries and their spouses and children and for providing rehabilitation services to the disabled.

Trust fund ratio. A measure of the adequacy of the trust fund level. Defined as the assets at the beginning of the year, including advance tax transfers (if any), expressed as a percentage of the cost during the year. The trust fund ratio represents the proportion of a year’s cost which could be paid with the funds available at the beginning of the year.

Unemployment rate. The number of unemployed persons seeking work as a percentage of the civilian labor force.

Wage base. See “Contribution and benefit base.”

Year of exhaustion. The year in which a trust fund would become unable to pay benefits when due because the fund has no assets.

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