

Characterization of Clocks and Oscillators

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This publication covers portions of NBS Monograph 140 published in 1974. The purpose of this document is to replace this outdated monograph in the areas of methods for characterizing clocks and oscillators and definitions and standards relating to such characterization.



U.S. DEPARTMENT OF COMMERCE, Robert A. Mosbacher, Secretary
NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY, John W. Lyons, Director

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PREFACE

For many years following its publication in 1974, "TIME AND FREQUENCY: Theory and Fundamentals," a volume edited by Byron E. Blair and published as NBS Monograph 140, served as a common reference for those engaged in the characterization of very stable clocks and oscillators. Monograph 140 has gradually become outdated, and with the recent issuance of a new military specification, MIL-0-55310B, which covers general specifications for crystal oscillators, it has become especially clear that Monograph 140 no longer meets the needs it so ably served in earlier years. During development of the new military specification, a process involving discussion and input from many quarters, a key author of the specification, John Vig of the US Army Electronics Technology and Devices, urged the National Bureau of Standards (now the National Institute of Standards and Technology, NIST) to issue a revised publication to serve as reference for the characterization of clocks and oscillators. With NIST having agreed to this task, the framers of the military specification used the nomenclature "NBS Monograph 140R" in their document, anticipating a revised (R) volume which had not yet been prepared.

Considering the availability of a number of newer books in the time and frequency field, the rewriting of a major volume like Monograph 140 seemed inappropriate. The real need has not been for rework of everything in Monograph 140, but only for those parts which provide reference to definitions and methods for measurement and characterization of clocks and oscillators, subjects which are fully covered in a number of papers distributed through a variety of conference proceedings, books, and journals. For the near term, we concluded that the most effective procedure would be to collect a representative set of these papers into one reference source with introductory comments which permit the reader to quickly access material required to meet particular needs. Thus, we arrived at this particular collection. The editors' challenge has been to select representative papers, to organize them in a convenient manner, and to deal with errata and notation inconsistencies in a reasonable manner. In the longer term, the material in this volume needs to be more completely integrated. This task would profitably await further developments in the area of phase noise measurements.

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<p>This is a collection of published papers assembled as a reference for those involved in characterizing and specifying high-performance clocks and oscillators. It is an interim replacement for NBS Monograph 140, <i>Time and Frequency: Theory and Fundamentals</i>, an older volume of papers edited by Byron E. Blair. This current volume includes tutorial papers, papers on standards and definitions, and a collection of papers detailing specific measurement and analysis techniques. The discussion in the introduction to the volume provides a guide to the content of the papers, and tables and graphs provide further help in organizing methods described in the papers.</p>				
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