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# COMPUTATIONAL SIMULATION OF THE NONLINEAR RESPONSE OF SUSPENSION BRIDGES

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#### ABSTRACT

Accurate computational simulation of the dynamic response of long-span bridges presents one of the greatest challenges facing the earthquake engineering community. The size of these structures, in terms of physical dimensions and number of main load bearing members, makes computational simulation of transient response an arduous task. Discretization of a large bridge with general purpose finite element software often results in a computational model of such size that excessive computational effort is required for three dimensional nonlinear analyses. The aim of the current study was the development of efficient, computationally based methodologies for the nonlinear analysis of cable supported bridge systems which would allow accurate characterization of a bridge with a relatively small number of degrees of freedom. This work has lead to the development of a special purpose software program for the nonlinear analysis of cable supported bridges and the methodologies and software are described and illustrated in this paper.

### INTRODUCTION

In light of the limitations of the current state of engineering and scientific knowledge, the seismic analysis of long-span bridges still contains many uncertainties. Important issues for these flexible, expansive structures include the effects of spatially varying ground motion, the influence of long period motion in the ground accelerations near a fault and the details of the complex dynamic response of a large structure with complex articulations. Practical limitations of experimental testing procedures, and lack of a large body of measured earthquake response data for these important structures, requires engineers to rely extensively on computational simulation in order to affect a design or retrofit. Additional research will be required in order to reduce the uncertainties and provide engineers and earth scientists with a clearer understanding and higher confidence in estimating the seismic response of these critical structures. A multi-year research project being conducted by the University of California and the Lawrence Livermore National Laboratory is investigating many of the scientific and engineering issues in the response of long-span bridges. The research project is considering the San Francisco-Oakland Bay Bridge as a case study (see Fig. 1). This 1930's vintage twin suspension span carries the highest daily traffic volume of any bridge in the Universite and is a critical transportation link in the San Francisco region.

The Bay Bridge resides in a hazardous seismological environment, with the Hayward and San Andreas Faults traversing east and west of the bridge site respectively. Because of the close proximity of major active earthquake faults, this site embodies many of the seismological issues related to long span bridge response. The multidisciplinary research project is addressing both seismological and engineering aspects of the seismic response of long bridges and is focusing particularly on the relationship between the bridge transient response and various physical parameters effecting the ground motion at the bridge site.

An essential component of this research is the development of computer based bridge models which possess sufficient computational efficiency to allow a large number of parametric and sensitivity studies. This paper briefly describes progress in the development of the special purpose bridge models, and describes application to the nonlinear response of a suspension bridge.



Figure 1. The San Francisco - Oakland Bay Bridge western crossing, San Francisco, USA.

## **ELEMENTS OF THE COMPUTATIONAL MODEL**

In cable supported bridge systems, geometric nonlinearities have the potential to significantly influence the system response (Nazmy and Abdel-Ghaffar (1990)). Gross changes in the overall bridge geometry, with a resulting change in the magnitude and direction of the restoring force vectors in the cable system, can appreciably perturb the global instantaneous stiffness matrix. Internal force resultants can vary nonlinearly with system displacements as a result of the eccentricity of large axial compressive forces acting in the bridge towers. Abrupt and severe nonlinearities associated with contact and impact can occur at structural articulations. Full inclusion of geometric nonlinearities, and the development of expedient nonlinear solution algorithms, was considered essential for rigorous and accurate representation of the transient response to major earthquakes.

The basic components of the computational bridge model are summarized in Fig. 2. The cable systems are represented by tension-only cable elements and the towers are modeled with cubic  $(C_1)$  fiber flexural elements. The deck system, which in the case of the Bay Bridge consists of a double deck, is represented with a reduced order composite model consisting of a combination of truss, orthotropic membrane and sway stiffness elements. Potential contact/impact locations, such as the interface between the deck system and the towers, are characterized with a simple node-to-node penalty function contact model. The commonality between all of the various finite element types representing the bridge system is the assumption of finite (large) displacements and infinitesimal (small) deformations. Rigid body motions are removed by introduction of an updated Lagrangian coordinate system which tracks through space with each of the individual elements in the bridge model. A driving force behind the element technologies was the development of the simplest and largest (i.e. physical size) elements which would lend the model to explicit time integration of the equations of motion. The models described herein have been implemented in the *SUSPNDRS* special purpose finite element program at the Lawrence Livermore National Laboratory and a detailed account of this work is given in McCallen and Astaneh-Asl (1997).

## Reduced model of the deck system

For the computational model development, a reduced order model of the deck system was constructed which resulted in significant reduction of degrees of freedom, yet still allowed adequate representation of the stiffening truss/deck system dynamics. The model which was devised is shown in Fig. 3. The stiffening truss is repre-



Figure 2. The five basic elements of the bridge model. a) Tower model; b) reduced order deck model; c) tension only cable model; d) contact element; e) rocking block foundation model.

sented with truss elements, the deck system, consisting of deck slabs, deck beams and stringers, was represented with a simple orthotropic membrane element. For the specific deck configuration of the Bay Bridge, the connectivity between the deck slabs and the stiffening trusses occurs through the weak bending axis of the deck beams (Fig. 4), which results in a very weak load path between the deck slabs and the stiffening trusses. A consequence of this is the membrane stiffness of the deck system is not fully activated by in-plane deformation of the stiffening truss. In addition, expansion joints located every third or fourth bay of the deck



Figure 3. Reduced order model of the bridge deck system (Bay Bridge configuration).

provide further interruption of the deck membrane forces. The effective membrane stiffness of the deck slab for this specific deck configuration is therefore significantly less than the full summation of the area of the deck slab and deck stringers. To develop an appropriate characterization of the membrane stiffness of the deck system, detailed models of multi-segment deck structures were constructed and the effective stiffnesses of the deck membranes were determined by in-plane loading of the detailed deck segment models, as indicated in Fig. 4. In the discrete deck models, the axial force resultants were applied directly to the truss chords so that the flexible connection between the truss and deck slab was accounted for.

The sway stiffness element was developed to account for transverse bending of the double deck configuration. Because of the lack of bracing in the transverse direction, horizontal shear between the upper and lower decks must be transferred through out-of-plane bending of the stiffening trusses. This results in significant bending moments in the frame consisting of the horizontal deck beams and the vertical posts of the truss system. The eight-by-eight matrix which characterizes the sway stiffness only multiplies nodal displacements (see Fig. 3)



Figure 4. Example of determination of effective membrane stiffness of a deck segment (bottom deck shown).

and thus allows the effects of frame bending to be approximated without including rotational degrees of freedom in the global model. The result is a significant reduction in global degrees of freedom. The deck truss, deck membrane and sway stiffness element each have a local updated Lagrangian coordinate system which tracks with the element, removing significant rigid body rotations of the elements as they move through space. The reduced order deck system model has been validated by comparison with detailed deck models. Figure 5 shows a comparison of natural frequencies and modeshapes for a pin supported, twenty bay deck segment of the Bay Bridge configuration.



Figure 5. Modeshape comparison of detailed and reduced order models of a twenty bay (223 m) segment of bridge deck.

## Representation of the bridge towers

Bridge towers are modeled with a finite displacement, inelastic fiber element which has recently been developed by McCallen and Astaneh-Asl (1997). The tower element employs a cubic approximation to the transverse displacement field. The fiber element cross section is divided into a number of fiber zones and uniaxial, nonlinear constitutive relations are applied in each fiber zone. The subdivision of the element into zones allows the characterization of the complex open cell cross section construction typical of bridge towers built-up from steel plating. The element matrices are developed by applying numerical integration along the length of the element and three point Lobatto integration is employed. The advantage of Lobatto quadrature over standard Gaussian quadrature is that the Lobatto integration employs numerical integration points at the extreme ends of the fiber element, which are the locations where yielding typically initiates. To account for finite rotations, the tower fiber model utilizes three local element coordinate systems (Fig. 6). One coordinate system translates and rotates with the principal section axes at either end of the element, and the third system translates with the overall element. The incorporation of three coordinate systems provides a simple means for updating the nonvectorial large rotations of the fiber element and computing rotational deformations of the element as described in McCallen and Astaneh-Asl (1997).

The fiber tower element currently admits classical elasto-plastic behavior with kinematic hardening. The tower fiber element has been evaluated for both large rotation and inelastic problems by comparison with independent beam and shell element formulations. Figure 6 shows a comparison between the fiber tower element and a elasto-plastic shell element model for the nonlinear response of a simple wide flange section, and a comparison with the general purpose nonlinear program *NIKE3D* beam element for the analysis of a simple portal frame undergoing extreme displacements and rotations. The fiber model accurately represents the inelastic response of the wide flange section and captures the smooth transition from an elastic to a fully plastic section as yield-ing progresses across the wide flange section. The fiber element also accurately represents the large rotation behavior of the frame system.



Figure 6. The fiber flexure element used for bridge tower representation. a) Updated coordinate systems of the tower element; b) performance of the element for elasto-plastic wide flange bending; c) performance of the element for flexural bending with large displacements and rotations.

#### Representation of the cable systems

The cable system model consists of a simple two-force, tension only element which accounts for finite displacements and initial stress in the cable element. For cable systems, if the appropriate unstretched length of the cable is defined initially, a crude approximation to the initial cable geometry can be employed and the finite displacement model can then be used to equilibrium iterate to the final cable geometry. Figure 7 shows a comparison between computed and experimental response (Irvine and Sinclair (1976)) for a sagging cable. The initial geometry utilized in the cable model consisted of two linear segments and the cable elements were initalized with an approximation of the cable tension in order to render the initial stiffness matrix positive definite. The SUSPNDRS model rapidly converges to the appropriate gravity shape after approximately five equilibrium iterations. Application of a point load at the point indicated also closely matches the experimental data.



Figure 7. Simulation of the response of a sagging cable. a) Initial geometry guess; b) static solution for gravity loading; c) static solution for gravity plus 19.6 N point load.

#### Contact and impact in the bridge system

In bridges with expansion joint connections, significant impact can occur under seismic excitation. Accurate simulation of bridge response must account for potential impacts and the dramatic influence impact may have both as a damage mechanism and as a mechanism for effecting the accelerations and dynamics of the structure. For the specific case of the Bay Bridge, impact can occur between the bridge decks and the towers and the bridge decks and the caisson at the central pier (Fig. 8). A node-to-node penalty contact element has been implemented in the *SUSPNDRS* program for consideration of impact at the critical locations. The element monitors closing between specified nodes of the model and when the separation reduces to the specified stand-off distance, contact is assumed to occur and an interface stiffness is incorporated in the model. Inclusion of the possibility of a large specified stand-off distance is essential for this application because bridge elements are modeled base on line diagram geometry and the physical width of many impacting members (e.g. the bridge towers) is quite large. Figure 8 illustrates a computation in which node-to-node contact occurs between the tips of two vertical bars. An initial stand-off distance is specified and when the two members close within the stand-off distance, contact occurs. The plot shown in Fig. 8 demonstrates the force-displacement behavior with and without contact respectively.



#### **Rocking** foundations

The large, stiff piers of the Bay Bridge will be idealized as rigid blocks with the capability for uplift and rocking. This feature of the *SUSPNDRS* program is currently under development. The basic methodology will follow that outlined in McCallen and Romstad (1994).

# NONLINEAR SOLUTION ALGORITHMS AND MODEL INITIALIZATION

Cable structures present unique problems from the standpoint of appropriate initialization of the computational model. For cable supported bridges, the static initialization procedure must ensure that when full gravity load is applied to the model, the model assumes the appropriate geometric configuration, structural members such as the elements of the deck stiffening truss have the appropriate force distribution, and the cable system has the appropriate geometry and tensions in the various cable segments. This typically requires understanding of the bridge construction sequence and the original design objective of the construction sequence. In the case of the Bay Bridge for example, the cables were spun into place (with appropriate jacking of the towers to ensure the towers would be straight and vertical once the full deck system was in place) and the stiffening truss of the deck was lifted segmentally into place. The joints of the stiffening truss were not rigidly connected until the truss system was entirely in place and the deck steel was added. The design objective of this procedure was to have the truss posts as the only significantly stressed truss members under full gravity load. Ideally, the truss diagonals and chords are subjected to stresses only under application of live load. This procedure has important implications for initialization of the computational bridge model. It is necessary to provide a numerical solution which arrives at the appropriate large vertical roadway curve specified for the main spans of the bridge, and the appropriate linear design grade for the side spans. The initialization must also provided vertical towers which are subjected to axial forces only, and a deck system with dead load stresses in the vertical posts of the truss.

### Static initialization of the global bridge model

For static analysis, a Newton-Raphson procedure is incorporated in the SUSPNDRS program. Complete reformation of the instantaneous stiffness matrix occurs for each equilibrium iteration. Convergence was based on driving the norm of the force residual vector to an acceptably small value. To initiate the equilibrium iterations in the global bridge model, it is necessary to define both an initial geometry of the bridge structure and an initial estimate of the tensile field in the cable elements. The initial lengths of the main cables must be estimated from



Figure 9. Initialization of the nonlinear bridge model. a) Pre-gravity geometry based on parabolic cable approximation; b) postgravity model with appropriate geometry and stress field (4 Newton Raphson equilibrium iterations).

construction or design records. The main cable/tower system is analyzed under dead load in order to estimate the final geometry of the main cable under full load. Once the geometry is established, the unstretched length of each suspender cable can be determined based on the difference between the elevation of the main cable under dead load and the design elevation of the deck system. Once all cable lengths are determined, the initial bridge geometry is generated with these cable lengths. To expedite convergence, a parabolic approximation of the initial bridge geometry has been employed for the *SUSPNDRS* program (Fig. 9a). For the static initialization of the computational model, the stiffness and residual vector contributions of the stiffening truss chord and diagonal elements are neglected, and after equilibrium iterations are completed a new precise element length is computed for these elements in the deformed configuration. The computed length becomes the new unstretched length of the member. This ensures these elements will be unstressed after gravity initialization is completed. During the gravity equilibrium iterations, the main cables are allowed to slip horizontally over the tower tops so that no fictitious shear and bending moments are introduced in the towers. This model initialization procedure has been automated in the *SUSPNDRS* program and the methodology guarantees the appropriate model configuration at the end of static equilibrium iterations. Natural modeshapes and frequencies for a segment of the gravity initalized model of the Bay Bridge of Fig. 9 are shown in Fig. 10. Experimental observation of the



Figure 10. Selected modeshapes of a gravity initialized model of a section of the Bay Bridge.

vibration of this segment of bridge performed by Carder in 1937 (Carder (1937)), utilizing fairly crude instrumentation, yielded a fundamental period of vibration of 9.2 seconds, which is in reasonable agreement.

## Transient dynamic analysis of the bridge system

Temporal integration for structural models subjected to earthquake ground motions is predominately carried out with implicit time integration (e.g. Newmark- $\beta$  integration). This is reflective of the fact that conditionally stable explicit integration schemes typically require too restrictively short a time step to be practical for earthquake loadings of twenty or more seconds in duration. However, the simplicity of the element technology developed for the *SUSPNDRS* bridge model, coupled with the fact that the elements of the bridge discretization have physically large dimensions, lends the *SUSPNDRS* computational model to temporal integration with explicit integration schemes. Potential advantages of explicit integration, particularly for problems with pervasive nonlinearities, are well know and include the ability to capture contact and impact without the stiffness reformations and large number of equilibrium iterations. Explicit integration is significantly more reliable when strong nonlinearities are prevalent in the system (implicit schemes often fail to converge or require excessive computational effort associated with solving large systems of equations) and computer memory requirements are minimal. For the temporal integration in the *SUSPNDRS* program, central difference formulas have been implemented with the velocity lagging by one-half time step in order to avoid equation solution when spectral damping matrices are utilized.

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