Technology & Model-Based Conceptual Assessment: Research in Students’ Applications of Models in Physics & Mathematics

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Summary

To improve the communication between students and teachers, particularly in large classes, many universities have begun using technology-based response systems. These systems enable an instructor to pose questions and see, within a few minutes, the students’ responses to those questions. Another similar approach is to assign homework that is submitted, graded and returned quickly via the World Wide Web. Both of these technology-based systems offer instructors the opportunity to record each student’s responses in a database. Thus, the instructor can track students' understanding much more completely than with traditional homework and quizzes and can use the resulting data to investigate more deeply how students understand the scientific and mathematical concepts. In addition to seeing the present level of each students understanding the instructor can learn how the students change their thinking by making comparisons of responses throughout the learning process. A present, the analyses of these responses generally tell instructors when the students are obtaining the right answers. However, for students who are not answering correctly, the present systems do little more than indicate that the student is not applying the scientific theories and models correctly. Still missing is an analysis tool that is based on contemporary educational research and can provide robust quantitative information on the students’ difficulties with the underlying scientific models and theories, and can track how the students' understandings of these models change during instruction.

These tools must go beyond correct answer analysis and analyze students’ incorrect answers by incorporating theories of learning into the systems. This project will begin with a model for students' conceptual learning processes and with existing work on assessing students' conceptual understanding in physics and mathematics. Then, research will be conducted on students’ applications of scientific models and mathematical concepts, on how the students’ thinking and applications change during instruction, and methods to present the results of these assessments to teaching faculty who are using in-class, real-time response or on-line homework systems.

By constructing sets of questions in which incorrect answers provide insights into the scientific and mathematical models that students are applying, the project's results will lead to a deeper understanding of students' abilities to learn physics and mathematics and the contexts in which that learning occurs most effectively. The analysis will also provide insight into students' abilities to transfer knowledge between physics and mathematics courses.

The major objectives of the project are to

- measure, with real-time feedback, students’ understanding of fundamental concepts and the application of those concepts,
- trace changes in those understandings and applications during instruction,
- investigate how students’ conceptual understanding depends on the context in which a new concept is studied,
- create analysis tools that can be used effectively in many educational environments,
- provide information about the transfer of knowledge between physics and mathematics, and
- investigate how students and instructors interact with this teaching environment.

The result of reaching these goals will be a system that will have a large impact on the teaching of science and mathematics. The impact will be particularly great in large enrollment classes where instructors are often very detached from their students because, frequently, such information becomes available only after students take an exam. Of particular importance for the instructors is knowledge of when students have begun to change their thinking but still sometimes revert to pre-instructional applications of scientific or mathematical concepts-- a mixture of understanding and a lack of understanding. Such situations are recognized to be an important intermediate step in the learning process. By knowing the extent of this mixture the instructors can plan the next step in the learning process based on the students’ present physical or mathematical understanding and the contexts which aid fundamental change in students’ thinking. Thus, the project will provide both information and tools to help science and mathematics instructors learn about the present knowledge of their students and how to use that present knowledge constructively to improve the students’ scientific and mathematical thinking skills.

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> *Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp new concepts and information presented in the classroom, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom. This finding requires that teachers be prepared to draw out their students' existing understandings and help to shape them into an understanding that reflects the concepts and knowledge in the particular discipline of study.* (Bransford, Brown, & R. Cocking, 1999; Donovan, Bransford, & Pellegrino, 1999)

This "key finding" from the National Academy of Sciences reports *How People Learn* emphasizes the importance for teachers to be aware of their students' states of understanding when they begin to discuss a new topic. This statement was based primarily on research conducted with pre-college students. However, the authors emphasize that while the statement above "assumes that the learners are children, … the principles apply to adult learning as well." Certainly, this statement is consistent with the recommendations for college-level science and mathematics teaching contained in the NSF report *Shaping the Future*. (George et al., 1996)

These reports are based on a large body of research on the teaching and learning of science and mathematics. In the past 15 years significant advances have been made in probing students' understanding of science concepts, development of new pedagogy, which is based on this research, to enhance learning, and the development of tools to measure student learning. A number of research universities have specialized groups who conduct research on learning, develop research-based curricula, then implement and test the outcomes. (A list of the physics education groups is available on University of Maryland web site http://www.physics.umd.edu/rgroups/).

Likewise in mathematics research in teaching and learning has focused how preparing students to apply their knowledge to solve non-routine problems. The recently released National Council of Teachers of Mathematics (NCTM) standards for mathematical instruction include training in problem solving as one of the 10 standards for school mathematics instruction (NCTM, 2000) and continuing the emphasis placed on problem solving in the original 1989 standards. (NCTM, 1989) Traditionally, mathematicians have asked students to carry out various standard calculations and assumed that any student who could successfully perform these algorithms had developed an understanding of algebra, trigonometry, or whatever mathematical concepts were presented. This assumption is now known to be incorrect, and mathematics educators are working to better understand how students can be taught problem solving and the ability to apply what they know. (Dubinsky, Mathews, & Reynolds, 1997; Schoenfeld, 1985) It is clear that the ability to apply mathematics requires conceptual knowledge as well as basic computational skills. The proper balance between instruction in basic skills, problem solving techniques, and conceptual development is a matter of controversy. A quantitative study of how students' computational skills and understanding of concepts develop in real-time during the course of instruction would be extremely valuable in answering questions about the proper balance of instructional time, particularly in view of the current emphasis in California that standards should be based on quantitative experimental research (Sowder, 1998).

Although the comments above and the body of published research suggest specific guides for teaching and learning in science and mathematics, a number of research questions warrant closer examination:

- Is it possible to monitor student understanding of scientific concepts in a way that reflects their state of understanding and their transitions from one state to another rather than if they have correctly mastered the concept?
- Can we monitor the process of transfer of knowledge from one subject to another (e.g. mathematics to physics) to understand better how this transfer does or does not occur?
- Can we use modern technologies and develop analysis tools, which are based on contemporary learning research, to monitor student understanding during the learning process?
- Is it possible to provide fast feedback to science and mathematics teachers so they can clearly see the results of the research themselves and how it applies to the students and courses they teach?

Hypotheses and Research Goals

To address these issues we propose to refine a method, *Model Analysis,* of measuring students' state of understanding of scientific models and science and mathematics concepts. We will couple this method with contemporary technology -- both classroom response systems and on-line homework to develop a conceptual assessment tool that will monitor student understanding of scientific models and conceptual understanding in physics and mathematics *during* the learning process. The tool will be placed in a science and mathematics teaching environment, and will enable teachers to monitor in real-time the development of the students' states of understanding.

Our primary hypothesis is that we can create a method of analyzing student responses in a way that will lead to a deeper understanding of their ability to learn physics and mathematics and the contexts in which that learning occurs most effectively. Further, this analysis method can be made in such a way that it will be useful for teaching faculty. We will gather evidence through research that underlies the Model Analysis and research on faculty use of our systems.

Our goals are to

1. measure and trace students' states of understanding and changes in those states during instruction for time periods ranging from a single class session to several weeks of instruction,
2. provide real-time feedback on the state of student conceptual understanding during an individual class period and/or between two consecutive class sessions,
3. develop tools which will promote research in ongoing classes (sometimes called action research) among instructors with interests in learning more about their students' states of understanding and in using that knowledge to improve their teaching,
4. create these tools so that they can be used effectively in classes ranging from small seminars to large lectures,
5. provide researchers and teachers with information about the transfer of knowledge between physics and mathematics, and
6. provide an opportunity to study how students and instructors interact with a new teaching environment that enables rapid feedback on the level of student understanding and transfer.

By reaching these goals we expect to create a system that will have a particularly large impact on the teaching of science and mathematics in large enrollment classes. Currently, instructors, especially in large universities, are often very detached from the knowledge of their students' state of understanding. Frequently, such information becomes available only after students take an exam. This situation leads to frustration for both the students and the instructors. Real-time, rapid feedback can provide instructors important information on the effectiveness of instruction and, more importantly, the mixed states of understanding of the students, which are recognized to be an important intermediate state in conceptual change. By knowing the mixed state the instructors can plan the next step in the instructional process based on the students' present physical model or ability to apply the results of mathematics education.

Research Issues

One of the most important goals of the teaching and learning of science and mathematics has been to enable students to learn how to apply scientific models. The students' abilities to complete such applications have been the subject of research for several years. However, the research and assessment tools have frequently focused on a binary question: Have the students applied the model correctly or have they not? When research of this nature has been reported, it has frequently been in the form of X-percentage of the students is able to apply the model as we presented it. Alternate frameworks or misconceptions are frequently reported. Researchers seldom look at the context in which the model was presented or consider whether students apply the accepted model in some situations and alternative models in others.

Mathematics students who haven't been trained in problem solving do very poorly on non-routine problems (Eisenberg & Dreyfus, 1985). Selden, Selden and Mason in a pair of papers (Selden & Mason, 1989; Selden, Selden, & Mason, 1994) examined the abilities of C students and A/B students on non-routine problems. The C students failed to solve a single non-routine problem while the A/B students solved, on average, slightly less than one problem apiece, demonstrating that while a good grasp of basic skills is necessary for solving non-routine problems, it is certainly not sufficient. It has become clear that knowledge of basic skills is also not sufficient for developing conceptual understanding. For example, Grossman (Grossman, 1983) observed that while over half of the students taking a New York University entrance exam were able to carry out computations with decimals correctly, less than a third could accurately identify the smallest decimal number in a list. On the other hand, some knowledge of basic skills is required for conceptual development (Sierpinska, 1992).

A large part of the problem for students facing non-routine problems is an inability to recognize the mathematical structure of the problem. Students often try to classify problems by the surface structure of the problem, confusing contextual information with superficial features. (Gliner, 1989, 1991). This same confusion of mathematical and surface structure was found to have a large effect on attempts to transfer problem-solving procedures between different disciplines (Bassok, 1990; Bassok & Holyoak, 1989) When problems were given in similar contexts, students were able to transfer their knowledge between different disciplines. When the surface details were different, students were unable to recognize that they had already seen similar problems in the past.

Clearly a deeper conceptual understanding is necessary for students to go beyond the surface details to succeed in transferring knowledge between mathematics and physics. Conceptual development in mathematics has focused largely on the concept of function, as the fundamental building block for later mathematical development. (Dubinsky & Harel, 1992). Many different models for development of the function concept have been proposed. These models often focus on distinguishing between functions as equations, functions as actions on entities, functions as representations of real-world phenomena, and functions as objects in their own right (Dubinsky & Harel, 1992; O'Callaghan, 1998; Sfard & Thompson, 1994; Williams, 1998). These models differ from the models in physics in that there is not a concept of “right” and “wrong” models. All models are correct in different situations, and students must learn which situations are best understood with which model and how to translate ideas between the different models.

Likewise in assessing the results of learning in a class, instructors usually look at the right answers and determine the fraction that understands the results. Partial credit is given for showing that students can begin in the correct way but do not bring the solution to a successful close. Seldom do instructors look in detail at the incorrect responses to determine what model or mixtures of models their students hold.

When teachers or educational researchers look only at the correct answers, they are not taking advantage of a vast amount of information that is hidden in both right and wrong answers. The lack of investigation into students’ alternative models is understandable. Completing such an analysis has been extremely time consuming. Until now the only method available to analyze the data further was one-on-one interviews with the students. With the development of research-based modeling of students' thinking combined with real-time analysis of students’ responses, the situation can be changed.

The use of technology in collecting and analyzing student responses enables us to change this situation significantly. In some cases the technology is a real-time response system that is located in a classroom. In this case the teachers can pose questions to students that are answered quickly, and these answers are analyzed by a computer system. As with much of the research described above, the analyses of answers are generally in a binary form. The students either obtained a right answer, or they did not.

A few investigators have moved beyond the right-wrong approach to in-class response systems. (Dufresne & Gerace, 1994; Dufresne, Gerace, Leonard, Mestre, & Wenk, 1996; Zollman, 1996) Two of the best-known approaches are Mazur (Mazur, 1997) and Thornton and Sokoloff. (Sokoloff & Thornton, 1997) Mazur uses a two-step process in which students give individual answers and then resolve differences and conflicts through peer interactions. Thornton and Sokoloff ask students to make predictions about a demonstration and then resolve any cognitive conflict by completing measurements during class. In both cases the classroom interactions are based on creating an intellectual disequilibrium. They have not yet built into the system a mechanism by which they can learn more about how their students are applying the concepts.

A similar approach has been taken with some rather low technology forms used for collecting real-time classroom responses. Many teachers have distributed cards to students and asked the students to hold up a certain color or a certain letter depending on their answers to the questions. (Moore, 1998) By looking over the class the teacher can determine quickly if the students have responded with a correct answer -- again a binary decision

Out of class applications of scientific models have also begun to be delivered and assessed through technology. On-line homework systems involve students completing problems and getting immediate feedback on their responses. (Kashy et al., 1993; Orr & Lewis, 2000; Titus, Martin, & Beichner, 1998) This approach enables faculty, even in very large classes, to obtain results from a variety of students and to provide feedback on a somewhat individualized basis. While some effort has been made to provide students with feedback in addition to "right" or "wrong" for the answer, little has been done at this point to try to analyze how the students apply scientific models to the situation or to understand the contexts in which they apply it correctly and those in which they do not.

In the research proposed here, we will use several different technologies with a technique that has been developed to analyze the way in which students understand and apply models related to scientific concepts. The research will investigate how one can use existing hardware and software systems to learn about students’ applications of scientific models and how a faculty member can develop means of analyzing their students’ situations in terms of the context and the frequency with which the students are able to apply different components of the model.

Mental Models and States of Understanding

Students' alternative conceptions and the issue of conceptual change have occupied a great deal of the science education research community's time and effort. However, it is important to remember that the way students think about science is more than just a collection of alternative conceptions. In a constructivist paradigm it is assumed that the students build knowledge structures in order to understand scientific concepts and ideas. The term *mental model* is used frequently in science education research to describe the way students understand various scientific concepts and ideas. We might for example talk about a student's 'mental model of light'. Four views of mental models are discussed here.

As part of a discussion on constructivist approaches to science, Driver (Driver, 1995)) describes students' mental models in science in terms of psychological theories used to explain how human beings construct knowledge. These psychological theories encompass ideas including cognitive structures built by people in order to perceive the outside world and the procedural and contextual nature of knowledge. Driver's use of the term mental model in science education research is then based on a theory of students building cognitive structures from their interaction with their environment. If we were, for example, to refer to a student's mental model of shadows, we would be describing a cognitive structure or schema the student has built as a result of their experience with shadows. Mental models are complex and dependent on many factors. Glasersfeld writes:

> "What determines the value of conceptual structures is their experimental adequacy, their goodness of fit with experience, their viability as means for solving problems, among which is, of course, the never-ending problem of consistent organization that we call understanding."
> (Glasersfeld, 1989)

Driver proposes that from a very young age children build mental models about science, even when they have no formal science instruction. As examples she quotes, throwing and catching a ball, turning on a light and watching plants grow. All of these experiences are stored in schemata and when formal science instruction begins new concepts are added to these mental models.

Redish (Redish, 1994) states "people tend to organize their experiences and observations into patterns or mental models." He sees schema as the basic building block of mental models and lists six properties:

1. They consist of propositions, images, rules of procedure and statements as to when and how they are to be used.
2. They may contain contradictory elements.
3. They may be incomplete.
4. People may not know how to "run" the procedures present in their mental models.
5. Elements of a mental model don't have firm boundaries. Similar elements may get confused.
6. Mental models tend to minimize expenditure of mental energy.

Driver and Redish have similar views of mental models in that they view them as an organized representation of students' ideas on a subject, accumulated over a long period of time, from formal and informal interactions. It includes concepts, facts, definitions, images, examples and procedures. Both researchers recognize that a change or modification of students' mental models is a complicated and long-term process.

Another prominent example of the effort dedicated to developing a theory of student knowledge and its development in physics is the work of diSessa. (diSessa, 1993) He proposes a theory of "knowledge in pieces" to describe the development of students' intuitive understanding in physics and its relationship to the knowledge structures of expert physicists.

DiSessa's theory is based on hypothetical knowledge structures called *phenomenological primitives* or *p-prims*. An example of a p-prim as used by diSessa is "Maintaining Agency." This p-prim is an element of cognitive structure that connects the ideas of motion and force. A common observation from physics education research is that students tend to believe that motion implies a force. DiSessa has observed that students *activate* this p-prim in certain contexts. The activation of p-prims is an important element of this learning theory. He describes the difference between expert and intuitive understanding as the extent to which a person has developed a "sense of mechanism," and modified the activation conditions for elements of knowledge.

This learning theory questions the validity of the alternative conceptions in that it disputes that cognitive structures need to be changed, rather the contexts that activate the structures need to be modified and reinforced by a "sense of mechanism." In explaining this distinction Hammer (Hammer, 1996) states, "rather than describing students' knowledge in terms of conceptions inherently inconsistent with expert knowledge, this perspective posits knowledge elements that, appropriately organized, contribute to expert understanding."

The work of Minstrell (Minstrell, 1992) helped connect these ideas to how students apply different components of their knowledge to the study of physics. To organize his studies he developed facets for different concepts in physics. By dividing a concept into a set of facets, Minstrell was able to create systems in which he could analyze how students were understanding and using different components (facets) of a particular concept. His approach was, in fact, a decision tree in which students were directed along different paths depending on their ability to respond to questions about each of the facets. Thus, he was able to analyze some of the students' abilities at a depth beyond "right" or "wrong."

Minstrell's work emphasized how students apply scientific models or components of these models. In effect, he could begin to understand students' states of understanding of the scientific models. More recently, Bao and Redish have developed a tool for analyzing students' state of understanding of scientific models known as Model Analysis (referring to the scientific not the mental model). Both the components of the concept and the contexts in which they are used are part of analysis. In the Model Analysis approach the students are assessed in how they are able to apply a "facet" of a concept and how this application varies as the context is changed. In the Model Analysis approach one does not simply say that a student can or cannot apply a given concept. Instead, one states that the student is able to correctly apply the concept in a certain percentage of all of the problems related to a concept. Further, one can begin to understand which contexts are difficult for the student to see how to apply the model. Thus, the researcher can start to build a picture of the students' states of understanding with respect to the concept and how those states change during instruction.

Research on Conceptual Understanding

In recent years students' conceptual understanding of science has been investigated in a variety of circumstances. (McDermott & Redish, 1999) Physics education researchers, in particular, have looked closely at a large number of different concepts and how students can apply those concepts to novel situations before and after instruction. Frequently, this research has been completed by interviewing students in one-on-one situations. The conclusions have led to concerns that students are not learning physics in the way that faculty have thought or hoped. Somehow, the knowledge did not "stick" or was not learned in the classes.

A similar situation exists in the relation between mathematics and physics courses. Students learn algebra, trigonometry, and/or calculus. They do well on exams. Yet, when faced with applying the techniques of these classes to a problem in physics, they have trouble transferring the knowledge. Again, the instructors concluded that the students knew the material, but knowledge did not stick or was not transferable.

In each of these cases the students were able to display some level of understanding at a particular time and in a particular context. When the time, place and context changed, the knowledge did not seem to be there. Because science is a system in which one applies models to understand and explain a phenomenon or observation, we can conclude that students were able to apply a model in one context but not in another. For the physicist or mathematician this context dependence means that the students did not really know the concept. However, for the students this dependence just means that they have not learned all possible situations.

Part of the problem is based on the tools used to determine student understanding. The research and assessment tools have frequently focused on a binary question: Have the students applied the model correctly or have they not? When research of this nature has been reported, it has frequently been in the form of X-percentage of the students are able to apply the model as we presented it. Likewise with course exams – the students receive a grade. Seldom, do researchers or instructors look at the context in which students were asked to apply the model and determine if the students could have reached some partial level of understanding.

An excellent example of the quandary is the *Force Concept Inventory*, a popular assessment tool for student understanding of mechanics. (Hestenes, Wells, & Swackhammer, 1992) In the past ten years this inventory has been administered to a very large number of students and has been used to investigate a wide range of methods of teaching. (Hake, 1998) Factor analysis of students' responses to the questions has caused some researchers to question what the Force Concept Inventory actually measures. (Huffman & Heller, 1995) However, the Inventory continues to be used for a variety of assessments and diagnostics of teaching in introductory physics courses. Essentially all conclusions based on the student responses have been based on total scores.

Recently, Schecker and Gerdes (Schecker & Gerdes, 1999) analyzed the Force Concept Inventory as a tool for understanding the model that students applied in dynamics problems. They assumed that students would generally hold one of three models -- Aristotelian, Impetus or Newtonian. To determine the students' model they needed to look beyond the right answers and see which wrong answers the students selected. Then, they needed to determine if the students consistently selected the wrong answer associated with the same model. However, the Inventory did not lend itself well to such an analysis because all three models were not represented in each of the questions about forces. Thus, it was not possible to use an analysis of wrong answers to determine the students' preferred models.

Schecker and Gerdes also investigated briefly how the context of the question may affect the students' responses. One of the questions on the Inventory asks students to select an answer to describe the forces on a golf ball after it has been hit and is traveling in the air toward a green. They modified the question slightly and asked the students to describe the forces on a soccer ball after it has been kicked and is traveling through the air toward a goal. For the golf ball problem 42 of 87 students included a force in the direction of motion. However, when faced with an identical problem involving a soccer ball 23 of these 42 students selected either only gravity or gravity plus air resistance. A similar behavior was noted on another question. The authors concluded that the model which students apply to a situation is dependent on the context.

The lack of consistency was also evident in the models that students applied to problems that involved the same physics but were not simple variations of each other. The choice of model depended on the context and the situation presented. This lack of consistency led the authors to conclude that these students were in a mixed state (Mischzustand) when they applied dynamical models. Sometimes the students applied one model; sometimes, another.

(This discussion is not presented as a critique of the *Force Concept Inventory*. The Inventory was not created to assess students' applications of models. Thus, an analysis of the distracters in the way described above was never intended. However, because it is the best-known Inventory for assessing student conceptual understanding of mechanics, it provides a starting point for a discussion of the type of inventories that we wish to create. Schecker and Gerdes' attempt at this type of analysis, using an instrument not made for this purpose, points to a direction that is an appropriate next step for assessing student conceptual understanding.)

The mixed learning state implies that students may simultaneously hold more than one physical model in their minds. Which model they choose to apply is likely to depend on the context of the situation with which they are presented and their experiences with similar contexts. While these mixed learning states have not been the subject of much research, they should not come as a surprise. Ample research shows us that students develop naïve frameworks or models as a result of everyday, concrete experiences. These models prove sufficient to explain most of the students' observations until they encounter formal instruction in physics. At the time of that instruction, we should not expect the students to change suddenly to a new, more generalizable model. Instead, the change is likely to be gradual. Students will apply the newly learned model to some situations while retaining the previous model for others. The mixed learning state will occur as the students make the transition.

A similar, although not identical, situation is seen when students in a physics class need to use techniques learned in a mathematics course. For some students the application of the mathematics knowledge is straightforward. For others, the context is totally different, so they see no immediate relationship. In this case we suspect that a similar analysis can be completed. Because knowledge is new, the students are in a mixed learning state. They are not always able to transfer techniques from one course to another.

The project proposed here will create tools to help physics and mathematics instructors assess the state of their students' learning in real time and guide the instructors toward strategies that can help decrease the amount of the mixture. The foundation for these tools will be Model Analysis.

The Methods of Model Analysis

Model Analysis begins with our knowledge of student learning that is obtained from research and experience. We know that for any different physical concept students are likely to apply several different models. These models may be consistent with the accepted physical model and they may not. For example, when discussing the relation between force and motion students will use the Aristotelian Model (object's natural state is to be at rest), Impetus (a force must be in the direction of motion) or Newtonian (force is proportional to acceleration). These models are well documented in the literature and, thus, can form the basis for multiple-choice questions about the concept.

We then select a scenario for which the models can be applied. Each physical model or mathematical concept will have several components to be probed. For example, the relation between motion and mass, force, acceleration or shape of object. With these components we design a model-based multiple-choice test, which is validated by research and can be easily implemented in large classes. The key elements in the creation process of this method include the following:

1. Based on previous research and additional student interviews (if necessary), common student models are identified and validated.

2. Multiple-choice questions that are designed to measure how student models are developed. The effectiveness of the questions is validated through research.
3. The full responses (including the wrong answers) and the context in which they were given are analyzed. The results provide explicit information on the students' state of understanding.

Model Analysis is based on both qualitative and quantitative methods. The qualitative research identifies the students' models that reflect the majority of different types of student understandings. Quantitative methods are used to analyze the multiple-choice instruments (see below). Thus, we use interviews to identify the students' actual reasoning that is common to a large population and research-based multiple-choice instruments to measure the students' use of this popular reasoning in learning. The combination of the two methods provides an effective and stable tool to probe large classes. Once a reliable package is developed, instructors with limited training can easily implement Model Analysis instruments to obtain immediate feedback from students with comparatively rich information on the students' actual understanding.

It is often argued that by putting in our knowledge we also limit the framework for student possible models. In Model Analysis, we always include an option that will allow any possibilities that may be missing when the test is designed. If a large number of students select this option, they are alerting us that some student models are not included in the original design of the test. Therefore, Model Analysis provides a set of tools that can be used to investigate different levels of student understanding and the applications of models.

Assessing Students' States of Understanding

In problem-solving situations, students are found to use their models in a variety of ways, some of which can be inconsistent. We call the collection of the ways in which students apply their models their states of understanding. With a set of questions designed for a single physics concept, we can measure the probability for a single student to use different models in solving these questions. For each student, the distribution of probabilities will be different; we use this probability distribution to represent student state of understanding. Thus, the student state of understanding is defined as a specific configuration of probabilities for a student to use different models in problem-solving contexts related to a particular physics concept. A student is considered to have a consistent state of understanding when the student uses one model consistently in responding to questions related to a single concept. A student is considered to have a mixed state understanding when this student applies different models to a situation that would seem to require identical models to explain. Mixed states can be context dependent or can seem to be random.

For an entire class of students Model Analysis enables one to make statements such as 87 percent of the students were able to apply the model correctly 64% of the time. Looking at the details of the percentage where the concept was not applied correctly provides us with detailed knowledge about student understanding.

In designing Model Analysis inventories we wish to address three questions:

1. Are the students consistent in their application of models?
2. If not, is the mixing of different models context dependent?
3. For mixed states or consistent but incorrect states, do particular variables trigger a student's choice of model?

The number of questions about a single concept determines how well we can determine the level of mixing and its context dependence. Just to detect the mixing of two physical models requires a minimum of two questions. To determine how the context is important may require many more questions. For example, the students in Schecker and Gerdes' study applied different models to a golf ball and soccer ball. A hypothesis is that they have had concrete experience with soccer balls. To investigate that hypothesis more carefully would require questions about other types of projectiles and, perhaps, several other situations. By looking at the patterns from a large number of such questions we could determine the nature of the context dependence. Therefore, to obtain robust measurement of student states of understanding, we need to include questions with diverse context settings. This approach will provide a stable measurement on mixed states, and will reduce possible bias when students may be doing well on one type of context settings and still have difficulties with other contexts.

When multiple physical features are involved, we need to make the number of questions larger than the number of physical models with that physical feature. For example, our preliminary work (Bao, Zollman, & Hogg, 2000; Bao, Zollman, Hogg, & Redish, 2000) has shown that several variables will trigger students' application of Newton's Third Law in a collision. Some students will focus on the relative size of the two objects; others, on the speed; still others on the active-passive nature of the objects, and so forth. To obtain an accurate measurement of a single student state of understanding, the number of questions needs to be larger than the number of variables.

The Model Analysis technique has clear uses in research on student learning and student development of applications of scientific models. However, we wish to move the Model Analysis approach beyond the educational research laboratory and address the question: Can this type of analysis be used effectively in real-time response and automated homework systems?

Thus, we will prepare an assessment technique that enables faculty who are using either in-class response systems or on-line homework to determine their students' states of understanding quickly and easily. Thus, we will focus on the analyses of student responses and how they can be used more effectively in the classroom and with on-line homework as well as faculty reaction to those responses and how they can become an effective part of the learning process. This part of the research will include investigating ways to present the results of the assessment with visualization tools.

Formalities Model Analysis

(This section assumes some knowledge of matrix algebra.)

Figure 1 shows a schematic of the procedures for assessing the states of understanding for a class. (Bao & Redish, 1999; Bao & Redish, 2000) We first calculate single student state of understanding by analyzing individual students' raw responses. For example, we administer a set of three questions related to one concept. Each student's state is a 1x3 matrix representing the probability of his/her using different models. A class model density matrix is obtained using all of the single student vectors. We then calculate the eigenvectors and eigenvalues of the class model density matrix.

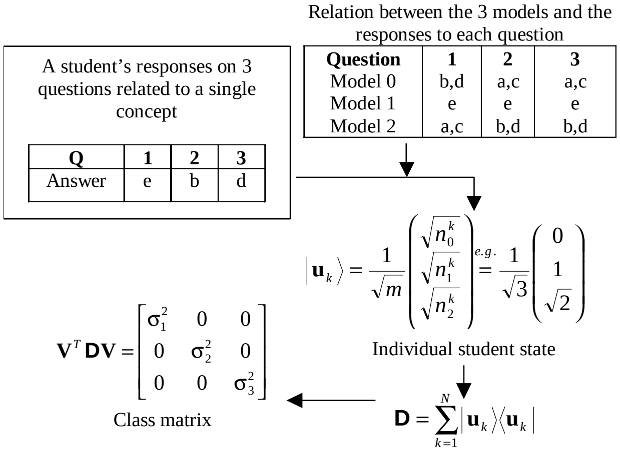

Figure 1: A schematic representation of calculating the density matrix of understanding (student models) and extracting the class' state of understanding. In this case students may hold 1 of 3 models for the concept.

The eigenvectors of a class density matrix reflect the set of unique features of the models held by the individual students in the class, whereas the eigenvalues reflect the popularity of the corresponding physical models as used by the class. When considering the shift between two common models, which is a quite common situation in many classes, we often represent the class states and eigenvalues on a model plot. As shown in Figure 2, a particular state of understanding as well as its eigenvalues can be represented with a point, e.g. "P," on a model plot. A class state with a large eigenvalue will result in a point close to the upper boundary, which indicates that a large number of students in the class have states of understanding similar to this class state. When individual students are consistent in using their models, which results in "pure" single student states of understanding, the point representing the class state of understanding will be in either Model 1 or Model 2 regions. Consistency does not necessarily imply that the model used by students is correct. When individual students are inconsistent in using their models, which results in "mixed" single student states, the point representing the class state of understanding will be in the mixed model region.

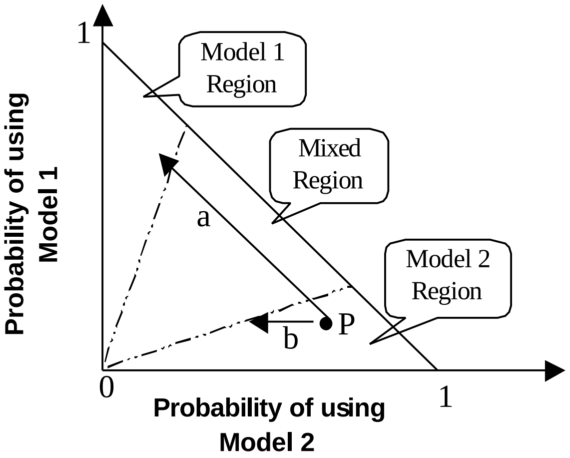

Figure 2: Graphical representation of a class' state of understanding and possible transitions (paths a and b) in that understanding.

As suggested by many researchers, the stage of mixed states of understanding is often a significant intermediate step for a complete, favorable concept change. (Thornton, 1994; Vosniadou, 1994) However, such information is difficult, if at all possible, to obtain with score-based assessment or by counting the frequency of students' using different models. For example, frequency counting only provides the percentages of the class' responses associated with the different models, which are the diagonal elements of the class model density matrix. It does not, however, give any information on whether the individual students in the class are consistent in using their models. Such information is stored in the off-diagonal elements of the density matrix and is reflected in the eigenvectors. Therefore, Model Analysis can extract this information from students' responses on multiple-choice questions. As indicated from research, the initial states of students often show a consistent incorrect model and therefore can be represented with point "P" in the Model 2 region (see Figure 2). When the students are experiencing an appropriate conceptual change, the class states of understanding usually follow a path such as the one shown by *a* in Figure 2. On the other hand, when students are not going through appropriate conceptual changes, the class states often move along other paths, such as *b*.

Model Analysis provides a convenient tool to analyze the content of student understandings. It allows the assessment on the probabilities of students' using alternative ideas in problem-solving situations. With Model Analysis, the state of student conceptual understanding can be quantitatively evaluated which provides explicit feedback to instructors and researchers on the states and changes of student conceptual understandings in education environments.

Model Analysis and Factor Analysis

Factor analysis is a popular tool in education research. It can give the correlation of student behavior on different test items; however, the results are usually based on student scores or the correct responses to a set of questions. As discussed above, focusing on correct response and/or scores often fails to provide complete information on students' states of conceptual understanding. Further, because of the context dependence of student reasoning, students can produce inconsistent responses on questions related to a single concept and considered equivalent by experts. Thus, they may answer some items related to a concept correctly and others incorrectly. In such cases, factors might not show a connection to conceptual understanding.

Unlike factor analysis, which tries to draw the components of student reasoning, Model Analysis uses information from qualitative experiments into the analysis to determine the state space. Therefore, Model Analysis provides a set of tools that can be used to investigate different possibilities for the students' states of understanding and how those states are related to accepted scientific models.

With factor analysis, even when a factor is found, the physical interpretation for such a factor is still uncertain and not unique. With Model Analysis, the possibilities of student models are identified through qualitative research. This can provide a clear physical meaning for the student states extracted from the analysis.

The following example demonstrates how these two methods are different. Suppose we give 4 multiple-choice questions to a class of 100 students ($m = 4$, $N = 100$). All four questions probe understanding of a single physics concept. We have learned from previous work that students use one of two models -- Model A and Model B -- when explaining situations related to this concept. Consider two situations:

- Case 1: All students in the class are self-consistent. Half of them use Model A on all four questions, and the other half use Model B.
- Case 2: All students' applications of the models are context dependent. They use Model A and Model B equally, so each student applies Model A to two questions and Model B to the other two.

In case 1, the results from Model Analysis will give two states with equal weights indicating that the class has two groups, each consistently using one of the models. The results from factor analysis will give a single factor, which shows that all the students are consistent. Because the students give either all correct answers or all incorrect answers, everyone is consistent in producing scores. However, the result from factor analysis does not determine in which way the students are being consistent. The same factor can be obtained when the students are consistent regardless of the number of students who respond to the questions correctly or incorrectly. Therefore, in this case, factor analysis can only provide limited and indirect information on student understanding of the concepts.

When the individual students are inconsistent as in case 2, factor analysis often fails to provide useful information. When students are equally mixed between two models, the probability for a single student to use either Model A or Model B is equal for all of the questions. Then, the results from Model Analysis will show a single mixed class state. On the other hand, since the students are inconsistent in answering the questions, factor analysis will give insignificant random-like correlation between different questions and show no dominant factors. This example shows that score-based factor analysis cannot deal with the randomness in students' use of conceptual models.

Methods of Investigation

Both Kansas State and Ohio State Universities have much of the necessary hardware and software to complete this research. Both universities have installed computerized response systems (Classtalk) in the large classrooms that are regularly used by their Physics Departments. To be able to collect research data in smaller classrooms we are requesting funds to purchase portable wireless response systems. In the trigonometry classes at KSU, the Mathematics Department uses a homework submission system that enables students to include some graphical responses and functional dependencies as well as simple answers. In addition, KSU maintains an on-line teaching environment that enables responses to multiple-choice and textual questions. Thus, we can build on an existing foundation and only need to add a small amount of hardware and modify some present software in order to create the system upon which our research will be completed.

Developing the Model Analysis tools will begin with the basic structure that has been created by Bao and Redish. (Bao & Redish, 2000) In physics this structure will be greatly enhanced by the work of Minstrell who has developed facets for most of the concepts that are taught in a traditional introductory physics course. In mathematics we will rely on similar research and develop the equivalent of facets for a topic. In addition, we have available a large number of concept tests which are not model based. (Beichner, 1994; Fekete, 2000; Hestenes & Wells, 1992; Hestenes et al., 1992; Marx, 1999; O'Kuma, Maloney, & Hieggelke, 2000; O'Kuma, Maloney, Hieggelke, & Heuvelan, 1998; Sokoloff, 1993, 1997; Thornton, 1992; Thornton & Laws, 1995, 1998a, 1998b; Thornton & Sokoloff, 1995; Thornton, 1994) When necessary, we will conduct additional interviews that will probe student understanding and carefully investigate situations and contexts in which students are able to correctly apply the model and those in which they have difficulty applying the model. Of course, we will not have a situation in which we can say all students can apply Model A in context X and not in context Y. Instead, we will understand better the mixed state in which the model, the context, and the student background will all influence how each student is effective in understanding the application of the physics or mathematics.

Based on all of this work we will create a series of questions that can be presented with an automated real-time response system or on-line homework. Each question will probe student understanding of an aspect (or a facet) of student understanding. Additional software created as part of the project will enable faculty to quickly assess the state of their students and see in what mixture "pure intellectual understanding states" they are.

An important component of the research will be to develop different representations both of individual student's understanding and of a class' understanding. The mixed state of student and class understanding must be presented in a way that faculty can quickly recognize the result and develop alternative teaching-learning strategies. The representations can be built on the research into visualization of complex data sets such as the work of Ben Shneidermann.(Card, Mackinlay, & Shneiderman, 1999; Shneidermann, 1998) Better Education, Inc., manufacturers of Classtalk and distributors of the Personal Response System, has agreed to help in the process of converting their computer code to be compatible with the Model Analysis approach. Thus, we will be able to create various visual representations relatively easily and analyze their usefulness through research with teaching faculty who use the Model Analysis system.

Because we will be applying our analysis tools to both physics and mathematics courses, we can also look at the transfer of knowledge between the two subjects. Teachers of mathematics frequently use concepts from physics to demonstrate the value of a mathematical solution. (For example, simple oscillations are demonstrated using a spring and a weight when students study second order differential equations.) Ample antidotal evidence tells us that students do not make the connection between the same concepts as presented in different courses. With our method of analysis we will be able to focus on components of the learning and try to determine which facets are blocking the transfer.

The central focus of our research effort will be the refinement of the Model Analysis approach and its combination with contemporary technology to develop a deeper understanding of student learning. We realize that this approach to analysis and assessment of student learning can be somewhat abstract for the average teaching faculty member. Another thrust of our research will be to uncover means by which the analyses can be made understandable and useful to a broad spectrum of teaching faculty, including those who are not well versed in the methodology of physics education research. Thus, we intend to complete fundamental research on student learning and build better models of the learning process as well as create a system that can be used effectively with emerging technologies such as in-class, real-time responses and on-line homework.

Contributions to the NSF's Goals

This research will increase our knowledge on student conceptual learning processes in classroom settings and create new strategies for using contemporary teaching technologies. Research on ways to assess students' knowledge state, the development of model-based assessment tools, and ways to present this information to teaching faculty will improve the teaching environment and give instructors a new means to help students learn difficult concepts. This project will also increase our knowledge about the use of in-class and web-based response systems and provide guidance to their development. Thus, this research is consistent with several of NSF's education goals including

- making high quality science and mathematics education available to every child in the United States,
- ensuring that the instructional workforce has the disciplinary and pedagogical skills necessary to provide an excellent education to every student in science and mathematics, and
- ensuring that those who select careers in science, mathematics or engineering have the best professional education at the undergraduate and graduate levels.

As stated in the ROLE announcement, many existing educational approaches and tools were developed without the benefit of a strong research foundation. Therefore, the main goal of this project is to conduct research on the student application of models in physics and mathematics. These pedagogical issues can form the basis for further development of the emerging classroom response and on-line homework technologies.

This project also fits many NSF research goals: It can significantly "increase our knowledge of student conceptual learning in complex education settings" that are equipped with modern teaching technologies or low-tech variations of them. We will "develop research-based learning tools, pedagogical approaches, and materials that enhance SMET education at all levels" and "conceptual learning assessment instruments and their effective utilization." The refinement of the quantitative Model Analysis will increase the number of ways to use methods for assessing student learning in science and mathematics.

Advanced Technologies

We will be working with contemporary technologies that are used in assessing student understanding either in real-time classroom environments or through on-line homework submission. We will not be developing any major new hardware or software for these environments. Instead, we will complete research which will help us better understand how students process knowledge as they try to apply newly learned scientific models to a variety of contexts and how they transfer knowledge from one course to another. Our research will also result in better ways to represent the students' state of understanding to teaching faculty so that they can quickly make assessments at a deeper level than whether they know the answer.

Impact on Learners

This research will provide us with a better understanding of students' application of physical principles. Development that is based on the research will enable teachers and researchers to analyze more completely the state of student understanding and the contexts in which they are able to use or not use their knowledge. The focus on using contemporary technology to complete this type of analysis provides a means to accomplish this type of effort for relatively large groups of students. Thus, the teachers will have a much better and deeper understanding of both the students' successes and difficulties while learning physics and mathematics. In turn the teachers will be able to adjust the teaching-learning process to address the difficulties and build on the successes.

The immediate impact will be on learning physics and mathematics, primarily at the university level and on the development of the next generation of technology for in-class and on-line response systems. The longer-range impact could be on similar efforts in all of the sciences and perhaps on an even wider range of disciplines.

Personnel Qualifications and Management Procedures

Dean Zollman, Professor of Physics and Distinguished University Teaching Scholar, will be the principal investigator for this project. He will be responsible for the overall management of the project and for overseeing the research design and analysis. His efforts will include coordination of the research on students' states of understanding in physics with those in mathematics and for the coordination of the efforts at the two universities. Prof. Zollman has over 27 years experience in developing innovative educational activities and projects for the teaching of physics and for teacher preparation and enhancement. He is a highly regarded member of the science education community and has received a number of awards for his teaching, research and development in physics education, and development of innovative software. He has also managed several other projects that are equal in magnitude and complexity to the one proposed here. As discussed in the Results of Prior Support at the end of this proposal, these projects have come to a successful conclusion and most of them are now available as commercially distributed products by the private sector. As part of each of these projects he and his co-workers have completed research on understanding and learning of a variety of topics in physics. Subjects of this research have included students and in-service teachers.

Andrew Bennett, Associate Professor of Mathematics, will be the primary investigator for the Model Analysis in the on-line homework in mathematics and in conducting research on the transfer of knowledge between mathematics and physics. He received his Ph.D. in 1985 from Princeton University. He has won teaching awards at both Kansas State University and the University of Texas at Austin. For the past eight years he has served on the planning committee for the Kansas City Regional Mathematics Technology Expo, an annual conference where mathematics teachers from colleges and universities, community colleges, and high schools share information on teaching with technology. He is also past-chair of the Kansas section of the Mathematical Association of America.

Dr. Kirsten Hogg, Research Associate at Kansas State University, received a PhD in physics education research from the University of Sydney in 1999. Her doctoral research investigated students' understanding of wave phenomenon, particularly interference and diffraction. Since completing her degree she has been a post-doctoral research associate with the physics education research group at Kansas State University. During the past year she has worked with future teachers in two physics courses and has completed research on their understanding of physics and their confidence to teach the subject. At present she is implementing a Web-based distance education course that will offer in-service teachers a means to update their knowledge of quantum science. She will work with Prof. Zollman on the research and development in the physics classes at KSU and with Prof. Bennett on the issues of knowledge transfer between physics and math courses.

Lei Bao, Assistant Professor of Physics at Ohio State University, received a Ph.D. in 1999 from the University of Maryland. His dissertation research investigated student understanding of some aspects of quantum physics and included the beginnings of the Model Analysis approach. Since completing his degree he has been a post-doctoral research associate with the physics education research group at Kansas State University. He has continued work on developing the Model Analysis approach and has collected data, using a paper and pencil inventory, from future elementary school teachers who were enrolled in a physics course at KSU. In addition he has developed instructional materials for teaching quantum physics to second-year physics majors. Dr. Bao will have primary responsibility for the research that will be conducted at Ohio State. He will work closely with Prof. Alan van Heuvelan, leader of the physics education research group at OSU, to select appropriate classes in which to conduct the research. He will also take on major responsibility for refining the Model Analysis approach.

Fedor Andreev, mathematics education post-doctoral research associate, will implement the data gathering approach for the on-line homework and classroom response systems. His background includes Web-oriented learning. Working with Prof. Bennett, he has written on-line quizzes for differential equations and trigonometry. He has all required knowledge in programming languages, server-side scripts, databases and Internet technology.

Graduate students in physics and mathematics education will work under the direction of Professors Bao, Bennett and Zollman at both KSU and OSU. In addition to the two research associates described above, a part-time research associate at OSU will be added to the project. Thus, we will provide experiences for the next generation of SMET education researchers.

Project Timeline

The starting date for this research project is January 1, 2001, with duration of three years. During the first year of the project, we will concentrate on refining the Model Analysis approach and creating a system for using it in classrooms. Simultaneously, we will review all of the existing concept inventories in physics and mathematics to ascertain how they can be modified for our research. Once this initial work is completed, we will focus on creating teaching strategies that can provide faculty with the results of Model Analysis in a manner that is not a significant intrusion into their classes. As part of the research, faculty at both Kansas State and Ohio State will use the system during the second year of the project. At that time we will take initial data on how students' states of understanding and applications of physical models change during instruction. By including a variety of instructors at two different universities in mathematics and physics, we will obtain data that should be generalizable to many other teaching-learning environments. The data will also enable us to see ways to modify the questions, software and other aspects of the system to improve both the research and delivery aspects.

In developing the Model Analysis inventories, we will begin with mechanics in physics and trigonometry in mathematics. The use of vectors provides the link between the two topics that will enable us to study the transfer of knowledge. As the project continues into the second year we will create inventories for other topics such as wave motion, electromagnetism and more advanced mathematics so that instructors will be able to use the results of our research throughout introductory courses. In the final year of the project we will offer one or more workshops at professional meetings to introduce faculty to the research-based system that we have developed and to encourage them to try it in their classes. An approximate schedule for the project is displayed in Figure 3.

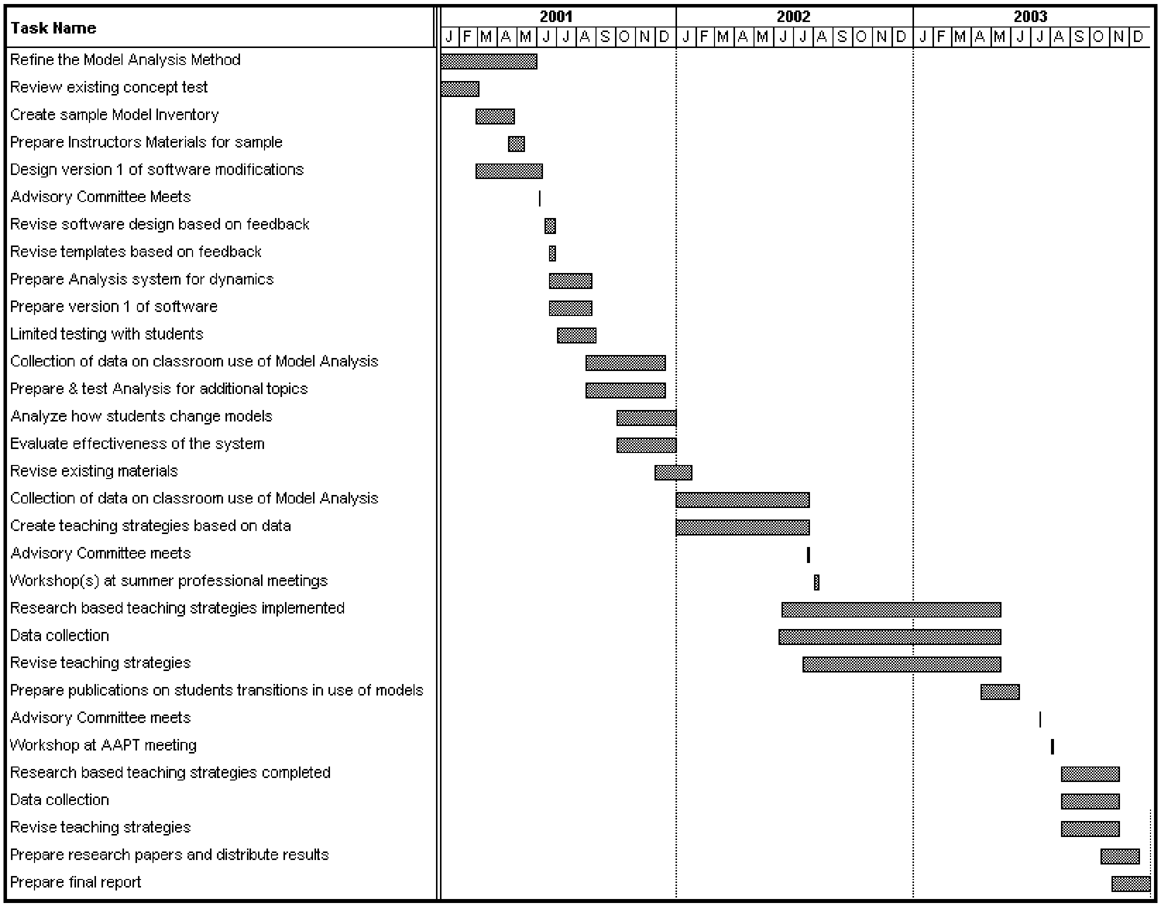

Figure 3: Approximate timeline for major tasks in the project.

Advisory Committee

The project’s Advisory Committee will review all aspects of the project development and provide advice on direction for our efforts. We plan to have annual meetings of this group so that they can review our efforts and offer timely advice. The members of the committee have been selected so that they will bring expertise in physics education, mathematics education, the application of cognitive science to physics and math education, assessment, and affective issues facing students of math and physics. The people who have agreed to serve are described below.

Genevieve Knight is Wilson H. Elkins Distinguished Professor for the University System of Maryland and Professor of Mathematics at Choppin State College. Dr. Knight is a winner of the Distinguished Teaching Award from the Maryland-District of Columbia-Virginia section of the Mathematics Association of America and a former Governor of the Association. She has received a lifetime achievement award from the National Council of Teachers of Mathematics. Her areas of specialization are mathematics education and preparation of mathematics teachers. She has long been an advocate for equity for women and minorities in mathematics and mathematics education.

Jose P. Mestre is a professor of physics and a member of the Scientific Reasoning Research Institute at the University of Massachusetts – Amherst. He is leading Assessing to Learn Physics, which is to study the implementation of continuous, formative assessment in high-school physics classes. Students who use the Assessing to Learn materials are actively processing questions and tasks, thinking about ideas, communicating with other students, and reflecting on their own ways of thinking. His recent papers include “Cognitive aspects of learning and teaching science” (Mestre, 1994) and “Minds On Physics: Materials to develop concept-based problem solving skills in physics.” (Gerace, Dufresne, Leonard, & Mestre, 1999)

Edward F. (Joe) Redish is a professor of physics at the University of Maryland, College Park. Since 1982 he has been actively involved in the subject of physics education and since 1992 has devoted his entire research effort to physics education. Among his recent publication are “Implications of Cognitive Studies for Teaching Physics”

(Redish, 1994) and "New Models of Physics Instruction Based on Physics Education Research." (Redish, 1996) He has received numerous awards for his work in physics education including the 1998 Robert A. Millikan Medal from the American Association of Physics Teachers. (Redish, 1999)

Elaine Seymour is the director of ethnography and evaluation research at the Bureau of Sociological Research, the University of Colorado, Boulder. She has also served as senior scientist at the National Institute for Science Education and the University of Wisconsin, Madison. Her research unit has focused on change in the education and career paths of undergraduate and graduate science, mathematics, and engineering) majors. Her recent book, *Talking about Leaving,*(Seymour & Hewitt, 1995) describes a major national study of reasons why undergraduates leave the sciences, math, and engineering. Her research on reform of undergraduate chemistry has led to the development of web-based student learning assessment tools. (Seymour, Wiese, Hunter, & Daffinrud, 2000)

Results for Prior NSF Support

Visual Quantum Mechanics, ESI-9452782, 1996-2000, $807,258

Professor Zollman was involved in the *Visual Quantum Mechanics* grant that recently concluded. This project is the one that has been completed most recently. It is related to the proposed project in that *Visual Quantum Mechanics* involved the creation of instructional materials, including visualization software and working closely with secondary teachers to understand how they could best use the materials.

The *Visual Quantum Mechanics* project applied physics education research, contemporary ideas on pedagogy, and instructional technology to develop, test and distribute teaching and learning materials about quantum science. These instructional materials combined traditional paper and pencil activities and hands-on activities with inexpensive equipment and interactive computer visualizations in an integrated package. By emphasizing experimentation and visualization rather than mathematics, the project was able to help students who would not otherwise be able to learn quantum mechanics come to an understanding of the basic concepts. The *Visual Quantum Mechanics* instructional materials are divided into instructional units, each of which requires between 8 and 12 hours of class time. The prerequisites for each unit have been kept to a minimum so that the instruction on contemporary quantum science can be included throughout the academic year rather than at the end of the study of physics as is done in a traditional physics course. By creating short instructional units we also provide teachers with the opportunity to maintain most of their present content and include some contemporary physics.

The complete instructional units were aimed at high school students who would not normally have the opportunity to learn about quantum mechanics. However, the completed products have proven to be rather versatile with some of the materials being used in middle school physical science classes and others in advanced undergraduate physics classes.

Field test versions were distributed to all interested teachers who agreed to use some of the materials in their classrooms. Field tests were conducted at high schools and colleges throughout the United States with the feedback being used to improve the teaching and learning qualities of the materials. Research on student use of the materials and their learning was also conducted during the field tests. Overall, the field tests indicated that the materials were able to reach their cognitive and affective goals as well as foster collaborative learning among students.

During the course of the project, several high quality interactive computer programs were created to help students develop and visualize some of the models used in contemporary physics. These models ranged from an energy level view of the atom to using potential wells to see how quantum mechanics leads to energy bands and gaps to a quantum mechanical view of the scanning tunneling microscope. In the past five years, this software has received seven awards from the American Institute of Physics for its quality.

To introduce teachers to both the pedagogy and the content, the project staff conducted a large number of workshops at professional science education conferences and meetings. At these workshops teachers were presented a "sampler" of the *Visual Quantum Mechanics* materials. These presentations were hands-on activities conducted in much the same manner as a *Visual Quantum Mechanics* classroom would be conducted. To reach teachers who are unable to attend professional meetings, the project collaborated with similar efforts at Ludwig-Maximillians University in Munich to develop a series of Web sites for distance education. Thus, a variety of means have been used to provide teachers with an introduction to *Visual Quantum Mechanics* and its pedagogical techniques.

In addition to the Web sites, the product of *Visual Quantum Mechanics* is a teaching package that includes student study sheets, a teachers' guide, inexpensive hardware, and a CD-ROM. Kansas State University is now in the process of seeking a commercial outlet for these materials.

Extensive testing of the instructional materials has been undertaken at high schools, community colleges and universities throughout the country. The units are being used in secondary schools and colleges throughout the U.S.

and in a few other places. We do not know all the places that it is being used because all the material has been on the Web, and people download it. We know that we have given materials to people in Southeast Asia and throughout various parts of Europe as well as the U.S. Most of our reports, however, have come from the U.S.

Approximately 175 different teachers in 160 different schools have used the materials in classes and reported results back to us. In addition, as part of the evaluation process members of our staff observed the teaching of the materials in a number of schools and a few college classes. All data (self-reported and observational) from the evaluations have been tabulated.

Students' attitudes toward these materials are very positive. They frequently make comments like, "I really like this better than our regular physics. Can we keep doing it?" Our observations indicate that in addition to being positive, the students are interactive with the materials and each other. End of unit tests based on the learning objectives indicate that they seem to be learning. Most of the teachers are also positive. A few are not. Many teachers who were willing to try the materials did not have a very strong background in quantum mechanics. Even though we are approaching quantum mechanics in a much different way than it is normally taught, some teachers still feel uncomfortable. Building the teachers' confidence is very important. Our continuing efforts to build a teacher education Web site (in collaboration with the group in Munich) are addressing that issue.

Some interesting observations came from teachers who used the materials in a demonstration mode. We had some teachers who decided that it is too much trouble to have the students work in a hands-on mode with all of these programs. So, they just show the programs to the students. In these cases learning went down; attitudes went down; everything went down. Hands-on activities make a difference. Of course we should not be surprised because we built the material for the students to use, not for the teacher to talk about.

Human resource development during the Visual Quantum Mechanics project was substantial. Two post-docs (Rebello and Thoresen) and two graduates students (Escalada and Gruner) now hold tenure track positions where they are expected to complete research in physics education. Another graduate student is a high school teacher. Two international visitors (Jolly and Euler) have continued research and development related to the teaching of quantum mechanics after they returned to India and Germany, respectively.

Overall, *Visual Quantum Mechanics* met its goals. We created interactive teaching materials on an abstract topic. These materials have had a positive effect on both student attitudes toward science and their learning of modern physics. The materials are now being used in a variety of physics related classes and several different levels of instruction. Commercialization of the instructional package will increase its distribution and availability.

The following publications in refereed journals or proceedings of conferences were a result of this grant.

N.S. Rebello, K. Sushenko and D. Zollman, “Learning the Physics of a Scanning Tunneling Microscope Using a Computer Program,” *European Journal of Physics* ***18***, 456-461 (1997).

L.T. Escalada, H. P. Baptiste, Jr., D. Zollman and N.S. Rebello, "Physics for All," *The Science Teacher* ***64***, 26-29 (1997).

N. S Rebello, C. Cumaranatunge, L.T. Escalada and D. Zollman, "Simulating the Spectra of Light Sources," *Computer in Physics* ***12*** (1) 28-33 (1998).

P. Jolly, D. Zollman, N.S. Rebello and A. Dimitrova, “Visualizing Motion in Potential Wells,” *American Journal of Physics* ***66*** (1), 57-63 (1998).

D. Zollman, “Hands-on Quantum Mechanics,” in *Hands-on Experiments in Physics Education* (G. Born, *et al*., eds) GIREP, Duisburg, Germany (1999).

R. Unal & D. Zollman, “Students’ Views of the Atom,” *Journal of Physics Education Research,* submitted 1999.

D. Zollman, “Quantum Mechanics for Everyone: Can It Be Delivered Through Technology?,” *Simulation and Multimedia in Engineering Education* pp 3-9, (H. Vakilzadian & C. Wie, eds.), Society for Computer Simulation, San Diego, 2000.

D. Zollman, “Using IT for Physics: Examples and Applications” in *Handbook on Information Technologies for Education and Training* (H. Adelsberg, B. Collis & J. Powkowski, eds). Springer-Verlag, Berlin, to be published, October, 2000.

In addition, during the course of the project, the staff received seven awards for outstanding educational software from *Computer in Physics*, a publication of the American Institute of Physics. Work related to the *Visual Quantum Mechanics* project resulted in three Masters theses and one Ph.D. dissertation.

Beginning in June 2000, staff at the Weizmann Institute in Israel will translate *Visual Quantum Mechanics* to Hebrew and adapt it for Israeli schools. More information about the results of this project, including the ability to run most of the software on-line, is available at http://www.phys.ksu.edu/perg/vqm/.

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