Understanding Math Classroom Affordances of Networked, Hand-Held Devices

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SUMMARY

Prior SimCalc work exploited the representational affordances of the computational medium to help democratize access to the core ideas leading to and underlying Calculus, important ideas that had previously been sequestered in a capstone course reached by at most 10–12% of the population. Simulations, new graphical ways of creating and editing functions, their derivatives and integrals, and dynamic visualization tools together enabled a reconstituting of these ideas into core curriculum beginning in the middle grades. We now propose to study the profound potential of combining the previously established *representational* innovations with the new *connectivity* affordances of increasingly ubiquitous hand-held devices in typical classrooms. In combination with the representational affordances and the availability of powerful, robust and inexpensive hand-helds, we see classroom connectivity as a critical ingredient to unleash the long-unrealized potential of computational media in education, because its effects are direct and at the communicative heart of everyday classroom instruction– but only if those effects are sufficiently understood to inform iterative improvement of technologies and classroom practices that support and do not impede learning, as well as design of teacher development and support structures.

With the support of three major corporate partners, Texas Instrument, Palm, and Nokia, we will work with teachers in ordinary grade 8–12 classrooms equipped with school-standard graphing calculators and newer devices wirelessly networked to each other and to a teacher's workstation. We will examine three Opportunity Spaces generated by classroom connectivity. (1) Assessment: principled diagnostic assessment routinely embedded in instruction based on students sending their responses to carefully designed thought-revealing probes and problems to the teacher for analysis and action. (2) Learning: new and highly engaging student activity structures involving (a) teacher-student and student-student challenges, and (b) student contributions to shared and publicly displayed constructs (e.g., students creating velocity functions on their hand-helds that, when uploaded, control their character in a class marching parade or dance). (3) Teaching: teacher classroom management support for distributing and collecting student work, viewing & annotating student screens, and generally managing the flow of information in the connected classroom.

We will use design experiments to produce a series of classroom-grounded case studies that embody theoretical frameworks and carefully structured accounts of classroom phenomena. These are intended to help guide further design and development as noted, as well as to support further research. The work will take place primarily in ordinary grade 8–12 classrooms taught by teachers in MA and CA. We will deliberately vary the experience of the initial teachers, and, over time, vary the technology platforms. We will study affordances and constraints at three different time-scales and levels of detail: (1) carefully-designed and closely observed 1–2 week teaching experiments common across sites; (2) semester-long observations of teachers and classrooms based on occasional visits, teacher reflective journals, and structured debriefing-interviews of teachers; and (3) longitudinal change of the both the teachers and the technologies as they mature together over the life of the project, including comparisons with novice teachers introduced in YR 3.

PRIOR WORK

Over six years, our joint work under the banner of the SimCalc Project has pursued a vision of exploiting the novel representational affordances of computers to enable many more learners to master increasingly advanced mathematical concepts. Over time, this work evolved from early formative design experiments to an increasing concern with cost-effective and affordable implementation with measurable effects. Now, a new emerging capability of hand-held devices, wireless connectivity, is becoming available. Within this capability, we see equally fundamental affordances for restructuring classroom patterns of activity and communication so as to deepen and extend mathematical learning possibilities for all children. Thus we now propose to extend our prior work in classroom-based design experiments that exploit wireless connectivity.

We begin the proposal below with an extensive review of SimCalc's prior work, for several reasons. First, it illustrates how our work engages and helps define historical trends and fundamental driving forces that situate mathematics teaching and learning. Second, it clarifies the mathematical content central to our investigation. We argue that this mathematics, while stretching the bounds of today's curricula, will become increasingly important through the new century. Third, it places the present proposal in a multi-year, multi-project vision. We presently propose early-stage, focussed, formative, design experiment research, as this is most appropriate given the novelty of technology and paucity of existing relevant research. We anticipate this work setting the stage for others' projects and products that utilize additional methodologies to increasingly address the translation of these opportunities into scaleable, implementable innovations with important, documented effects.

The Time-Scale and Deeply Institutionalized Form of the Challenge

The initial objective, addressed in the first SimCalc project (RED-9353507, $,1993–96), was to democratize access to the Mathematics of Change & Variation ideas underlying calculus using a combination of new representations, links to simulations, and new curriculum materials for grades 6–13. The outcome was a series of Mac-only software and curriculum materials together with proof-of-concept data that showed that students in these grades, including those who are least advantaged, could indeed learn these ideas if given appropriate instruction and access to the appropriate technology (Roschelle, et al., 2000). The second round of SimCalc project funding (REC-9619102,$3,079,982, 1997–2000) turned towards questions of large scale implementation: What kinds of technologies, curricula, pedagogical structures, teacher development and support, assessment practices, and implementation strategies, both local and national, including partnerships with the private sector, can enable the large scale infusion of our approaches?

We have come to recognize that we are confronting deeply institutionalized views of content, curriculum and representational forms, including and especially algebraic forms that were centuries in the making. The Mathematics of Change & Variation (MCV) began with geometrically oriented and very narrowly defined efforts by the Greeks, and then revisited as part of a much broader effort to mathematize change in the world by the Scholastics in the 1300's. In the centuries that followed, these attempts were pursued by the intellectual giants of our civilization and led to profoundly powerful understandings of the different ways quantities can vary, how these differences in variation relate to the ways the quantities accumulate, and the fundamental connections between varying quantities and their accumulation. Together with new symbolically mediated norms of argument and new relations between mathematics and experience, these efforts concomitantly gave rise to the development of such extraordinary representational tools as the algebraic symbol system and coordinate graphs, and the eventual formalization of such basic mathematical ideas as function, series, limit, continuity, etc. (details of this history are reviewed in Kaput, 1994). *All this work took place within the necessary semiotic constraints of static, inert media, and largely without regard to learnability outside the community of intellectual elite involved.* Both of these factors have changed, with the advent of dynamic, interactive media and socioeconomic shifts that require the broad population to learn the MCV. A third profound change is a shift in the nature of mathematics and science towards the use of computationally intensive iterative and visual methods that enable entirely new forms of dynamical modeling of nonlinear and complex systems previously beyond the reach of classical analytic methods– a dramatic enlargement of the MCV that will continue in the new century (Kaput & Roschelle, 1998; Romberg & Kaput, 1999). This is the context of the SimCalc Project.

The animus driving the SimCalc Project comes from another direction. Over the past two+ centuries the mathematical community's potent tools, methods, and products– the foundations of the science and technology that we utterly depend upon– were not only institutionalized as the structure and core content of school and university curricula and taken as the epistemological essence of mathematics, but became the measure by which academic success was defined. Mastery of these prerequisite tools became the gateway to all that academic success offers. These views are more recently institutionalized in standards for curricula and instructional materials, pedagogical practices, relations between layers of schooling (including the MCV largely sequestered in a capstone calculus course intended for 10–12% of the population), the nature of pre-service teacher education, the filtering processes of standard assessment practices, rapidly spreading accountability practices, and even in the ways educational policy as a discipline constructs this kind of interlocking universe of belief, practice and institutions (Popkewitz, 2000).

The Dual Nature of Our Response: Implement Specific Innovations & Infiltrate Widely

We have come to understand that the appropriate long term response to this multidimensional challenge cannot *only* be to develop and then try to implement specific instructional materials and software for various grade levels, appropriate teacher-support and professional development materials, assessment support, etc. As described below and in our publications, with a growing set of partners we are working on all this. However, another kind of response is also needed, one that infiltrates, integrates and transforms, almost in a viral infection-like manner. Therefore, we are infusing the representational innovations and simulations into other software systems, moving to infiltrate existing, widely used technology such as graphing calculators, creating new assessments to enlarge the pool of available items to include those in the MCV (in a separate IERI planning proposal led by Jeremy Roschelle), building and testing materials for teacher education and teacher-educators, and planning to exploit newly emerging connectivity technologies at their inception– which is what this proposal is all about. We are also working towards the extension of MCV content to 21st century dynamic mathematics and to understand learning of this mathematics in ways that include attention to neurophysiology and the phenomenology of this new learning by high school students and pre-service teachers (another ROLE proposal, led by Ricardo Nemirovsky). Our infiltration-integration approach extends to technology design, including the use of highly configurable software that can be modified by different levels of users ranging from programmers to teachers to fit a variety of use-contexts. This also drives our intention to port aspects of the software to as many device-types as possible. It is reflected in our attention to interoperative components (Roschelle & Kaput, 1996; Kaput, et al., 1998) that led to the ESCOT Project (REC #9804930; http://www.escot.org). This same approach defines our curriculum materials, which aim to integrate with and to transform existing core curriculum ideas while simultaneously building the MCV ideas underlying calculus, and which was also the approach taken in the MCV strand of the Investigations elementary school materials (Russell, et al., 1995).

Another reflection of our approach is adherence to "open architecture" (Roschelle, et al., 1996), including open *project* architecture whereby we have functioned as much as a center than as a project, sharing ideas and supporting collaborators and subcontractors wherever possible (especially less established researchers), building an open digital library of instructional and assessment resources, and working to ensure that other researchers can build upon the work we have begun. Indeed, we have embraced multiple views on education and research, ranging from randomized experimental design to open-ended, deliberately non-evaluative clinical work directed towards identifying the phenomenological aspects of student experience with the MCV and certain explorative devices. Similarly, we have worked to influence the evolution of national standards to include MCV ideas– see the new NCTM *Principles and Standards for School Mathematics* (NCTM, 2000), particularly the Technology Principle and the Algebra and the Representation Standards (to which the PI contributed). We also expect, in the future, to develop materials for administrators, parents, and for after-school programs. Finally, the need to reach multiple audiences in a sustainable way led us to partner with the university to form a commercial entity, SimCalc Technologies, LLC, in order to create (non-exclusive) commercial partnerships. The first of these involves Texas Instruments (TI), whose graphing calculators have become the standard in U.S. schools.

Six Core Representational Innovations to Support Reconstituted Content

Because they change *conceptual* infrastructure, some of the deepest changes in culture generally, and mathematics in particular, are representational– consider the placeholder system for numbers, algebraic notation, or Cartesian graphs, for example (Kaput, 1999a, 1999b). Thanks to the computational medium, over the past two decades the character string approaches to the MCV have been extended to include and to link to tabular and graphical approaches, yielding the "the Big Three." However, almost all functions continue to be defined and identified as character-string algebraic objects, especially as closed form definitions of functions– built into the keyboard hardware– and the overall structure of the curriculum remains unchanged. By contrast, our core web of representational innovations, which require the computational medium for their realization, are support for:

- Definition and direct manipulation of *graphically defined* functions, especially piecewise-defined functions, with or without algebraic descriptions. Indeed, many of the naturally arising and easily defined functions are very difficult to describe algebraically.
- Direct, hot-linkable connections between graphically editable functions and their derivatives or integrals– bringing the most fundamental ideas of calculus to the center of our design and to our longitudinal curriculum organization. Traditionally, connections between descriptions of rates of change (e.g., velocities) and accumulations (displacements) were mediated through the algebraic symbol system as serially executed derivative and integral procedures via formulas defined by the algebraic forms of the functions involved, and "linked representations" referred to representations of the *same* function.
- "Snap-to-grid" constraints on direct graphical manipulation of functions, allowing a new balance between model complexity and computational tractability. Hence we are able to introduce sufficient variation to model interesting situations, avoid the degeneracy of constant rates of change, while controlling (but not ignoring!) the messiness and conceptual challenges of continuous change.
- Direct linkage between the above representational innovations and simulations, especially motion simulations, to allow immediate construction and execution of a wide variety of variation phenomena, allowing us to expand the "Big Three" to the "Big Four" so that the mathematics is *about something!*
- Dynamic visualization tools to support students' ability to interpret change and accumulation, e.g., highly manipulable slope, area, and trace tools integrated with simulations.
- Two-way connections between physical data and cybernetic data, especially motion-data. These enable Computer-based Lab importing of physical data for mathematical representation, analysis, and reenactment in simulations; and exporting of cybernetic data to the physical realm, whereby mathematical functions defined via any of the above methods as well as algebraically, can be used to drive physical phenomena, e.g., cars on tracks.

At the experiential level, the result of these representational innovations is a qualitative transformation in the mathematical experience of change and variation. In less than a minute, using either rate or totals descriptions of the quantities involved, or even a mix of them, a student as early as middle school can construct and examine a variety of interesting change phenomena that relate to direct experience of daily phenomena with real conceptual change, including students from especially challenging home and school environments (Schorr & Goldin, in press), or bilingual situations (Bowers & Doerr, 1998). And in more extended investigations, newly intimate connections among physical, linguistic, kinesthetic, cognitive, and symbolic experience become possible– as described in a series of project publications (Bowers & Nickerson, 1999, in press; Nemirovsky, et al., 1998; Nickerson & Bowers, 1999; Noble, et al. in press; Kaput, & Roschelle, 1997; Roschelle, et al., 1998). Interestingly, with appropriate and comparable instruction we see middle school students achieving at similar levels to high school students on topics such as periodicity (Nickerson, et al., in press), suggesting that the learning experience taps into conceptual resources available to middle school students. However, we have also seen dramatic and difficult to explain differences between two groups of middle schoolers' achievement who received very similar instruction by the same instructor using parallel technologies and the same curriculum but in two different contexts: one group was self-selected in an after-school program "Introduction to calculus" and the others in mandatory computer lab sessions seen as "enrichment" of a very traditional math class (Kaput & Cabral, in preparation). The former reached surprising, near ceiling-level understanding of key ideas, and the latter showed at best half the pre-/post-test gains on the same items.

At a deeper level, the representational innovations have led to a major reconstitution of the subject matter of the MCV as reflected in our various curriculum materials and activity structures. Finally, led by Nemirovsky and colleagues at TERC, we have also created and studied the use of hybrid physical/cybernetic devices embodying dynamical systems behaviors, e.g., the Bouncing Cart, whose inner workings are visible and open to examination and control with rich feedback, whose quantitative behavior is symbolized with real-time graphs generated on a computer screen (Nemirovsky & Monk, 2000; Nemirovsky, et al., 1998; Noble, et al., in press).

Focus on Teachers and Teaching

As is well appreciated, teachers are in the critical path to the implementation of any innovation (Frykholm, 1999), and when the innovation involves teaching and learning experiences that were not part of the teachers' preparation, enabling the teachers to adapt the change is an especially challenging, widely reported matter (Thompson, 1992). We have worked intensively with both pre- and in-service teachers to understand what they bring to the teaching of the MCV, (Bowers & Doerr, submitted; Doerr, & Bowers, 1998, 1999) and have found repeatedly that they, especially middle school teachers, bring the same kinds of misinterpretations and confusions to graphs of rates and accumulations, for example, as their students, although, given time, respect and instructional support, they learn quickly (Nickerson, & Bowers, 1999; Schorr, 1999). Consolidating a series of teacher-education and teacher support materials that embody our research findings is an important activity of the last year of our current funding cycle.

Focus on Implementation and Objective Measurement of Effectiveness

We are concurrently proposing to follow up this prior work with an Interagency Education Research Initiative planning project that will experimentally address the widescale implementability of our existing SimCalc representational innovations, especially in their low cost and easily disseminable form on graphing calculators. Building on our focus on teachers and teaching, this proposed follow-on will enable a research focus on the nature of the support that enables a varied set of classroom teachers to successfully implement SimCalc innovations. An important emphasis of this work will be the development of measures of learning outcomes that are rigorously constructed and validated, and permit objective assessment of the effectiveness of a variety of techniques for teaching the mathematics of change. We see this IERI work continuing to move SimCalc *representational* innovations along the trajectory towards measurable large scale impact, while the proposal that follows begins work on exploiting the next level of technological capabilities, combined *representational and communicative* affordances with affordable, robust, widely adopted devices.

Introduction: The Next Major Step for Technology in the Classroom– Combining Representational Innovations With Connectivity and Robust Low Cost Devices

Our Larger Strategy: Combine Representational Affordances with Connectivity

As described above, the SimCalc Project to date has focused on the representational-simulation affordances of the computational medium in mathematics education. However, as we all have seen, the connectivity made possible by computational media constitute a second profoundly important set of affordances. After approximately a generation of growing computer use in the world of business, LAN and WAN connectivity *coupled with the integration of computation into all aspects of business practice* has paid off in surprising increases in economic productivity during the past decade, now approximately 4% annually. And, of course, the connectivity embodied in the WWW has led to even more startling impacts on the world outside of education. Indeed, this wider connectivity has changed the conditions of innovation in ways that compound and accelerate change (Bollier, 2000). We are poised to begin a comparable application of connectivity in education. The missing ingredients are at-hand computation (see, for example, Becker, et al., 2000) and connectivity at the epicenter of teaching and learning, the classroom. But of course, as was the case in business, the classroom connectivity ingredient can pay off only if coupled with the integration of computation into educational practice, particularly into daily acts of teaching, learning and assessment of substantive content.

Two changes are underway that, if coupled with appropriate R&D of the sort we propose to do, will help supply the missing ingredients: The proliferation of inexpensive, personal and robust computing devices that can run independently produced software, and the ability to connect these to each other and to other diverse computational devices *in the classroom,* e.g., cell-phone-based appliances, tablet-based displays, electronic books, LCD projectors, and network-aware lab gear. Indeed, the latest generation of existing graphing calculators can now run independently produced software (using Flash ROM technology) and local, classroom connectivity has been announced by TI and likely to be provided by other vendors as well, utilizing one or another existing wireless technologies. TI has also announced the integrative software system, TI Inter*Active!* that allows users to move calculator data between calculators and popular Microsoft computer documents and to a quasi-emulator of the calculator. Given the cost advantage that increasingly capable and connected small devices have over larger computers and space-consuming laboratory configurations (a fully equipped connected classroom is likely to cost well less than a third of a computer lab), we can anticipate the rapid deployment of diverse, connected technologies in classrooms over the next several years. However, little is known about how to use these connected technologies to advantage in typical classrooms with typical students and teachers. Moreover, present uses of such technology are either representationally sophisticated but connectivity-limited (e.g. our prior SimCalc work) or connectivity sophisticated but representationally impoverished (e.g. commercially available ClassTalk systems). Hence we propose to study the affordances and constraints of mathematics classrooms equipped with mixes of (mainly, but not exclusively) hand-held devices, wirelessly connected to each other and to at least one larger, projected, computer-controlled display, with a focus on powerful mathematical representations.

SimCalc Partnership with Major Vendors: Nokia, Palm, and Texas Instruments

The practical side of the proposed project is based in a unique partnership with three of the world's leading vendors of the technologies involved, Nokia, Palm, and long-term SimCalc partner, TI. With the help of these partners, we propose a program of basic yet immediately applicable research that builds (1) a base of organized experience and (2) a conceptual framework that, together, support explanatory power to guide further design, development, and deployment of new technology, new curriculum and new teacher-education and teacher-support programs, both for the Math of Change & Variation and more widely. Put simply, our larger aim is to help ensure optimal and synergistic use of the combined representational opportunities described above and connectivity opportunities illustrated below.

Building on the SimCalc Project's Knowledge, Technological and Partnership Base

As outlined in the research plan below, our proposed work will directly exploit and extend prior work– insights about students' understanding, curriculum products, software technologies, and professional and institutional relationships developed in the SimCalc Project. We will also exploit new commercially available software developed under private funding that moves our technological and curricular innovations from dependence on expensive computer labs *separate* from the classroom towards massively deployable, low cost implementations that run on TI graphing calculators *in* the classroom. In particular, we have recently completed an initial port of basic computer-based SimCalc MathWorlds software stand-alone functionality to the TI 83+ graphing calculator, which will be commercially available by summer. We had also previously created a prototype version for the PalmPilot. Thus, beginning with a TI prototype classroom network (see the letters from TI and Palm) we are uniquely positioned to begin building towards exploration of connectivity in time for prototype testing during the Fall, 2000 term. We will be able to phase in testing with other devices, beginning with PalmOS hand-helds, in the following 12 months.

Research Issues and Opportunities in Assessment, Learning & Teaching

Exploration of Three Opportunity Spaces and the Underlying Role of Representation

Our proposed work focuses on how new configurations and applications of connected devices can support or perhaps impede potentially profound progress in three opportunity spaces at the communicative heart of mathematics education in real classrooms:

- **Diagnostic assessment and evaluation**: We wish to study how teachers can use connectivity and analytic tools to exploit what we know about student thinking and learning in order to actively diagnose and efficiently respond to student thinking on a regular basis in the classroom.
- **Student learning and activity structures**: We wish to study classroom affordances and constraints of new activity structures that exploit the ability of students to design and pass structured mathematical objects (e.g. functions) and representations (e.g. graphs) among themselves and to the teacher.
- **Teaching and the classroom management of information flow**: We wish to study the classroom management implications of wirelessly connected hand-helds, with particular attention to teacher-specific tools to help organize the flow of the vast amount of information available to them (e.g. what each student is doing on their individual hand-held), decide among alternative actions (e.g. send every student an identical graph vs. call students' attention to a projected graph on a central display), and set policies on network communication (e.g can students send each other text messages? Only within their group? Only to an assigned partner?)

Research Questions. Across these opportunity spaces, we plan to collect and analyze data addressing the following broad questions:

- What uses of mathematical notations and representations, when shared across devices, lead to deep, intense or efficient content-oriented interaction and meaning-making among students and between teacher and students?
- Which characteristics of networked, hand-held devices (e.g. screen size, lack of color, availability of stylus, ease of beaming data, ability to move about the room) strongly enable or impede the ease, comfort, and effectiveness of mathematical conversations in the classroom?
- In what ways do networked, hand-held devices most strongly engage learners' cognitive strengths and solve practical, important teaching problems, or conversely, distract learners from the task at hand and impose new burdens on the teacher?

Additionally, across the course of the project we will be able to analyze how the answers to the above questions change as both the teachers' experience with connectivity, and the technologies themselves, mature. Our research questions reflect our belief that hand-held, networked devices will not necessarily have a simple causal effect upon learning outcomes. As has been the case historically, introducing technology into schools will not necessarily change practice (Cuban, 1986), and it is likewise well appreciated that many technological tools simply reinforce existing practice (Marx, et al., 1998; Means, 1994). Similarly, as argued by Roschelle & Pea (1999) regarding WWW connectivity with resources outside the classroom, the promise may be ill-understood or nearsighted. Hence, it is our goal to begin building a framework that can adequately describe *uses* which are likely to distinguish effective from ineffective practice. Likewise, we do not take for granted that manufacturers specifications (processor speed, communication speed, screen size) are the device characteristics that necessarily enable or impede use, and seek to build a *conceptual analysis ofdevice characteristics* that directly relates to observable classroom behaviors. Finally, we recognize that widespread impact from such devices is only likely if we identify the strongest ties to learner's strengths, solve difficult teaching problems, and introduce no serious new difficulties. Thus we emphasize developing our framework to clearly identify how these affordances and constraints play out in realistic classroom settings, and thereby to guide iterative design that magnifies the unique benefits and minimizes the newly introduced impediments as well as to design appropriate teacher development and support structures so that connectivity becomes a pedagogical support tool in the sense of Putnam & Borko (2000).

After elaborating our conception of each opportunity space below, we describe the research design used to address these questions.

Opportunity Space #1: Improving Diagnostic Assessment Processes in Classrooms

A long term goal of many educational technologists over the past four decades has been to make diagnostic assessment more precise, more scientifically based, and more timely. This goal has mainly taken the form of increasingly sophisticated computer aided instruction, where the feedback was between student and machine, where it was designed around some relatively formal model of the learner's interaction with the domain based in a theory of learning, often supplemented with heuristics of various sorts based on expert performance, and usually implemented independent of the flow of daily classroom instruction (see, for example, the historical review offered in Kaput, 1992). Indeed, the major successes of such uses of educational technology have been outside of classrooms and outside of schools, although in recent years, we see new combinations of CAI with teacher input (Zehavi, 1993) or combining tools-oriented systems with tutor-structures/agents (e.g., Ritter & Koedinger, 1996).

To consider the contribution of classroom networks towards comparable goals, imagine the following simple scenario: The teacher poses carefully designed, thought-revealing problems and the students respond– but in this case, the students upload their responses to the teacher's computer where they are subject to a variety of analyses and actions, by the teacher, by the computer, and, as the technology is developed, by the teacher and computer in collaboration**.** In a simple case, it might be "Make a velocity function to match this motion," where students upload their respective velocity functions to the teacher's computer to be compared and perhaps used to animate screen objects for discussion. Unlike CAI or ICAI, the primary feedback cycle is initiated by the teacher. Secondly, the event is situated to provide efficient, informed feedback in the classroom, indeed, in the typical classroom of ordinary American schools**.** However just as was the case with CAI and ICAI, the feedback can be designed to apply directly what we, collectively, have learned about student learning and thinking in the domain at hand– as well as what we are learning about teachers, teaching and diagnostic assessment. The result of the actions and analyses is an instructional judgment by the teacher, informed more rapidly and precisely than ever before.

Of course, this notion is not new. It was developed more than a 15 years ago by Jim Minstrell (1983) and then more concretely in (Minstrell, 1988) where he described his use of HyperCard stacks to instantiate existing understandings of student thinking in the areas of kinematics and mechanics as supports for teacher instructional actions. This work was further expanded upon in (Hunt & Minstrell, 1994). Given the confluence of interests and objectives, Jim Minstrell has graciously agreed to act as a consultant on the project (see the attached letter).

Other work in the same domains intended to make college lectures more interactive using the ClassTalk technology developed by Abrahamson, comes technologically even closer to what we have in mind by utilizing feedback devices held by students who send their responses to carefully designed multiple choice questions to the teacher who uses a computer to read, aggregate and display them in histograms as the basis for discussion (Abrahamson, 2000a). This applied research on student thinking in physics (Hestenes, et al., 1992) with very positive results reported by Mazur (1997) and Hake (1996). Recently, Abrahamson has begun to apply ClassTalk in elementary school classrooms to support the teaching of reading, with highly positive reports (Abrahamson, 2000b). Reaching beyond ClassTalk, we can now enrich the feedback beyond multiple choice questions to include functions, for example, which can be compared either with one another or with an "ideal" response or otherwise aggregated and viewed by the teacher in new ways in various representations (including as drivers of simulated motions).

Such diagnostic feedback, both on individuals and on the classroom population using soon to be commercially available classroom networks may provide the kind of improvement of instruction that has long been the goal of AI assisted tutoring, as well as provide the basis for further application of intelligent assistance within the classroom context. Moreover, by serving as a conduit for application of our knowledge of student learning, thinking and interaction, these new classroom networks linked to assistive evaluation tools in the teacher's hand, can become a focused means for continuing improvement of instruction by individuals *and by the field* (Roschelle & Jackiw, 2000), perhaps even re-applying some of the knowledge and technique previously used in ICAI.

Opportunity Space #2: Enhancing Students' Structured Engagement with the Subject Matter By Employing New Activity Structures

With a few pioneering exceptions (Wilensky & Stroup, 2000), the design and study of activity structures (in the sense of Linn, et al., 2000) exploiting classroom connectivity using hand-held devices are new and in need of intense research. While an enormous variety of collaboration and competition structures is possible, some of which occur in multi-user games using Internet connectivity, we will focus on two basic categories, one involving student-teacher communication and the other involving student-student communication.

In the first category, students and teachers use the teacher's computer as publicly viewable common ground. To utilize this space, student may create functions on their own device and publicly upload them to a shared, publicly displayed object on the teacher's computer for a variety of purposes, including:

- Parades & Dances: Functions designed to satisfy certain constraints defined by the semantics of the larger object, e.g., students might contribute their, or their group's, velocity function described on the hand-held by a compact velocity graph, say, to control their own character's motion in a marching parade or dance which is enacted on a visually rich public display. The structure of the mathematics can be made to interact with the social structure of the classroom in interesting ways, for example, by having one set of groups create a motion with a certain kind of velocity function and another set use a different kind of function, e.g., the opposite of the first, or a piecewise constant approximation of the first, etc. Many variations are possible even within this context. For example, students might import their own motion data using an MBL or CBR device serially to be aggregated and run simultaneously as a dance, where the motions need to be carefully planned and choreographed to interact in a certain way when run in parallel.
- Functions graphically displayed on common coordinate axes, e.g. functions systematically varying within a given family, or designed to pass through some given points, etc.
- Students upload survey or other data (e.g., probability trials) to a common data-set on the teacher's computer that is then aggregated and downloaded as a data object subject to further analysis by the students on their local devices (a non-MCV activity).

Also within the teacher-student category, we anticipate target activities between teacher and students where students upload responses to classroom challenges, where challenges and responses are shown on the teacher's display:

- Define a function to fit this data, or an equation for this curve, or a polynomial that has these roots, etc.
- How many of these "Green Globs" can you hit with one function? (Dugdale, 1982).
- Define a velocity (or position or acceleration) function to match this motion.

In addition, other, less structured options abound that will utilize the teacher's existing repertoire of classroom moves, e.g., pool solutions to an open ended problem and investigate solutions for generalities, optimality, etc. Understanding how and under what conditions teachers come to use such capabilities is an important goal of the project.

Our second category focuses on target activities among small groups of students. An important class of existing SimCalc activities challenge students to create functions to satisfy some constraint, such as: Make a velocity graph to match this motion; "make a function which has this slope function;" "make a velocity function that gets Clown to the same place as Dude at exactly the same time;" "make an algebraic formula for Dude so that Dude's motion is exactly the opposite of Clown's;" "design a pizza delivery trip so that … ." While we have provided these challenges both on- and off-line in our materials, we will soon be able to support students sending such challenges to each other. Furthermore, our document architecture for Calculator MathWorlds can enable the user to specify which tools and representations are available, so that, for example, students could constrain the "Match my motion" challenge so that the recipients of their document-challenge could see the challenge-motion, but not see the motion associated with their velocity graph trial, forcing them from a trial-and-error strategy to a more analytical approach. We expect that the design of challenges will be both engaging and instructive, but understanding how to scaffold such activity in real classrooms will itself be a project challenge. Fortunately, we have some experience in close analysis of student activity in the domain at hand (Roschelle, 1998; Teasley & Roschelle, 1993).

Opportunity Space #3: Teaching and Managing the Classroom Flow of Information

One severe limitation of stand-alone hand-held devices is the inability of the teacher to know what is happening on 20–30 devices, a problem that becomes more important as the devices increase in capability and become the locus of ever more intellectual work in the classroom. The ability of a teacher to call up, or for a student to send up, a screen for the teacher to react to and perhaps even annotate, may be an enormously important new support for the teacher. Of course, at this point we have virtually no classroom experience with such capability, although prototype forms of it will become available during the 2000–01 academic year. We need to determine ways to support the teacher, to manage student privacy, to control information flow (e.g., during quizzes), and so on. We need to determine how many devices a teacher, especially a teacher who has five or more classes a day, can fruitfully monitor and at what level of intellectual depth. We also need to understand the technical features of such environments. What distinguishes screens from mathematical objects? How do we enable the teacher to use a spatial layout, such as a seating plan, to organize interaction with the students' devices? How can we integrate with existing teacher databases for purposes of uploading quiz and homework data? How can we process data pulled from outside the classroom into forms that can be shared with students; and so on. Can teacher instruction become more precise, or the intensity of student engagement increase above today's typically low levels?

A Paucity of Prior Work Involving Networked Hand-helds in Classrooms

Prior work in the interaction between hand-helds and larger computers is relatively sparse, and generally was not intended to directly inform classroom design– although some informal, unpublished pilot work has been done by TI-supported teachers, and we ourselves have also used point-to-point connections between single calculators and between a calculator and a computer via TI's widely used GraphLink technology to support early versions of the aggregation activities described above. For workplace-oriented work, see, for example, (Myers, 2000), where the study centered on networked hand-helds (Palm Pilots) being used to control a PC, particularly to control a PowerPoint display, and more recently to support collaboration on a central PC display (Riddle, 2000) and even mobile collaboration. Other work has involved networked classroom computers, e.g., (Inkpen, et al. 1997) with anecdotal positive impacts described, particularly among young girls. Of course, stand-alone Palm Pilots have been fruitfully used in science by Soloway and colleagues, (Soloway, et al., 1999), particularly to promote authentic field-based data-gathering and analysis.

The already complex relations among student, teacher and machine (Sutherland & Balacheff, 1999), particularly given the diversity of student thinking resulting from interactions with idea-provoking simulations, is further complicated by interactions among students in a connected classroom. However, teachers' implementation of stand-alone graphing calculators is reasonably well understood, with good quantitative data on how differences between orientations to mathematics teaching (especially rule vs. concept-building orientations) play out in the use of the technology (Tharp, et al., 1997). We expect comparable orientations in our teachers. Similarly, student responses to graphing calculator affordances vary considerably within a classroom (Ruthven & Chaplin, 1997), and the role of the problems and activities is critical to determining student conceptual change (Guin & Trouche, 1999). Note that pre-service teachers' understanding of function and modeling strategies are considerably influenced by the availability of computational tools and the students' prior knowledge (Zbiek, 1998), and we will be working with pre-service teachers in exactly this area.

Perhaps the most important relevant work to date on connected classrooms is being conducted by Uri Wilensky and Walter Stroup in their NSF-funded Participatory Simulations Project, which, among other kinds of activities, involves students using connected hand-helds to play the roles of agents within simulations. This is done through a new architecture, HubNet (Wilensky & Stroup, 2000), which integrates a simulation authoring language, NetLogo (Wilensky, 1999), with the TI classroom network. We and Wilensky feel that such ambitious and intense uses of classroom connectivity are likely to be implemented through stages that include certain of the less intense and more routine uses that we will begin with. Hence we plan to collaborate closely with Wilensky at the Center for Connected Learning and Computer-based Modeling at Northwestern University, and Stroup at the University of Texas at Austin (and long-time SimCalc partner) to ensure that we gain maximal cross-benefit, conceptually, technologically and practically, between projects, and that together, we can develop bridging strategies from the disconnected classroom configurations of today to the high-end participatory simulation activities that might be common in the connected classrooms of the future (see the letter from Wilensky).

Specific Research Plans, Questions and Objectives

Overview of Plans, Questions and Deliberate Project Variation

Focusing on the three opportunity spaces listed above, we propose to use design experiment methods (Brown, 1992; Collins, 1992) to produce a series of classroom-grounded case studies that embody theoretical frameworks and carefully structured accounts of classroom phenomena. These are intended to help guide further design, development, and deployment of technologies, supports and practices that exploit diverse connected devices in classrooms. The work will take place primarily in ordinary grade 8–10 classrooms taught by typical teachers in MA and CA. We will deliberately vary the experience of the initial teachers, ranging from a teacher experienced with SimCalc and with graphing calculators and computers, to teachers new to SimCalc but experienced with technology, and finally, to teachers who are new to both. In addition, we expect to prototype and gather data in other settings that are convenient, including a class for pre-service teachers and especially a class taught by the PI (Ball, 2000) for academically disadvantaged college freshmen whose mathematics backgrounds are equivalent to early high school students. We will study affordances and constraints at three different time-scales and levels of detail: (1) carefully-designed and closely observed 1–2 week teaching experiments common across sites (Cobb, 2000); (2) semester-long behaviors of teachers and classrooms based on occasional visits, teacher reflective journals, and structured debriefing-interviews of teachers; and (3) longitudinal change of the both the teachers and the technologies as they mature together over the life of the project, including comparisons with novice teachers introduced in YR 3.

At each of these different time-scales, we expect to document lessons that use no technology, use SimCalc technology without network connectivity, use graphing calculators without SimCalc software, in addition to our planned uses of networked SimCalc-specific representations. Hence we will gather data for a variety of conditions within the practice of each teacher we observe; this will help us understand the unique contributions of the specific technology components against the background of "ordinary" practice for that teacher.

For each year's major teaching experiment, we will select a single topic and instructional unit that includes a sampling of the activity structures listed above as well as a special set of assessment activities for systematic classroom observation at each location each year. Wider sets will be made available for the less intensely observed classroom work, which will vary across sites to take advantage of natural variation in teachers and context as well as to maximally explore the three opportunity spaces.

As detailed in the SRI subcontract statement of work, we will use a variety of data collection methods to support triangulation, and will employ appropriate qualitative data analysis techniques, with an emphasis on building analytic categories, and using them to present detailed case studies. We plan to summarize the results of each years work in case studies, deliberately organized to allow readers to compare and contrast findings across situations and settings. We also plan a comprehensive longitudinal report, aimed at addressing our findings with regards to the research questions stated earlier:

- What uses of mathematical notations and representations, when shared across devices, lead to deep, intense or efficient content-oriented interaction and meaning-making among students and between teacher and students?
- Which characteristics of networked, hand-held devices (e.g. screen size, lack of color, availability of stylus, ease of beaming data, ability to move about the room) strongly enable or impede the ease, comfort, and effectiveness of mathematical conversations in the classroom?

- In what ways do networked, hand-held devices most strongly engage learners' cognitive strengths and solve practical, important teaching problems, or conversely, distract learners from the task at hand and impose new burdens on the teacher?

Software design and development will emphasize rapid iteration in response to feedback from the research process. We will begin with external funding in YR 1 (see below), and proceed through the life of the project on two fronts: development of customized software for the hand-helds and development of prototype customization for the teachers' computers to support all three types of uses, diagnostic assessment, student-teacher activity structures and teacher management (especially viewing and annotating student screens). We will begin in YR 1 with technologies and materials at or near commercial availability and move in YRs 2 and 3 to newer technologies, and hence will be able to gain a sense of the trajectory of both the growth of the opportunity spaces and the growth of teachers as they accommodate these new options. We will begin with TI-based technologies in YR 1 at the outset, but will introduce PalmOS technologies in Spring of YR 1, providing insight into the affordances of substantially different technologies. Further, through our relationship with Nokia we expect to have early access to prototypes of yet more varied types of devices (such as PDA/cell phone combinations, tablets, set top boxes, etc), and will prioritize these based on the accumulated research to date, and explore them as resources permit.

All project activity, including technological infrastructure development, empirical work, and data analysis, will be distributed across the UMD and SRI locations. While somewhat more classroom-based empirical work will take place at UMD sites, SRI expertise will be employed to help train the UMD data gathering team, and a common web-based database will be employed for all project data. Joint activity in the form of reporting and idea-generating workshops will be held annually and will include teachers and researchers, corporate sponsors, and special invitees from other domains (see below). Joint analysis of critical-event episodes will serve as the point of departure for each year's case studies, all of which will be jointly authored.

Specific Research Work Plans, Sites and Timeline

Choice of Curriculum Content: Our YR 1 choices of curriculum and software to customize for networked use are based on well-tested SimCalc curriculum materials, the first of which involves slope, intercept, and linear functions and equations that are at the core of traditional topic coverage in Pre-Algebra and Algebra I courses taken by virtually all students. A second topic choice involves velocity-position connections for variable-velocity functions, with the opportunity to include periodicity. We plan to vary the second choice during YRs 2 and 3, particularly to include more advanced content.

Choice of Teachers and Sites: We will choose two teachers from separate high schools in the UMass area, one of whom is an experienced SimCalc teacher, and the other is new to both the content and to electronic technology. The former will be an assistant in a 7 day summer 2000 workshop introducing SimCalc content and graphing calculator software, and the latter will be one of 15 teachers taking the workshop. It is possible that we may invite two teachers from this workshop if they share the same classroom and classes and can provide us additional useful variation– in particular, we would attempt to choose teachers whose orientation to mathematics teaching differs in the extent to which it is concept/understanding oriented vs. rule/how-to oriented as operationalized by (Tharp, et al., 1997), among others. The two classroom sites are large, educationally under-performing urban high schools (totaling 6000 students) that have been extremely slow to adopt instructional technologies in mathematics. Indeed, the 2000–01 Academic year will be the first for which substantial use of graphing calculators will be used outside of AP Calculus, and virtually no computers have been used. The CA site has not yet been selected (several candidates are available through prior ESCOT Project work), but will be a much more technologically sophisticated high school.

YR 1 Data-Gathering: After prototyping in the Fall of YR 1, including building and testing appropriate modifications of MathWorlds software on both calculators and computers, and training in data-gathering protocols led by SRI personnel, we will begin systematic data-gathering with near-commercial classroom network and graphing calculator technologies at each site in Spring of YR 1. Also, beginning in Spring 2001 we will explore the use of the target activities and technologies in a UMD course for pre-service middle and high school teachers. Finally, as noted, we will use a year-long course taught by the PI in a program directed by the PI for academically disadvantaged college freshmen as a prototype context for all new development. The summer will be used for building the first pair of case studies and continuing software development. Similar timelines will be followed during

YRs 2 and 3. Note that the 2 MA sites are on different block-scheduling regimes, so that constraints on scheduling of closely observed interventions are eased.

Staged Introduction of New Technologies: Needless to say, the hardest thing to predict is the exact form and schedule of technology availability. However, we are confident that we can realize all student activity structures, and basic diagnostic assessment activity specified above by Spring of YR 1 on TI platforms and network, and we are confident that we can reach most of them on Palm platforms by Fall of YR 2. Note that in YRs 2 and 3 we may opt to introduce new platforms in selected sites rather than all sites, depending on whether certain of these become commercially available. Note that in a given site a single classroom can actually serve multiple classes, and hence affords the opportunity for variation in either activities and/or teachers, and each site can serve more than one classroom if necessary (beyond the additional no-connectivity condition), particularly if it becomes feasible to continue one technological platform while introducing another. In any case, we will exploit the variation in the technology to help understand how differences in specific features affect usability, implementability, and student learning.

Why Focus on the Math of Change & Variation?– The Generality Issue

While in an ideal world, and probably in subsequent work, we could learn more by varying the domains under consideration to include, for example, geometry, beyond practical considerations we have decided to focus of the Math of Change & Variation for three reasons: (i) We can leverage a large amount of existing experience, curriculum material, and software technology, (ii) the domain is sufficiently wide and deep to provide considerable internal variation in content to support theory-building and validation, (iii) the domain is at the heart of the core math curriculum taken by essentially all students, so whatever is learned here is already of enormous practical value. Lastly, our annual meetings will include technology and curriculum leaders in: Geometry, Nick Jackiw, author of the multiple-platform Geometer's Sketchpad; Statistics and Probability, Bill Finzer, author of the powerful Fathom data software, and Cliff Konold, author of probability and statistics tools, including TinkerPlots; and Dynamical Systems and Agent Modeling, Uri Wilensky and Walter Stroup. In this way, our work can stimulate development in these other areas as well as provide us feedback on the generality of our emerging findings and constructs. (See letters from Finzer-Jackiw, and Konold.)

Relationships Among Project Partners

The following four private sector partners will contribute to the proposed project: Texas Instruments, Palm Technologies, Nokia, Inc., and SimCalc Technologies, LLC (SCT). (The PI is the CEO of SCT and the co-PI is a minor partner. The University of Massachusetts is also a partner.) The first three have parallel roles in providing the base technologies that we need to conduct our work, including technical support for software development on the respective platforms and monetary support for the annual meetings (see the attached letters from the partner corporations). In addition, TI will provide additional support to SCT to tailor software for additional devices and to integrate this software with its prototype classroom network. Any technology developed under these auspices will be made immediately available to the project, and, unless premature disclosure of proprietary information would result, they will be made public as part of normally accepted research practice. In the unlikely event that it occurs, such a restriction should not materially affect the execution of the project since the pre-commercial technologies needed and to be developed for YR 1 are not under any non-disclosure restrictions. Indeed, the project has the immediate benefit of cutting-edge technologies that would be very expensive to produce independently. Nonetheless, a significant amount of ad hoc development will serve research needs and not be commercially valuable, hence must be budgeted for. Obviously, care needs to be taken to ensure that private interests and public interests are separately served and that potential for conflicts be regularly monitored. Fortunately, a conflict management system is already in place and operating, and will be continued to oversee the proposed project, as reflected in the attached letter from the responsible UMD administrator, Richard T. Burke.

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