Part I.

Developing a Framework for Evaluating Professional Development

CHAPTER 1

What Are the Needs of Teachers Who Are Engaged in School Mathematics Reform?

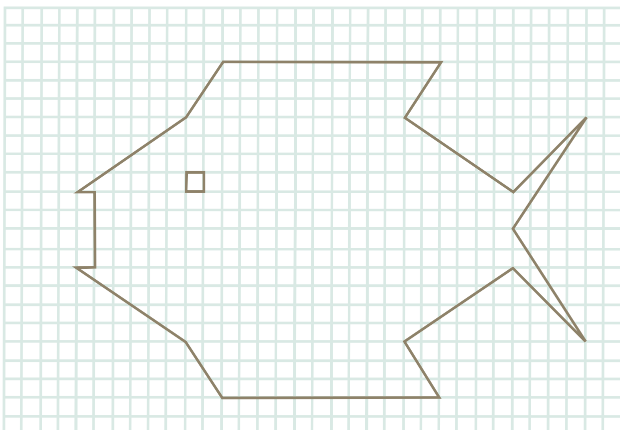
Knowing the needs that teachers engaging in school mathematics reform today are likely to experience is critical to help providers set goals for a specific professional development initiative and to evaluate the potential contributions of any given program. To highlight the extensive changes in teaching practices called for by the current reform and the challenges that these changes are likely to present teachers, we will first present an image of a reform-oriented mathematics classroom. The vignette that follows describes an actual classroom experience (see Callard, 2001, for a more detailed account of this experience). We will refer to this vignette throughout the chapter to illustrate key points about the learning needs of teachers engaged in school mathematics reform as identified in the research on teacher development and reform.

An image of a reform-oriented mathematics class

The instructional unit captured in this vignette was developed by Mrs. Callard, the classroom teacher, based on a set of instructional materials created to support an illustrative inquiry unit on area for middle school students (Borasi, 1994a). While these illustrative instructional materials provided an overall design for the unit, Mrs. Callard had to make a series of pedagogical decisions to adapt the unit to her own goals and to the constraints of her 8th grade mathematics class. For example, since she knew that her 6th grade colleagues had already worked with students on developing the concept of area and had introduced area formulas for rectangles and triangles, she decided to focus the unit on developing area formulas, drawing from the second part of the instructional resource materials.

Mrs. Callard began her four-week unit on developing area formulas with an activity that would invite students to review what they already knew about area with the goal of building on their prior knowledge and also identifying gaps and misconceptions in their understanding. The activity required students to find the area of a complex figure drawn on graph paper – the "fish" reproduced in Figure 1.

Figure 1
The "fish"

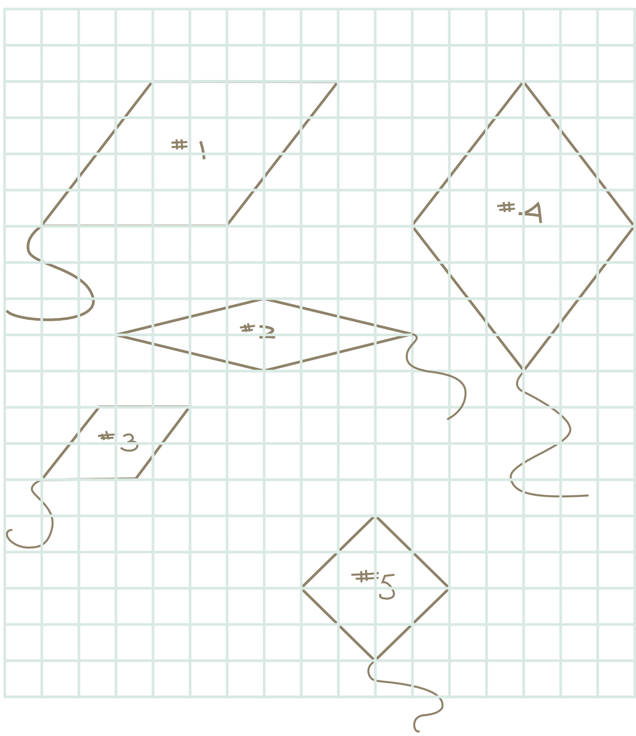


Mrs. Callard handed out a copy of this figure to each student, instructing the class to work on this task individually for a few minutes and then to share their preliminary results with a partner prior to a whole class discussion (so as to invite collaboration and scaffold their sharing in front of a large group). To further support the student's mathematical thinking and suggest alternative approaches, the teacher also made available a variety of tools, such as rulers, compasses, scissors, calculators, string, and tape, and even additional copies of the "fish." As the students worked, the teacher moved around the class for about 15 minutes observing, encouraging and supporting the students.

When most pairs reached solutions that satisfied them, the teacher asked volunteers to show their solution/strategies to the rest of the class. This sharing enabled students to appreciate the variety of approaches that could be used to solve this problem. These included strategies such as breaking the fish into rectangles and triangles and then adding the areas of these simpler figures, "boxing" the fish and then taking away the extra pieces, or simply counting the whole squares in the fish and approximating the partial ones. As each pair shared its solution, the teacher asked the students to articulate the strategy they used, and she recorded it on newsprint, so as to make each strategy explicit and to enable the class to later examine the strengths and weaknesses of the alternative strategies for computing the area of a complex figure. In this discussion, the teacher also pointed out the key role that the area formulas for rectangle and triangle played in several of the strategies identified.

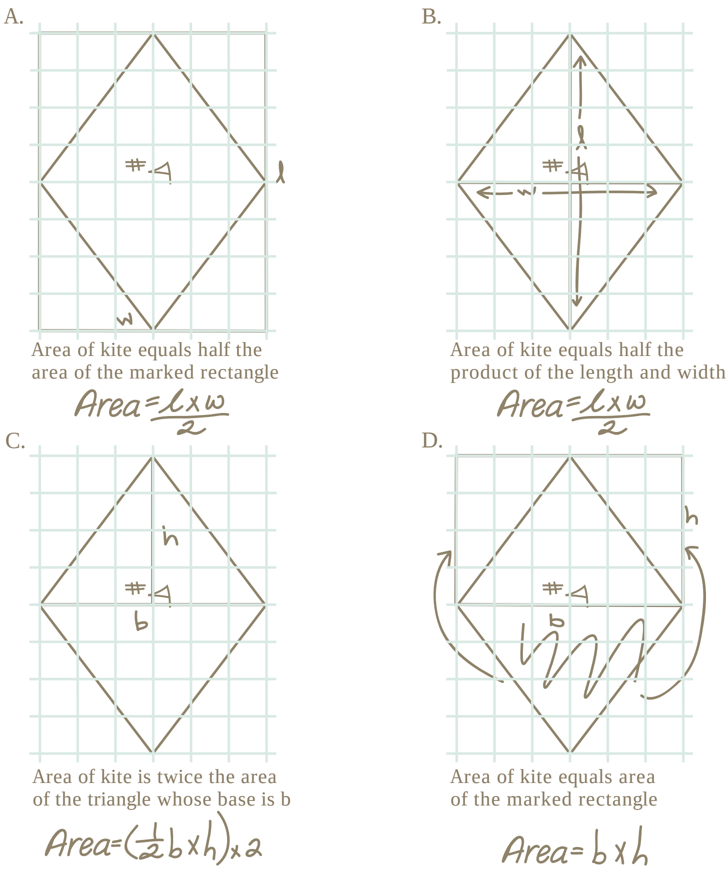
One of the teacher's main goals was to have her students appreciate that area formulas are not mysterious things to be memorized, but rather they are a short-hand notation that summarizes an effective strategy for computing the area of figures with certain common characteristics. This idea was further highlighted in the next activity, where students had to compute the area of different kinds of "kites" (see Figure 2).

Figure 2
"Kites"



Under the teacher's direction, the entire class attempted to develop an area formula that would work for all kites. Different students, by focusing on different characteristics of the kites in Figure 2, proposed the various procedures and formulas summarized in Figure 3.

Figure 3
Alternative area formulas for "kites" and their graphical explanation



A. Area of kite equals half the area of the marked rectangle

B. Area of kite equals half the product of the length and width

C. Area of kite is twice the area of the triangle whose base is b

D. Area of kite equals area of the marked rectangle

As the class critically examined these potential solutions, the teacher carefully facilitated the discussion, making sure that nobody was left out and everybody's contribution was seriously considered. She also occasionally asked questions to highlight important mathematical points, noting, for example, that students developed different yet equally acceptable area formulas depending on what they chose to measure and how they named their variables.

Mrs. Callard also took advantage of the controversy that erupted when one student observed that formula D “may not work for all kites.” Instead of resolving the student’s concern, she asked the class how they could decide whether something was a kite or not. Since a “kite” is not one of the standard figures usually defined in mathematics textbooks, this apparently simple question led the class to grapple with the challenging task of *creating* their own definition for kite and then defending it! Eventually, the class voted to define a kite as “a quadrilateral with perpendicular diagonals.” Based on this definition, the class concluded that formulas A and B were acceptable area formulas for kites, while formulas C and D worked only for special kinds of kites. This activity enabled the students to experience first-hand the power and excitement of “creating” mathematical formulas and definitions and also provided them with a deeper understanding of these fundamental mathematical concepts. To help the students reflect on and better appreciate the significance of what they had learned, the teacher then asked students to write answers to a few questions about mathematical definitions.

To help students synthesize and generalize what they had learned so far, Mrs. Callard led the class through a careful review of the steps they had followed over several class periods to come up with an area formula for a kite. She recorded each step on newsprint and later distributed this list (reproduced Figure 4) to the students as a reference for developing other area formulas in the future.

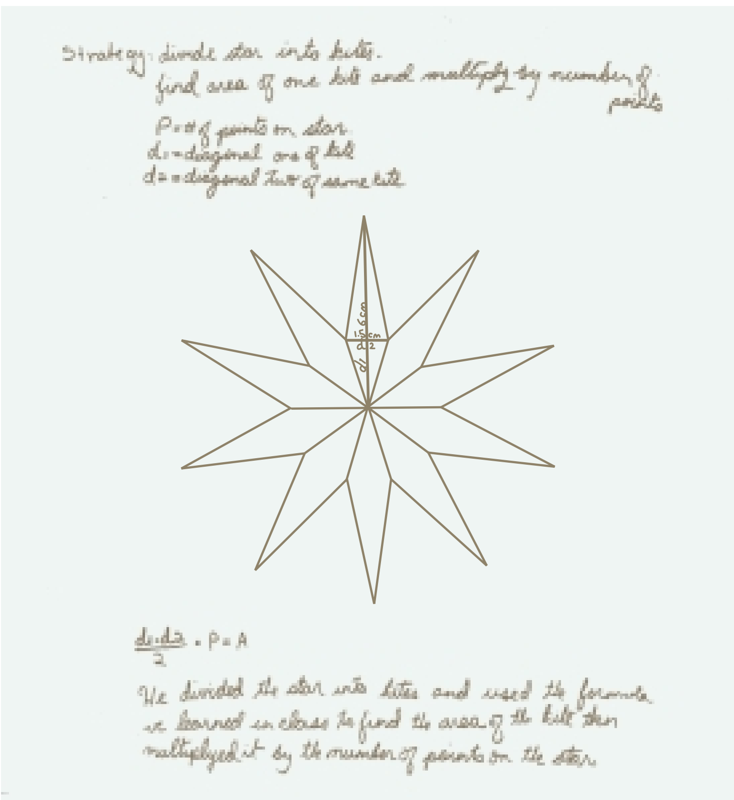
Figure 4
Key steps in developing an area formula

1. We started with examples of the figure and computed their area.
2. Shared strategies, ideas – discussed.
3. We checked to see if one strategy would work on all of the figures.
4. Tested the strategy.
5. Wrote a formula defining variables carefully.
6. Explained why the formula works.

As a culminating activity for the unit and as a form of performance assessment, Mrs. Callard asked students to create an area formula for a given “star.” She carefully assigned partners for this project, taking into account students’ different mathematical strengths, weaknesses and unique learning styles. Because students worked on part of the project in class, the teacher also provided more scaffolding for some pairs of students as needed. In the students' poster presentations at the end of the project, most of the pairs showed remarkable mathematical thinking abilities, and they communicated the results of their work effectively (see Figure 5 for an example).

To gather feedback about the learning achieved by individual students, the teacher also assigned two traditional take-home tests, one on developing area formulas and the other on applying known area formulas in practical situations. Students' grades for this unit also took into account their performance on homework and in-class assignments, so as to provide a comprehensive assessment of the students' learning based on data gathered from a variety of complementary tools.

Figure 5
Example of a "star" poster



Teachers' learning needs for implementing school mathematics reform

The previous vignette shows that the kind of school mathematics reform currently promoted by many constituencies involves much more than "superficial features" such as using manipulatives or introducing computers in the classroom. Rather, whenever we speak of "reform-oriented" practices in this monograph, we refer to a comprehensive approach to mathematics instruction that is centered on teaching for understanding and enabling students to engage with meaningful problems and "big ideas" in mathematics. This approach is characterized by a set of beliefs and theories about what counts as significant mathematics, how students learn and what conditions call such learning in a classroom environment, as articulated in the NCTM Standards (1989, 1991, 1995, 2000) and much of the current literature in mathematics education. At the same time, this does not mean that the most recent wave of school mathematics reform can be reduced to a prescriptive set of teaching strategies or "exemplary lessons." As argued throughout this monograph in the case of professional development, no single model of reform-oriented mathematics teaching will work for all, and all teachers will need to make decisions about what will be most appropriate and effective for their students.

Regardless of these differences, our vignette suggests that teaching mathematics in a reform-oriented way demands a lot more from teachers – even experienced teachers – than teaching a traditional mathematics lesson. However, teachers interested in reform should not be given the message that anything "traditional" is necessarily "bad" nor that they have done everything wrong so far and should abandon all their current practices. Teachers indeed bring valuable experience to reform, although they are asked to review their beliefs and practices critically in light of new instructional goals and pedagogical approaches. Identifying what teachers need to meet this enormous challenge, therefore, is a critical prerequisite to establishing worthwhile professional development goals and evaluating how specific professional development practices may contribute to achieving such goals.

Drawing from the literature on teacher development and reform (e.g., Friel & Bright, 1997; Fennema & Nelson, 1997; Darling-Hammond, 1997; Wilson & Berne, 1999), we grouped the main learning needs of teachers engaging in school mathematics reform into nine categories (see Figure 6), which we will examine in more depth in the rest of this chapter.

Figure 6
Main categories of teacher learning needs

1. Developing a vision and commitment to school mathematics reform.
2. Strengthening one's knowledge of mathematics.
3. Understanding the pedagogical theories that underlie school mathematics reform.
4. Understanding students' mathematical thinking.
5. Learning to use effective teaching and assessment strategies.
6. Becoming familiar with exemplary instructional materials and resources.
7. Understanding equity issues and their implications for the classroom.
8. Coping with the emotional aspects of engaging in reform.
9. Developing an attitude of inquiry toward one's practice.

Before engaging in this analysis, a few words about possible differences between elementary and secondary teachers of mathematics are warranted. Indeed, elementary and secondary teachers come to professional development experience with quite different preparation, background in mathematics and teaching experiences. Secondary teachers are usually specialists in their subject matter; most have completed the equivalent of a major in mathematics and teach only mathematics courses (often multiple sessions of the same two or three courses) to a total of 100 to 150 different students each year. Elementary teachers, instead, have been trained as generalists and usually teach all subjects to a class of 20 to 30 students; many of them have taken only one college-level mathematics course, although they may have had a wider exposure than their secondary colleagues to learning theories and innovative teaching practice as part of their training. It is also not uncommon for elementary teachers to express a greater interest and confidence in teaching language arts or almost any other subject matter! – than mathematics. These differences will undoubtedly play an important role in elementary and secondary teachers' expectations, responses and even attitudes toward professional development in mathematics, and it will be critical for every professional development provider to take them into serious consideration in their planning. At the same time, we believe that elementary and secondary teachers alike experience *all* of the learning needs identified in this chapter, although they may do so differently.

Developing a vision and commitment to school mathematics reform

Mathematical experiences such as the one described in the above vignette are not likely to happen unless teachers believe reform is important and understand what school mathematics reform calls for. Teachers interested in reform must thus become familiar with the new instructional goals and teaching practices proposed and understand their rationales.

Teachers need to develop a personal understanding of the reform recommendations articulated in the NCTM Standards (1989, 1991, 1995, and 2000) and other documents. Teachers also need "images" of reform classrooms in action, such as that offered in our vignette, because reform-oriented instruction is so different from the experiences of most teachers and students. By reading scenarios from actual mathematics classrooms, teachers can observe, in their mind's eye, the learning environment, typical activities and tasks that are taking place, and students' reactions. Several professional development projects have recently recognized this important need and responded to it by creating written and/or video images of reform-oriented mathematics lessons (e.g., Borasi, Fonzi, Smith & Rose, 1999; Ferrini-Mundy, 1997).

Because changing practices is not easy, teachers also need to be convinced that their students will benefit. Indeed, research on professional development efforts has shown that program outcomes, and teacher change in particular, correlate with the level of individual teachers' participation, effort and identification with reform goals and agendas (e.g., Clarke, 1994; Loucks-Horsley, 1997). At the same time, participating teachers initially may have only a limited vision of their needs and goals in terms of instructional innovation (Ferrini-Mundy, 1997). Thus, a professional development program should strive to *create a felt need* for reform while also taking into consideration the participants' *perceived needs* and actual constraints.

For some teachers, just witnessing students' active engagement and enjoyment of reform activities and seeing the depth of the mathematics learned in those lessons may be reason enough to want to offer similar opportunities to their own students (Fennema, Carpenter & Franke, 1997). Others, however, may need further evidence of the need for change, such as data on student achievement in comparative studies.

Developing a vision and commitment to reform among mathematics teachers is an ongoing and long-term goal for any professional development project. It is clearly the most critical element of any professional development program aimed at *initiating* the process of reform, although it should also continue to be an ongoing goal for any professional development project.

Strengthening one's knowledge of mathematics

Shulman's research identified subject matter knowledge and pedagogical content knowledge as key variables influencing teachers' decisions in the classroom:

> Prior subject matter and background in a content area affect the ways in which teachers select and structure content for teaching, choose activities and assignments for students, and use textbook and other curriculum materials. (Shulman & Grossman, 1988, p.12).

While developing teachers' knowledge of mathematics has always been considered a desirable goal of professional development, what counts as desirable mathematical knowledge has changed with the reform agenda. Reform-based curricula are informed by a different set of instructional goals. These include areas of mathematics that have been neglected in the traditional K-12 curriculum, such as probability and statistics. Even more importantly, there is a new emphasis on understanding "big ideas" in mathematics and on apprenticing students to the ways of thinking practiced by mathematics professionals.

Given their limited preparation in mathematics, elementary teachers are the ones often feeling the greatest need for learning more mathematics and deepening their own understanding of and confidence in the subject. However, despite their more extensive preparation in mathematics, secondary teachers also experience this need, as illustrated in our classroom vignette. In order to conduct the lessons on area formulas reported in the vignette, Mrs. Callard needed to know a variety of strategies for computing the area of complex figures, not just how to apply known formulas. She had to know how to develop area formulas, when to apply them and where mathematical definitions come from. These are aspects of mathematics that even teachers certified to teach secondary mathematics have not learned in their previous training (Fennema & Franke, 1992; Sowder, Philipp, Armstrong & Schappelle, 1998).

Furthermore, research on teachers' beliefs about mathematics (Thompson, 1992) documents the impact on curricular decisions and instructional practices of teachers' views on the following key topics: the nature of mathematics as a discipline; what constitutes legitimate mathematical procedures, results and justifications; and what constitutes desirable goals and acceptable outcomes for school mathematics instruction. Most teachers, regardless of whether they are generalists or specialists, never had the opportunity to make their beliefs explicit in traditional teacher preparation. Readings and discussions about the discipline of mathematics are notably absent from school mathematics and even college-level mathematics courses. Nevertheless, because they studied in traditional mathematics classes, most teachers hold deep-seated beliefs that mathematics is a body of absolute truths with little room for creativity or personal judgment. This means that, as teachers, they are likely to value correct answers over tentative conjectures, standard procedures over personal approaches to solutions, and facts and algorithms over inductive problem solving and reasoning skills.

Since these views conflict with the most recent calls for school mathematics reform (Borasi, 1996; NCTM, 2000), professional development programs designed to promote reform must provide opportunities for participants to critically examine their views of mathematics as a discipline and offer alternative perspectives grounded in reform.

Understanding the pedagogical theories that underlie school mathematics reform

Research shows that most mathematics teachers, including prospective teachers, have strongly-held beliefs about student and teacher's roles, desirable instructional approaches, students' mathematical knowledge, how students learn and the purposes of schools (Thompson, 1992). These beliefs have mostly developed as a result of the teachers' own schooling. Although rarely made explicit, the following views of knowledge, learning and teaching lie behind what takes place in most traditional classrooms:

- ***Knowledge*** is a body of established facts and techniques that can be broken down and transmitted to novices by experts *(positivistic view of knowledge)*.

- ***Learning*** results from acquiring isolated bits of information and skills through listening, watching, memorizing and practicing *(behaviorist view of learning)*.

- ***Teaching*** is the direct transmission of knowledge from teacher to student; it takes place as long as the teacher provides clear explanations for the students to absorb *(direct instruction view of teaching)* (Borasi & Siegel, 1992, 2000).

In contrast, the teaching practices recommended by the NCTM Standards (NCTM, 2000) and illustrated in our classroom vignette are grounded in views of knowledge, learning and teaching informed by a constructivist perspective (e.g., Brooks & Brooks, 1999; Davis, Maher & Noddings, 1990; Fosnot, 1996). Although different interpretations of constructivism exist, current school mathematics reform efforts are generally characterized by the following constructivist assumptions:

- ***Knowledge*** is socially constructed through human activity, shaped by context and purposes, and validated through a process of negotiation within a community of practice. Thus, it is always tentative rather than absolute. However, although knowledge is provisional in this paradigm, it does not mean that "anything goes."
- ***Learning*** is a generative process of making meaning that builds on personal knowledge and social interactions. This process may be stimulated by perceived dissonance. Prior knowledge, context and purpose play critical roles in the shaping of learning situations.
- ***Teaching*** is facilitating students' learning by creating a learning environment conducive to inquiry, setting up problem-solving situations to stimulate both student interest and cognitive dissonance about important mathematical ideas, and supporting students' attempts to solve problems and make sense of mathematical concepts (Borasi & Siegel, 1992, 2000).

To fully appreciate the constructivist pedagogical approach recommended in the NCTM Standards, teachers need to identify and understand the non-traditional theories of teaching and learning mathematics and the research supporting such approaches.

Understanding students' mathematical thinking

One of the main challenges that the teacher in our vignette experienced during her inquiry on area was interpreting her students' thinking and responding appropriately, especially when students proposed new strategies or formulas for computing area and explained how they got their results. The teacher benefited considerably by having already investigated a range of possible strategies and solutions to the open-ended tasks she posed – although some of the students' strategies still took her by surprise! Indeed, understanding students' mathematical thinking is especially critical in any constructivist approach if teachers are to design instructional experiences that help students build on their existing knowledge (Confrey, 1991).

Research on Cognitive Guided Instruction (CGI) has provided both theoretical arguments and empirical evidence claiming that mathematics teachers benefit from knowing about their students' prior knowledge and ways of learning specific mathematical concepts (Carpenter & Fennema, 1992; Fennema, Carpenter & Franke, 1997). Knowledge of child-constructed procedures is a crucial prerequisite for designing learning experiences that capitalize on, rather than override, the informal mathematical knowledge children bring to school. For example, many elementary teachers are surprised to learn that children often develop their own procedures for solving simple arithmetic problems *before* they enter school. Knowing this fact can help teachers rethink how arithmetic operations might be introduced.

Further empirical support for the value of teachers' knowing how students think comes from the *Integrating Mathematics Assessment* (*IMA*) project. This project focused on making teachers aware of the key features of student thinking about fractions. As a result, students made significant gains in solving problems involving fractions (Gearhart, Saxe, & Stipek, 1995).

While the results of studies like CGI and *IMA* are compelling, it is reasonable to ask whether we should expect teachers to acquire research-based knowledge about student thinking in all the mathematical areas they will teach, especially when most topics taught in secondary school are not as well researched as basic arithmetic and rational numbers. Rhine (1998) suggests that rather than trying to create such a knowledge base among teachers, it may be more important to foster a new attitude, one that values analyzing student thinking as part of teachers' everyday practice and provides strategies to help them do so.

Learning to use effective teaching and assessment strategies

One element that most distinguished the inquiry on area in our classroom scenario was the extensive use of teaching practices that are usually absent from traditional mathematics instruction. These included, for example, orchestrating group work using a variety of techniques, such as the initial "think-pair-share" activity; facilitating class discussions in which students shared results and jointly constructed new knowledge; using effective questioning techniques to synthesize key mathematical ideas; and assessing students' learning in multiple ways, such as the performance assessment in which students created an area formula for a star.

The pedagogical recommendations articulated in the NCTM Standards (NCTM, 1991, 2000) call specifically for teaching practices like these that are not currently used by many mathematics teachers, especially at the secondary level (for comprehensive lists of such practices, see Koehler & Grouws [1992] and Borasi & Fonzi [in preparation]). Non-traditional practices include not only facilitating what goes on in the classroom as lessons develop but also planning and assessing lessons effectively. Assessment has received special attention recently (e.g., Bright & Joyner, 1998; Lesh & Lamon, 1992; NCTM, 1995; Webb & Coxford, 1993) because determining what students know is necessary for teaching effectively within a constructivist paradigm. It is also critical for documenting the outcomes of reform efforts.

Learning to use novel teaching practices appropriately is not easy. Research on how people learn complex tasks may shed some light on what it takes teachers to adopt a new teaching practice. For example, Collins and his colleagues (1989) have suggested the following three-phase process for learning a complex task:

1. ***Modeling*** – The learner observes and examines how an expert engages in the task.
2. ***Scaffolded practice*** – The learner engages in the task himself/herself, but with the help of an expert and/or of other supporting structures.
3. ***Independent practice*** – The learner engages in the task without support.

Clearly, using new teaching practices effectively goes far beyond simply knowing they exist. While mathematics teachers should learn about a variety of teaching strategies to enrich their repertoire of resources, they should also have the opportunity to personally experience these practices in supported situations in order to evaluate fully their pedagogical potential. It is also critical for teachers to learn not only to use specific practices well but also to appreciate their strengths and limitations so they can choose practices most appropriate to an audience and to unique instructional goals.

Becoming familiar with exemplary instructional materials and resources

When reading about a well-designed, complex experience such as the inquiry on area described in our vignette, teachers might feel daunted by the prospect of creating similar lessons on their own. Fortunately, today's mathematics teachers are not expected to always create innovative units on their own as they may take advantage of the many exemplary instructional materials informed by the NCTM Standards that have been produced in recent years. As argued by Russell, this by no means demeans the professionalism of teachers:

> Curriculum materials, when developed through careful, extended work with diverse students and teachers, when based on sound mathematics and on what we know about how people learn mathematics, are a tool that allows the teacher to do her best work with students... . It is not possible for most teachers to write a complete, coherent, mathematically sound curriculum. It is not insulting to teachers as professionals to admit this. (Russell, 1997, p. 248)

Exemplary instructional materials may consist of replacement units, which are individual units designed to replace parts of the traditional curriculum while expanding the instructional goals and introducing some effective teaching practices or of comprehensive curricula. These consist of a sequence of units intended to totally replace the current mathematics curriculum at either elementary, middle or high school. Among the latter group, a set of instructional materials consistent with the NCTM Standards has been recently developed with support from the National Science Foundation (NSF) (see Figure 7 for a complete list of these comprehensive curricula and their websites' addresses). Additional exemplary mathematics curricula have been identified in a study by the U.S. Department of Education (U.S. Department of Education's Mathematics and Science Education Expert Panel, 1999).

Figure 7
NSF-funded exemplary comprehensive mathematics curricula

Elementary school (K-5):
- Everyday Mathematics *(http://ars-www.uchicago.edu/ucsmp-el/)**
- Investigations in Number, Data and Space *(http://www.terc.edu/investigations)**
- Math Trailblazers *(http://www.math.uic.edu/IMSE/MTB/mtb.html)**

Middle school (5-8):
- Connected Mathematics Project (CMP) *(http://www.math.msu.edu/cmp)**
- Mathematics in Context (MiC) *(http://www.ebmic.com)**
- MathScape *(http://www.edc.org/mcc/cscape.htm)**
- Middle Grades Math Thematics *(http://www.math.umt.edu/~stem/)**
- Middle School Mathematics through Applications Project (MMAP) *(http://mmap.wested.org)**

High school (9-12):
- Contemporary Mathematics in Context (CORE-Plus) *(http://www.wmich.edu/cpmp)**
- Interactive Mathematics Program (IMP) *(http://www.mathimp.org)**
- Math Connections *(http://www.mathconnections.org)**
- Mathematics: Modeling our World (ARISE) *(http://www.comap.com)**
- SIMMS Integrated Mathematics *(http://www.montana.edu/~wwwsimms)**

* web addresses are current at time of publication

In order to be considered "exemplary," a unit or comprehensive curriculum must be consistent with the NCTM Standards, designed by groups of specialists in mathematical content and pedagogy, and revised based on field tests in various instructional settings.

Exemplary instructional materials are much more than a textbook for students. They usually include a rich collection of documents to support learning experiences. The documents may include suggestions for planning lessons and orchestrating class discussions, examples of student work, tools and rubrics for assessment, and opportunities for teachers to learn more about the mathematical concepts to be taught.

While there is certainly a value for teachers to create their own innovative lessons and units, the results of the multitude of Teacher Enhancement and Local Systemic Change projects supported by the NSF in the last two decades suggest that the use of exemplary comprehensive mathematics curricula is critical to the success of systemic reform. That is, if the goal is to reform the entire mathematics program within a given school or district, not just to improve the practices of a few committed teachers, it is very difficult to achieve significant success unless the system adopts a coherent curriculum that ensures that students engage in a well-constructed sequence of worthwhile mathematics experiences, and frees teachers to focus their energy on improving their instructional practices and evaluating their students' learning.

While exemplary instructional materials can revolutionize the way we approach school mathematics reform (Ball & Cohen, 1996; Russell, 1997), they also require considerable time (and, in some cases, special expertise) to be used efficiently. Therefore, professional development programs should include opportunities for teachers to become familiar with at least some exemplary instructional materials, selected so as to maximize the participants' opportunities to implement reform in their classes.

Teachers also need to learn about high quality software and other technological tools if they are to implement mathematical learning experiences consistent with the most recent calls for reform. Indeed, new technologies such as graphing calculators, spreadsheets, and programs like the "Geometer's Sketchpad" and statistical packages like "Fathom," have radically changed the way certain mathematical topics can be taught in school (e.g., Dunham & Dick, 1994; Rojano, 1996). Teachers need to become proficient users of these technologies and to learn to consider how using these tools could affect not only their teaching practices but also their instructional goals.

Understanding equity issues and their implications for the classroom

At the forefront of the current call for school mathematics reform is the directive that *all* students should have opportunities to learn mathematics (NCTM, 1989, 2000; Secada, Fennema & Adajian, 1995). The underachievement of some ethnic minorities and women has been the cause of serious concern and one of the reasons that led to the recent critical scrutiny of curricula and teaching practices (Chipman & Thomas, 1987; National Science Foundation, 1986; Oakes, 1990; Secada, 1992). Students with disabilities may also perform much better in mathematics if they have appropriate learning opportunities and support (Silver, Smith & Nelson, 1995; Thornton & Langrall, 1997).

Because the new instructional goals and teaching practices articulated in the NCTM (2000) Standards are meant to recognize and respond to student diversity, researchers and policy makers are confident they will help bridge the achievement gap. Our vignette is evidence of how mathematical tasks can be designed to provide access to students with diverse learning styles, strengths and background experiences. An open-ended task, such as finding the area of a "fish," offers many more opportunities for success for all students than traditional tasks that recognize only one correct solution and one way to achieve it. Multiple forms of assessment, as exemplified in our vignette by the combination of a group performance assessment and more traditional paper-and-pencil tests, may also help students with different strengths and learning styles to show more easily what they know.

However, taking on new instructional goals and teaching practices will not be enough for teachers to fully address equity issues in school mathematics. Each teacher must first gain a good understanding of the many issues related to equity and diversity and their implications for mathematics instruction (Darling-Hammond, 1998). Teachers must also become aware of their own biases and privileges and learn how these may affect their relationship with students who are different with respect to race, class, gender, primary language, sexual orientation, etc. (Weissglass, 1996). Teachers must also believe that all students can learn mathematics when they are provided with ample opportunities, conditions conducive to learning and high teacher expectations.

Teachers also need to know how to identify their students' unique needs and how to differentiate instruction to address those needs. For example, it was important for the teacher in our vignette to recognize the different strengths and abilities of her students in order to place them with an appropriate partner for the final project; the same knowledge enabled her to offer additional scaffolding for some students who needed it. To respond to students with specific learning disabilities, teachers may need knowledge that is even more specialized.

Coping with the emotional aspects of engaging in reform

Several reform projects have noted that emotions, both positive and negative, inevitably accompany efforts to change one's teaching practices (Clarke, 1994; Ferrini-Mundy, 1997). A participant in one of our professional development projects aptly described her initial experiences in instructional innovation as an "emotional roller-coaster"; at times she felt elated by her students' success and the depth of their mathematical thinking, but she could also sink into dejection from an unsuccessful instructional experience she had spent hours putting together or from the opposition presented by a parent or administrator. Some teachers may suddenly feel inadequate after years of perceiving themselves as successful teachers and may even blame themselves for “doing it wrong.”

Studies of learning and problem solving show that behavioral changes often engender strong feelings of anxiety, frustration and elation (McLeod, 1992). Teachers need to know that conflicting feelings will inevitably arise and they need to find ways to cope with these feelings. If emotional needs are not directly addressed, teachers may even drop out of professional development programs and reform efforts. Weissglass (1993) has suggested that “any reform that does not provide methods for people to systematically and profoundly address their feelings, emotions and values related to reform will be inadequate.” (p. 3)

For teachers to recognize and deal constructively with feelings, they need, among other things, to break the isolation that so often characterizes teachers’ work. The need for teachers to share ideas and feelings with other teachers involved in research and reform has been long recognized in the teacher education literature (e.g., Clark, 1994). Quality professional development programs should strive to meet this need by creating opportunities for teacher collaboration.

Developing an attitude of inquiry toward one’s practice

Several researchers have identified teacher reflection on their practice and student learning as critical to the success of school mathematics reform. Darling-Hammond (1998) writes:

> . . . teachers need to be able to analyze and reflect on their practice, to assess the effects of their teaching, and to refine and improve their instruction. They must continuously evaluate what students are thinking and understanding and reshape their plans to take account of what they have discovered. (p. 2)

Barnett (1998) echoes Darling-Hammond’s call for teacher reflection and inquiry:

> Teacher inquiry plays a central role in many of the prevailing conceptions of teacher learning including critical reflection, reflection in and on action, personal and pedagogical theorizing, narrative inquiry, action research and teacher research. (p. 81)

Both researchers are building on the foundation laid by Schon (1983, 1987), who was one of the first to point out that teachers, just like