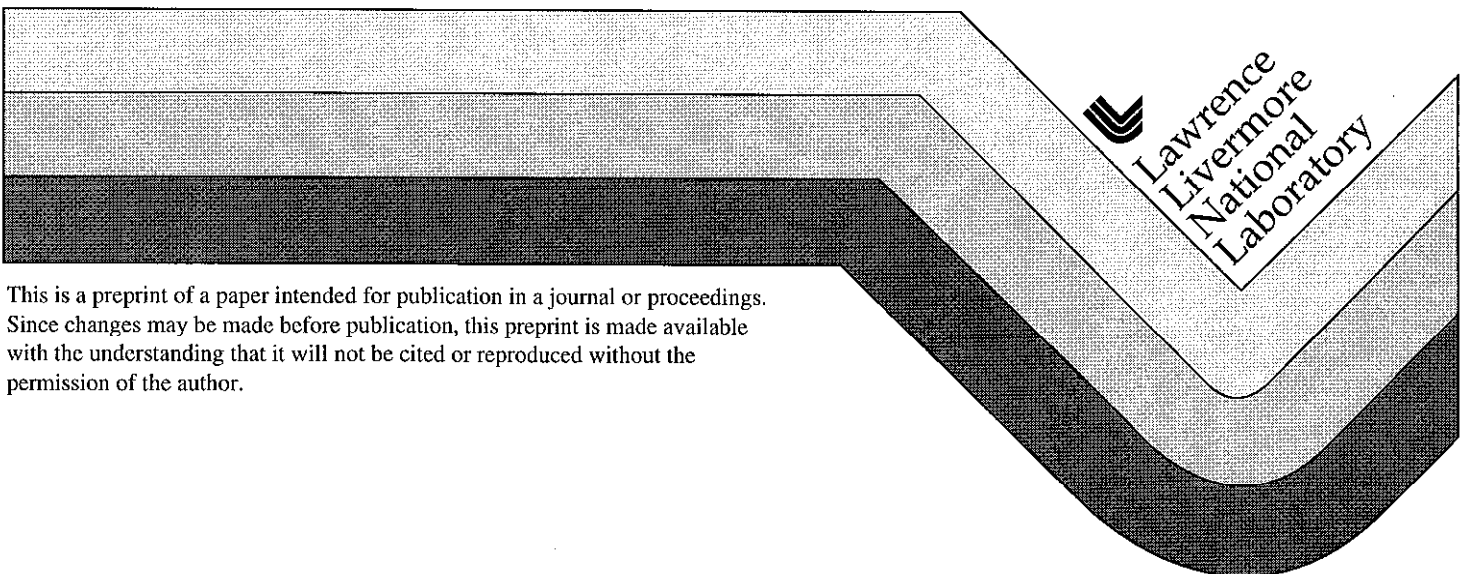# Elastic Modeling Initiative, Part III: 3-D Computational Modeling

S. Larsen
J. Grieger

This paper was prepared for submittal to the
1998 Society of Exploration Geophysicists Conference
New Orleans, LA
September 13-18, 1998

March 1998

DISCLAIMER

Elastic modeling initiative, Part III: 3-D computational modeling

Shawn Larsen*, Lawrence Livermore National Laboratory, and John Grieger, Phillips Petroleum Company

Summary

A 3-D finite-difference elastic wave propagation code that incorporates a number of advanced computational and physics-based enhancements has been developed. These enhancements include full 3-D elastic, viscoelastic, and topographic modeling (anisotropic capabilities are being added), low-level optimization, propagating and variable density grids, hybridization, and parallelization. This code takes advantage of high performance computing and massively parallel processing to make 3-D full-physics simulations of seismic problems feasible. This computational tool will be used to generate an elastic subset of the SEG/EAEG acoustic data set. The acoustic and elastic data will be compared to examine pitfalls with traditional processing, and to test the effectiveness of using elastic data as an aid to seismic imaging.

Introduction

Traditional imaging and seismic modeling techniques assume that the earth can be described as a simple medium in an acoustic half-space. This assumption was used for the generation of the large SEG/EAEG Acoustic Numerical Model data set (ANM), which simulated 3-D seismic surveys in synthetic subsalt and overthrust geologic models (e.g., House et al., 1998). However, the acoustic assumption does not accurately depict important characteristics of seismic wave propagation. These characteristics arise from elastic effects, viscoelastic attenuation, anisotropy, and topography.

The most significant limitation in the acoustic assumption may be the failure to model elastic wave propagation (e.g., Ogilvie and Purnell, 1996, Kessinger and Ramaswamy, 1996). For example, acoustic energy is partially converted into elastic energy at the free-surface and at boundaries within the geologic medium. The reverse is also true. This mode converted energy has two consequences for seismic modeling and imaging. The first consequence is "elastic noise", where an elastic signal is mistakenly interpreted as geologic structure. In addition, acoustic amplitudes are incorrectly modeled because some energy is lost to mode converted waves. Elastic phenomena need to be modeled and understood so that these pitfalls can be avoided. The second consequence is that elastic modeling contains additional information about the subsurface geologic structure. It may be possible to incorporate this information into the seismic imaging process, and hence improve the ability to resolve complex structure.

Elastic and other nontraditional techniques are numerically more intensive than acoustic modeling. Hence, there is a need to apply advanced computational technology and utilize high performance computing and massively parallel processing. This is important because the oil and gas industry is requiring greater use of sophisticated modeling techniques to find and develop hydrocarbon resources from increasingly subtle and complex geologic settings.

Methodology - basic implementation

A robust 3-D finite-difference wave propagation code capable of realistically simulating elastic waves in large 3-D geologic models has been developed at Lawrence Livermore National Laboratory. This code, known as E3D, is being used for a number of diverse geophysical research projects. These projects include seismic hazard evaluation, comprehensive test ban treaty verification, and oil exploration. E3D contains physics-based and computational enhancements. Physics-based enhancements allow more accurate simulations of wave propagation, and include full 3-D elastic and anelastic (attenuation) modeling, and the capability to incorporate the effects of surface topography on seismic energy. Anisotropic modeling capabilities are being incorporated. Computational enhancements make it possible to simulate larger problems more efficiently, and include low-level optimization, propagating and variable density grids, hybridization, and parallelization. E3D runs on a variety of platforms, from desktop workstations to Massively Parallel Processors (MPP).

E3D is based on the elastodynamic formulation of the full wave equation on a staggered grid (e.g., Madariaga, 1976; Virieux, 1986; Levander, 1988). In this formulation, the velocities v_i and the stress tensor components τ_{ij} are solved by an explicit finite-difference scheme (the indices correspond to the x, y, and z Cartesian coordinates). Using summation notation over repeated indices i, the basic equations are given by:

$$\dot{v}_i = \frac{1}{\rho} \cdot (\tau_{ij,j} + f_i) \tag{1}$$

and

$$\dot{\tau}_{ii} = \lambda \cdot v_{ii} + 2\mu \cdot v_{jj} + \dot{m}_{jj}, \tag{2a}$$

$$\dot{\tau}_{ij} = \mu \cdot (v_{i,j} + v_{j,i}) + \dot{m}_{ij} \tag{2b}$$

$$\tag{2c}$$

where ρ is the density, μ is the rigidity, and λ is the Lame parameter. Body force functions f_i and/or seismic moment rates $\dot{m}_{ij}$ are used as source terms to drive the velocities and stresses.

3-D elastic modeling

Equations 1 and 2 form a first-order differential representation of the full-elastic wave equation. As such, all mode converted waves are simulated. These equations are discretized over a staggered grid. In this system, each variable is staggered by 1/2 grid point spacing from the other variables (except that all normal stresses τ_{ij} are computed on the same grid). A 4th-order spatial stencil is applied to each differential term on the right side of Equations 1 and 2. A staggered grid implementation is beneficial because the spatial differencing stencils are centered around each variable, which minimizes the computational burden for a given level of accuracy.

An explicit finite-difference technique is used to update the time derivatives on the left side of Equations 1 and 2. The velocity updates (EQ 1) depend only on the stresses and the stress updates (EQ 2) depend only on the velocities (in addition to the source terms). Hence, each set of equations can be solved independently at 1/2 time-step intervals. These updates are 2nd-order in time, which means that all equations can be updated in place. This reduces the memory requirement.

Three-mechanisms are used to minimize artificial reflections at the side and bottom boundaries of the numerical grid: 1) absorbing boundary conditions using paraxial extrapolation (Clayton and Enquist, 1977); 2) sponge boundary conditions using an amplitude reduction coefficient applied over a swath of grid nodes parallel to the boundary (Cerjan et al., 1984); 3) attenuation boundary conditions using a low Q factor of about 5 applied at nodes near the grid boundary.

A run-time visualization utility is featured, making it possible to visually inspect the seismic wave field as the simulation progresses. This is particularly useful for identifying the characteristics of mode converted waves.

Physics-based enhancements

The free-surface is modeled as either a horizontal plane or as a boundary of topographic relief. In the first case, symmetry and stress-free conditions are used to correctly satisfy the stress tensor at the surface (e.g., Graves, 1996). In the second case, a density extinguishing approach is used to accurately model the interaction of seismic energy along a realistic topographic boundary (Schultz, 1997). While topographic modeling is not significant in marine surveys, it may play an important role in land surveys over rugged terrain.

Attenuation is optionally included in the basic equations using a relaxation mechanism scheme, whereby memory variables are added to the right side of Equation 2 (Robertsson et al., 1994). These memory variables are updated at each time-step. Attenuation modeling makes it possible to simulate the viscoelastic seismic damping that occurs in many geologic environments. This is particularly important at high frequencies.

An anisotropic modeling capability is being incorporated into the finite-difference code (e.g., Mora, 1989, Carcion, 1996). Initially, this capability will be used as a reservoir characterization mechanism for investigating the intrinsic attributes of seismic wave propagation through fractured media.

Computational enhancements

E3D incorporates low-level code optimization. This is particular useful when simulations are made on vector computer architectures, and on scientific workstations based on modern RISC technology. Low-level optimization improves performance 2 to 5 times.

Propagating grid mechanisms are applied in the code. Regions of the numerical grid void of meaningful seismic energy are not active in the computations. This is most useful in non-parallel environments, such as scientific workstations. Depending on the problem, a propagating grid can reduce run-time 2 to 4 times.

Element spacing in uniformly-spaced finite-difference grids, and hence problem size, is constrained by the lowest seismic velocity in the medium (see below). This is unfortunate for geologic models containing large velocity contrasts, because high velocity regions will be over-sampled. A multiple grid framework can be constructed, where densely sampled grids are mapped to low-velocity regions and coarsely sampled grids are mapped to high-velocity regions (e.g., McLaughlin and Day, 1994). This is sometimes referred to as static grid refinement, and in principle, is similar to adaptive grid algorithms found in computational fluid mechanics problems (Berger and Colella, 1989). E3D contains a layered grid structure. A more rigorous framework that is fully 3-D is being implemented. For typical problems, a variable grid framework can reduce run-time and memory costs by a factor of 10.

Hybridization is a method where output from one seismic modeling technique is incorporated as input into another technique. This is useful when a geologic regime contains regions that are relatively homogeneous (or layered), and other regions that are heterogeneous and complex. A computationally efficient scheme in the simple region (e.g., reflectivity), can be integrated with a robust but computationally intensive scheme in the complex region. This improves overall efficiency because computationally intensive methods are not being applied to the entire region of interest. E3D contains hybridization in the 2-D version of the code.

High performance computing

E3D is implemented on several high performance computing architectures, and on massively parallel processors in particular. A massively parallel machine typically has between 32 to 1024 nodes. Each node contains one or more CPU's, and

maintains it's own distributed memory. This memory is not directly accessible by the other nodes. A scientific problem can be decomposed across the nodes of a parallel machine so that each node simultaneously operates on a small portion of the problem. Hence, larger problems can be computed faster. Where necessary, internode communication is handled by message passing routines. These routines are explicitly implemented into the code, which can make the programming task difficult. E3D uses MPI (Message Passing Interface), among other constructs, to handle interprocess communication on massively parallel machines. In addition, the code runs on workstation networks using PVM (Parallel Virtual Machine).

Extremely powerful massively parallel computers are being developed for the Department of Energy's ASCI initiative. Within the next few years, there will be machines with 10 TBytes of internal memory (10 trillion bytes) and observable speeds of 10 TFlops (10 trillion floating point operations per second). For comparison, these machines will be approximately 100,000 times more powerful than typical scientific workstations presently available. This is tremendous computational power. For example, it may be possible to conduct 25 hz full-physics seismic simulations over 3-D geologic models tens to hundreds of kilometers in dimension.

Computational constraints

For 3-D fully heterogeneous elastic problems, E3D requires 12 floating point variables at each grid node: 3 medium parameters, 3 velocities, and 6 components of the symmetric stress tensor. For every time-step, 141 floating point operations are required for each updated grid node. This is significantly greater than 3-D acoustic modeling, where in the case of the SEG/EAEG modeling effort, 3 floating point variables (the acoustic velocity and the pressure at two time-steps) and 53 floating point operations per time-step were required at each grid node. Memory and the floating point count will increase if attenuation, anisotropy, or topography is included with the 3-D elastic simulations.

Problem size for 3-D seismic modeling is constrained by the available computing resources and the number of nodes in the finite-difference grid. The number of nodes ultimately depends on the spacing between the grid elements. For the finite-difference scheme discussed above, it is necessary to have at least 5-10 grid points per seismic wavelength. The wavelength depends on the frequency of the waves and the medium velocity. Hence, the maximum allowable size fundamentally depends on the seismic frequency and the lowest velocity in the medium. For 3-D models, doubling the frequency or reducing the lowest velocity by 1/2 requires an 8-fold increase in computer memory.

In addition, problem size is limited by the computational run-time of the simulation. Run-time depends on the speed of the

computer, the simulation time, the number of grid nodes in the model, and the time-step increment. The time-step increment is constrained by the Courant condition, which depends on the maximum velocity in the medium and the element spacing in the finite-difference grid. Hence, the maximum allowable run-time ultimately depends on the seismic frequency, the lowest velocity in the medium, and the highest velocity in the medium. For 3-D models, increasing the maximum velocity by a factor of two doubles the run-time. More significantly, however, doubling the frequency or reducing the lowest velocity by a factor of 2 increases the computational run-time by 16 times.

Example - SEG elastic modeling

Elastic finite-difference modeling will be used to generate a subset of the acoustic SEG/EAEG model data. The two data sets will be compared to examine the significance of elastic noise in acoustic imaging. Also, the elastic data will be used to examine their effectiveness as an added source of information for the imaging process. Ideally, the influence of other physical phenomena such as viscoelastic attenuation, anisotropy, and topography can be considered.

Elastic modeling is computationally more intensive than traditional acoustic techniques. In part, this is due to the greater complexity of the elastic wave equation, which necessitates more floating point operations per grid element. In addition, elastic modeling imposes an increased memory requirement on a computer system due to the need to store more parameters describing the model. More significantly, however, is the effect of shear wave velocity. In the SEG/ EAEG salt model, for example, the lowest acoustic velocity was 1500 m/s (water) and 1700 m/s (sediment). The finite-difference grid spacing was designed around these parameters. Shear wave speeds in the low velocity sediments may be 500 m/s (or slower). Accurate resolution of mode-converted energy in these low velocity sediments will require much finer grid resolution, and greater computation time. In all, elastic simulations may be 200 times more computationally intensive than the simulations used to produce the acoustic SEG/EAEG synthetic data.

The increased complexity of the elastic problem demonstrates the need to utilize advanced computational techniques and high performance computing. In addition, it emphasizes the reason why only a subset of the acoustic data can be elastically generated. It will be important to recognize the trade-offs between an accurate elastic model with low shear wave velocity and the computational burden. The E3D code has been validated against other finite-difference codes, and against other seismic modeling techniques. Nonetheless, it will be necessary to validate the elastic synthetics against the SEG/EAEG acoustic data to verify that they are simulating corresponding models.

3-D elastic modeling

Conclusions

The E3D explicit finite-difference code is capable of simulating seismic wave propagation in large geologic models. This code incorporates a number of advanced features including full 3-D elastic modeling, viscoelastic modeling, free-surface topography, propagating and variable density grids, hybridization, and parallelization. The capability for anisotropic modeling is being implemented. The code runs on a number of platforms, from desktop workstations to high performance computers and massively parallel processors.

E3D will be used to generate a subset of the acoustic SEG/EAEG model data set. The acoustic and elastic synthetic data will be compared to identify potential pitfalls when acoustic modeling assumptions are used to image seismic data. These pitfalls take the form of "elastic noise", where mode-converted energy is incorrectly interpreted as geologic structure, and where acoustic amplitudes are reduced because energy is lost to mode converted waves. In addition, the elastic synthetics will be used to investigate the effectiveness of using elastic data in the imaging process.

Three-dimensional full-physics seismic modeling requires significant computing resources. For example, elastic simulations over the SEG/EAEG models will require approximately 200 times more computer power than comparable acoustic simulations. Incorporation of attenuation, anisotropy, and/or surface topography will be even more computationally intensive. The necessary computational resources can be obtained with enhanced algorithms, and with high performance computing and massively parallel processing. In the next few years, the world's most powerful computers will have 10^{13} bytes of internal memory, and will be capable of performing 10^{13} floating point operations per second. This is approximately 100,000 times more powerful than typical scientific workstations. It is important for the oil industry to be prepared to utilize this great increase in computational power.

Acknowledgments

References

Berger, M. J., and P. Colella, 1989, Local adaptive mesh refinement for shock hydrodynamics, J. Comp. Phys., **82**, 64-84.

Carcion, J. M., D. Kosloff, and R. Kosloff, 1988, Wave propagation simulation in an elastic anisotropic (transversely isotropic) solid, Q. J. Mech. Appl. Math., **41**, 319-415.

Cerjan, C., D. Kosloff, R. Kosloff, and M. Reshef, 1984, A nonreflecting boundary condition for discrete acoustic and elastic wave equations, Geophysics, **50**, 705-708.

Clayton, R., and B. Engquist, Absorbing boundary conditions for acoustic and elastic wave equations, Bull. Seism. Soc. Am., **67**, 1529-1540.

Graves, R., 1996, Simulating seismic wave propagation in 3-D elastic media using staggered-grid finite differences, Bull. Seism. Soc. Am., **86**, 1091-1106.

House, L., K. Wyatt, W. Kessinger, S. Larsen, 1998, Elastic modeling initiative, Part I: Objectives, Soc. Expl. Geophys., this volume.

Kessinger, W., and M. Ramaswamy, 1996, Subsalt imaging using mode converted energy and acoustic depth migration, Soc. Expl. Geophys., Expanded Abstracts, **66**, 566-569.

Levander, A. R., 1988, Fourth-order finite-difference P-SV seismograms, Geophysics, **53**, 1425-1436.

Madariaga, R., 1976, Dynamics of an expanding circular fault, Bull. Seism. Soc. Am., **66**, 639-666.

McLaughlin, K. L., and S. M. Day, 1994, 3D elastic finite-difference seismic-wave simulations, Computers in Physics, **8**, 656-663.

Mora, P., 1989, Modeling anisotropic seismic waves in 3-D, Soc. Expl. Geophys., Expanded Abstracts, **59**, 1039-1043.

Ogilvie, J. S., and G. W. Purnell, 1996, Effects of salt-related mode conversions on subsalt prospecting, Geophysics, **61**, 331-348.

Robertsson, J. O. A., J. O. Blanch, and W. W. Symes, 1994, Viscoelastic finite-difference modeling, Geophysics, **59**, 1444-1456.

Schultz, C. A., 1997, A density-tapering approach for modeling the seismic response of free-surface topography, Geophys. Res. Lett., **24**, 2809-2812.

Virieux, J., 1986, P-SV wave propagation in heterogeneous media: velocity-stress finite-difference method, Geophysics, **51**, 889-901.