Part II.

Analyzing Promising Professional Development Experiences for Mathematics Teachers

CHAPTER 4

Engaging in Mathematical Experiences-as-Learners

In the type of professional development experience we describe in this chapter, teachers engage as *genuine learners* in mathematical learning experiences. While the nature, content and duration of these learning experiences may vary considerably, they all model effective instructional and/or learning practices promoted by school mathematics reform. Reflection is a critical part of these activities because it helps teachers analyze the experiences in light of their own beliefs and practices.

Theoretical rationale and empirical support

The benefits of teachers experiencing mathematics as learners go well beyond the important, rather obvious one, that teachers learn more mathematics. Research shows that teachers' beliefs about mathematics and about teaching mathematics are formed mostly as a result of having been students in traditional mathematics classrooms (Thompson, 1992). Since traditional mathematics is informed by pedagogical beliefs and practices that are radically different from those promoted by the current reform efforts, many teacher educators argue that before classroom teachers can change their beliefs, they must have personal experience of alternative pedagogical approaches (Brown, 1982; Schifter & Fosnot, 1993).

Further support for the value of experiences-as-learners for teachers comes from research on the learning of complex tasks. As we discussed in Chapter 1, Collins and his colleagues (1989) identified *modeling* as the first of three phases in the process of learning a complex task. When the complex task is learning a novel approach to teaching mathematics, we believe that facilitated "experiences as learners" activities offer an especially effective vehicle for such modeling. First, teachers observe an expert mathematics teacher educator teach mathematics in a non-traditional way. Second, because teachers participate in this instructional experience as learners themselves, they are in a unique position to examine how their students may *feel* about the new approach. As a result, they are in a better position to evaluate its potential advantages and drawbacks.

Simon's "learning cycles" model of teacher learning, which we described in Chapter 3, clarifies further the multiple roles that this type of activity can play in a professional development program. In Simon's first phase of the learning cycle, teachers must participate in situations that engage them actively as learners and that evoke cognitive dissonance. In this way, they are stimulated to construct new meanings. In the second phase, through sharing and discussing these constructions with a group, teachers come to consensus and make generalizations. This model suggests to us that good mathematical learning experiences for teachers need to invite active engagement, provoke cognitive dissonance, and encourage social as well as individual construction of meaning. Simon's model further claims that what is learned in one cycle can be used to stimulate another cycle of learning. We suggest that reflecting on these mathematical learning experiences can become the catalyst for teachers to begin yet another "learning cycle," this time focusing on the nature of mathematics as a discipline, how people learn and what can best support such learning.

Research corroborates the benefits of teachers experiencing mathematics as learners articulated above. This type of professional development experience plays a central role in several professional development programs with documented success (Simon & Schifter, 1991; Schifter & Fosnot, 1993; Borasi, Fonzi, Smith & Rose, 1999). A systematic study conducted by Simon and Schifter (1991) in the context of one of these programs has specifically shown changes in teachers' beliefs and practices toward a more constructivist approach to teaching mathematics. Since mathematical experiences-as-learners were not the *only* kind of professional development experience employed in these professional development programs, the results may not be considered conclusive. However, case studies and anecdotal evidence (Schifter & Fosnot, 1993; Borasi, Fonzi, Smith, & Rose, 1999) further confirm that experiences-as-learners were a critical element in changing the beliefs and practices of several participants in these programs.

Illustration 1: A facilitated inquiry on area for teachers

We derive the illustration in this section from one of the Introductory Summer Institutes in the Making Mathematics Reform a Reality in Middle

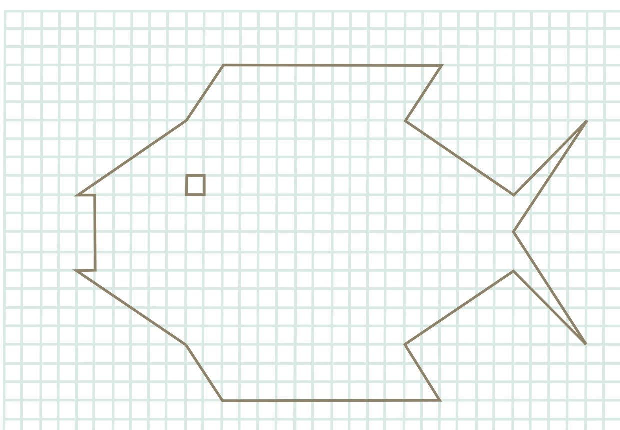
School (MMRR) project described in Chapter 2. This experience-as-learners was designed to help teachers analyze how an inquiry approach to teaching mathematics involves a radical rethinking of both mathematical content and pedagogical practices. It was also intended to introduce teachers to an “illustrative inquiry unit” they might be teaching in their own classes later -- a unit on area formulas designed for middle school students (the same unit featured in the classroom vignette included in Chapter 1). This experience-as-learners thus engaged participants in an inquiry similar to one they might be using with students.

The participants in the implementation described here included elementary teachers, secondary mathematics teachers, and special education teachers at the middle school level. It took about seven hours over three consecutive days to complete.

The instructor began by asking participants to take off their “teachers’ hats” and become learners in a series of activities about the concept of area. The instructor warned participants that this was *not* going to be a simulation in which they should pretend to be elementary or secondary students. Rather, the content would challenge everyone at their own level of expertise, so they should participate as genuine learners and use all they knew to deal with the tasks presented to them.

The first task was to find the area of a “fish” similar to the one middle school students worked with in the classroom vignette (see Figure 8).

Figure 8
The “fish”



Each teacher worked on this task first individually, then with a partner. The pairs then shared their results with the whole class. Most secondary mathematics teachers broke the fish into simpler figures, computed their areas using formulas they knew, and then added up those areas. A special education teacher had used a similar approach, yet made more efficient by using the symmetry of the fish and folding the figure in half. An elementary teacher showed instead how she had "boxed" the fish and then subtracted the area of the "extra pieces." Another elementary teacher "admitted" that she had simply "counted the squares," matching partial squares as best as she could to form whole squares.

Everybody was surprised by the variety of these approaches and by the fact that non-mathematics specialists had proposed the most creative solutions. A lively discussion surrounded this sharing, and participants came to appreciate the value of alternative strategies for finding the area of complex figures and the role that area formulas played in some of these strategies.

Next, the instructor challenged the participants to develop some area formulas on their own. First, she modeled this novel process by creating, together with the participants, an area formula for "diamonds." Later in the activity, she defined a diamond as "a quadrilateral with perpendicular diagonals." This task, and the reflection that followed it, highlighted important elements in the process of developing area formulas.

Participants then worked independently in small groups to develop area formulas for "regular" stars. The next day, they shared the area formulas they had created and explained the process they had used to derive them. Once again, everyone was amazed by the variety of area formulas thus created and by the creativity shown by several class members who had little mathematical background.

To help participants further appreciate the complexity of the mathematical concept of area, the instructor asked them to grapple with some thought-provoking questions for homework:

- Why are squares chosen as the "unit" to measure areas? Could other shapes be used? Why or why not?

- How do we choose the "size" of the squares to be used as units? Can this choice affect the value of the area of a given figure?
- Area formulas essentially enable us to compute the area of a two-dimensional figure by taking only linear measures (i.e., the length of the height, base, radius, etc.). How is this possible? Does this mean that you can measure area with a ruler?
- Can we ever find the area of a curved figure EXACTLY? For example, does $A=\pi r^2$ give us the exact value for the area of a circle or just a good approximation?

The difficulty they encountered responding to these apparently simple questions astounded the teachers. In all of their years as students of mathematics, not even the secondary mathematics teachers had been asked to think about questions like these, because learning about area had been reduced to memorizing and applying area formulas.

In the next session, the group discussed these questions in depth. At the end of this discussion, the facilitator handed out a mathematical essay on area as a follow-up reading assignment, to both validate some of the conclusions the group had reached and expand them further.

A number of follow-up activities encouraged the participants to reflect on this unusual learning experience and to analyze it from different perspectives. Participants listed "what they had learned" about area from this experience. This list was quite detailed and complex. Interestingly, although the teachers included a few technical facts, such as learning area formulas for diamonds and stars, they primarily identified elements related to mathematical processes and the nature of mathematics. For example, they highlighted the importance of learning to develop area formulas on their own, of understanding the role played by the choice of unit in measuring area, and of recognizing that mathematical problems could have more than one acceptable solution. Several participants also mentioned gaining increased confidence as learners of mathematics as a result of this experience.

The facilitator then began a discussion on the instructional goals that should inform a unit on area *for their students*. Not surprisingly, the group established quite different goals for their students than it is traditionally the case, such as: Students should understand the concept of area (how it is useful, what is actually measured); students should understand the concept of scale; students should discover that there is more than one formula for a given figure; students should be able to derive formulas.

A day later, as a culminating experience for the whole Summer Institute, the facilitator asked participants to reflect on this experience-as-learners on area and another experience-as-learners on tessellations they had engaged in a few days earlier. This time, the participants were asked to identify the *teaching practices* that the institute instructors had modeled in these experiences. As individuals shared their reflections with the whole group, the facilitator probed responses to elicit the rationale for, and potential effects of, these practices in *their own classroom instruction*.

For example, when someone identified the think/pair/share technique for the "fish" activity, a teacher pointed out how helpful it had been for her to work individually on the task first. Others corroborated this observation, noting the value of getting personally engaged in a task before interacting with others. In contrast, one participant expressed his relief at knowing that this individual stage would last only a few minutes, since he initially believed he would never be able to compute the area of the "fish" alone. This discordant opinion invited some considerations about differences in individuals' preferences and learning styles. Other people then commented on the power of the whole group discussion and how it had helped them go well beyond what they had achieved working with just one partner. The group agreed on the value of being able to explain one's strategies and solutions to another person first, and all the participants felt that this stage had been beneficial not only for gathering courage to report their ideas to the whole group but also for clarifying and expanding ideas by talking with a partner.

This reflective session also enabled the participants to recognize and discuss the role of less evident yet equally important pedagogical decisions, such as starting the unit with the complex, open-ended task of finding the area of the "fish." Participants noted the marked contrast between this decision and the traditional practice of assigning complex problems only *after* students have learned specific procedures that are presumed to be prerequisites for solving problems efficiently. This insight led to discussing the different assumptions about *learning* that distinguish constructivist/inquiry-based mathematics from traditional practices grounded in behaviorist learning theories.

Illustration 2: Working alongside mathematicians in a real-life setting

We adapted the illustration in this section from the *Growth in Education through a Mathematical Mentorship Alliance Project* (*GEMMA*) (ENC, 2000; Farrell, 1994).

As part of the *GEMMA* project, teachers participated in an eight-week summer internship in local businesses heavily involved in the use of mathematics and science, such as consumer marketing companies, scientific consulting firms, and automobile and other manufacturing companies. Each teacher was assigned a mentor in a company, and they worked together solving authentic problems that confronted the business. These projects included analyzing market surveys, testing fan blades for engines, researching the operation of a microwave that was being installed on a factory production line, determining and graphically displaying the relationship among molecules in a new material, and creating a computerized model of transportation systems. The companies expected teachers to be fully contributing members of the problem-solving team. In doing so, teachers had to learn about current industry practices for solving problems and to identify where and how mathematics was used.

During the internship, teachers attended a series of seminars where they discussed what they were doing, what mathematical applications they were learning, and what new instructional practices they were generating from their experiences with industry. By the end of the summer internship, teachers were expected to have designed some applied mathematical problems that they would pilot in their own classrooms. The project goal was to create a booklet of such "applications problems" to share with the other mathematics teachers.

The outcomes far surpassed the *GEMMA* project directors' expectations. They hoped the teachers would discover applications for the kind of mathematics they taught, which they did. However, the directors found that the internship experiences also introduced and/or reinforced many of the current reforms in pedagogy. In their final papers, for example, teachers wrote that they teach with a greater purpose and that they feel a need to integrate mathematics and science. They also wished to create collaborative learning environments in their classrooms and to give students much more responsibility for their learning.

Main elements and variations

The previous illustrations highlight several of the elements we believe need to be a part of any high-quality experience-as-learners.

Some of these elements have to do with the nature of the *mathematical learning experience* for the teachers. In order to be effective, we believe that these mathematical experiences need to accomplish the following:

- ***Challenge the participants intellectually,*** regardless of their mathematical backgrounds or the grade levels they teach. Only under these conditions can teachers be genuine learners and benefit fully from participating in these instructional experiences.
- ***Be mathematically sound and address key concepts.*** In order to strengthen teachers' knowledge of mathematics and invite them to rethink the goal of school mathematics, these experiences must offer opportunities to learn worthwhile and significant mathematics.
- ***Allow for mathematical reflection and discussion in addition to mathematical problem-solving.*** Doing so is essential to ensure that teachers revise and enhance their current understanding of key mathematical concepts and procedures, and do not just engage in "activities for activity sake."
- ***Model non-traditional ways of learning and/or teaching mathematics.*** Participants must experience alternatives to traditional school mathematics in order to appreciate their potential for student learning.

Another set of characterizing elements involves the *reflections* that follow the mathematical learning experience itself. As both illustrations show, these reflections are critical to the success of any experience-as-learners in initiating teachers' rethinking of their views of mathematics, teaching and learning. The following list captures the characteristics of optimal reflective activities:

- ***Reflective activities should occur after the learning experience is over, not during it.*** In this way, participants may find it easier to abandon their teacher roles as they engage in the mathematical learning experience and be genuine learners in it.

- ***There should be opportunities for individual reflections as well as group discussion.*** Participants need to make personal sense of the experience as well as hear other people's insights and perspectives.

Despite these common characteristics, successful experiences-as-learners can also differ in substantial ways, as reflected by our two illustrations. Important variations can occur along any of the following dimensions:

- ***Duration and complexity of the mathematical experience.*** Both of our illustrations included intense mathematical experiences – a 7-hour inquiry on area in Illustration 1, and a summer-long project in Illustration 2. In contrast, there are examples in the literature of shorter mathematical experiences, involving the solution of a problem or other isolated mathematical tasks.
- ***Diversity of participants.*** Participants may be a rather homogeneous groups of mathematics teachers teaching at the same level of schooling or they may include mathematics specialists and non-mathematics specialists at different grade levels (as it was the case in Illustration 1).
- ***Facilitator's role.*** The facilitator may purposefully model some innovative teaching practices (as in the inquiry on area reported in Illustration 1) or simply work alongside teachers in a joint task (as expert mathematicians did in Illustration 2).
- ***Scope and structure of follow-up reflections***. Reflective activities may be open-ended or focused explicitly on specific aspects of the learning experience. For example, facilitators may ask teachers to reflect on the teaching practices modeled, the reactions of different learners to the experience, or their views of mathematics. Leaders may also elicit individual reflections in different ways, such as asking teachers to respond in writing to written prompts, to write in journals or to brainstorm ideas with a partner before having teachers share and discuss them.

Experiences-as-learners can also take place in a variety of professional development formats. They can be part of an after-school workshop, a summer institute, a university course, an on-site study group, or even an immersion situation in which teachers become mathematics-learners and problem-solvers alongside mathematicians in real-world settings. In many cases, part of the participants' mathematical experience may require projects or other assignments that are undertaken by each teacher independently.

Experiences-as-learners can be conducted by facilitators with a variety of backgrounds. Although mathematicians might seem to be ideal facilitators for this type of professional development, they may need to work collaboratively with experienced teachers or mathematics educators who can complement their subject matter expertise with experience in instructional innovation. Conversely, experienced teachers playing the facilitator's role may benefit from coaching on the differences between teaching adults and K-12 students and from readings about the "big mathematical ideas" that form the core of any experience as learners. Regardless of their affiliation, facilitators leading experiences-as-learners need both a strong mathematical background and the ability to model innovative teaching practices.

Teacher learning needs addressed

Experiences-as-learners have the potential to address many of the teacher learning needs we identified in Chapter 1, yet the extent to which they do so depends on how the activity is implemented. In this section, we discuss what specific variations of experiences-as-learners can best help meet the needs of teachers who are interested in pursuing school mathematics reform and how.

- ***Developing a vision and commitment to school mathematics reform.*** Mathematical experiences-as-learners can be powerful to help teachers understand what school mathematics reform really mean and why it should be promoted. When a skilled mathematics teacher educator designs the activities to demonstrate the kind of mathematics instruction promoted by the reform movement, teachers can appreciate the vast difference between traditional and constructivist-based practices. For example, the inquiry on area reported in Illustration 1 allowed the teachers themselves to learn about a traditional mathematical topic by focusing on big mathematical ideas, solving problems through inquiry and constructing knowledge with others. It also illustrated concretely the new roles that teachers and students must play when a constructivist view of learning informs mathematics instruction.

The personal success and enjoyment that participants experience in novel mathematical activities are powerful motivators toward instructional innovation. Committed teachers want their students to experience the same positive emotions about mathematics. We have observed this happen, especially with teachers who have bad memories of being students in traditional mathematics classes. Even teachers who were successful students in traditional settings, however, can experience vicariously their colleagues' delight when they share such thoughts as "I never knew I could do mathematics! If only I had been taught this way!" This kind of response is especially common when the group includes non-mathematics specialists.

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- ***Strengthening one's knowledge of mathematics.*** Experiences-as-learners are ideal for strengthening teachers' knowledge of mathematics. However, the nature and extent of this learning depends on the duration and design of the learning experience. For example, immersion experiences (as shown in Illustration 2) expose teachers to mathematical tools and applications used in business, not in the traditional school curriculum. By seeing what mathematical knowledge and skills are really needed to solve real-life problems, teachers may begin to question what their students should learn. Consequently, they may rethink the goals of the mathematics courses they teach.

Teachers can also learn something new about topics that are currently in the K-12 curriculum, as shown in the area inquiry in Illustration 1. There are many benefits to doing so, since teachers – even those who have taken several college-level mathematics courses – often lack the deep conceptual understanding of mathematical topics in the K-12 curriculum that are necessary to implement reform lessons. As reported earlier, several teachers in the inquiry on area had never questioned the significance of using squares as units when measuring area, nor had they really understood what area formulas are or where they come from. However, the mathematical insights these teachers gained might not have been achieved at the same level without the reflection and discussions that followed the learning experience itself. Follow-up reflective discussions, such as the "What I have learned" analysis that followed the inquiry on area, are critical to challenge participants' views of mathematics as a discipline and their perceptions of themselves (and their students) as learners of mathematics.

- ***Understanding the pedagogical theories that underlie school mathematics reform.*** Experiencing mathematics as learners has also the potential to help teachers understand better the pedagogical theories that inform current reform efforts. As Simon's (1994) model of learning cycles suggests, this kind of professional development activity not only provides an experiential basis for new learning approaches but also stimulates teachers to reflect on, and inquire further about, the theories of learning and teaching on which these approaches are based. To ensure a thorough understanding of learning theories, however, personal reflections need to be augmented by specially designed follow-up readings and/or presentations, something that was missing in our illustrations.

- ***Understanding students' mathematical thinking.*** Because experiences-as-learners focus on the *teachers*' learning, they are not an ideal vehicle to pursue an understanding of *students*' learning and thinking processes. However, these experiences do help teachers become aware of their own – and other adults' – mathematical thinking and problem-solving strategies. This awareness can be eye-opening for many teachers, and it can inspire them to examine their students' thinking in the future.

- ***Learning to use effective teaching and assessment strategies.*** Experiences-as-learners are especially appropriate for modeling effective teaching practices, at least when the facilitator has the expertise to do so. As we argued earlier, modeling is a critical part of learning complex tasks (Collins, Brown, & Newman, 1989). To be most effective, modeling should not stop with the expert performing the novel task in front of the novice. Rather, it should be accompanied by *explicit reflection* on the teaching practice that was demonstrated so that participants can recognize and internalize its key elements. We believe, therefore, that a focused follow-up reflective session is necessary to help teachers identify the teaching practices modeled and to analyze the implications for mathematics instruction (as shown in Illustration 1).

- ***Becoming familiar with exemplary instructional materials and resources.*** Depending on the content of the mathematical learning experience, experiences-as-learners may or may not help participants become familiar with exemplary instructional materials and resources. Teacher educators who want to introduce participants to an exemplary curriculum series or to a replacement unit that teachers will be expected to implement later in their classes need to select mathematical tasks from these materials and adapt them for an adult audience. This is what happened in the inquiry on area we featured in Illustration 1, and it is a practice used in many projects designed to support the implementation of NSF-funded curricula.

- ***Understanding equity issues and their implications for the classroom.*** By doing mathematics in a group, teachers are inescapably confronted with the diversity in learning styles and approaches that exist. This is especially the case, though, when the mathematical task is open-ended and there are opportunities to share different solution processes. The experience can be especially powerful when the group is highly diverse and the implications of the differences are addressed explicitly. However, it is our experience that given an appropriate mathematical task, any group of learners will produce enough diversity in responses to begin a conversation. Facilitated experiences-as-learners are also ideal for modeling strategies for differentiated instruction based on diverse learning needs and, then, discussing participants' reactions to these strategies.

- ***Coping with the emotional aspects of engaging in reform.*** Coping with the emotional aspects of engaging in reform is not a central goal of engaging teachers in experiences as learners of mathematics. Nevertheless, using this kind of professional development experience early in a program can be instrumental in creating a bond among participants and engendering a "community of learners" that can offer emotional support as the participants undertake instructional innovation in their classrooms later on. It is also important to recognize that for some elementary and special education teachers just engaging as learners in a mathematical task may evoke painful memories of failure and raise anxiety levels. Acknowledging and addressing these feelings within the context of an experience as learners may help these teachers overcome their fears, thus mitigating emotional obstacles to their individual efforts at instructional innovation later on.

As teachers critically analyze the experience they participated in as learners, they begin to appreciate the power of reflecting on instructional practice.

- ***Developing an attitude of inquiry toward one's instructional practice.*** As teachers critically analyze the experience they participated in as learners, they begin to appreciate the power of reflecting on instructional practice. These reflective sessions can also model ways for teachers to structure their own reflections to make the process more productive. Therefore, experiences-as-learners can be valuable in addressing this teacher learning need, provided that the follow-up reflective sessions are designed to achieve that goal.

Summary

Our analysis shows that activities in which teachers become learners of mathematics can be a powerful way to accomplish multiple professional development goals, especially when they are thoughtfully designed and led by a capable facilitator. Any variation within this type of professional development experience can promote the learning of new mathematics and challenge teachers' beliefs about what students should learn and how. These experiences can also help teachers develop a vision for school mathematics reform, examine pedagogical theories and effective teaching practices and become aware of diversity in approaches to problem-solving and learning styles. However, we caution that these benefits depend on whether a facilitator carefully models novel teaching strategies and orchestrates focused reflections on these experiences. The length of the activity, the complexity of the tasks, the design of the format, and the structure of the follow-up reflection may also determine the extent to which this kind of professional development experience can meet various kinds of teacher learning needs.

Suggested follow-up resources

If you are interested in learning more about exemplary professional development materials that can help teacher educators plan and facilitate mathematical experiences-as-learners, we recommend the following resources:

Corwin, R.B., Price, S.L., and Storeygard, J. (1996). *Talking mathematics: Resources for developing professionals*. Portsmouth, NH: Heinemann.

This multi-media package is intended to support teacher educators in planning professional development for elementary teachers to help them promote and facilitate in their classes the kind of mathematical discourse recommended by the NCTM Standards. A main component of the proposed professional development program are experiences-as-learners where the teachers engage in a number of mathematical problems, chosen because they are mathematically rich and "engaging" yet accessible to elementary students. The materials include a facilitator guide, videotapes providing images of elementary classrooms engaged in mathematical discourse and a book for the participants. The facilitator guide provides considerable support for setting-up and facilitating the suggested experiences-as-learners.

Friel, S.N., and Joyner, J.M. (Eds.). (1997). *Teach-Stat for teachers: Professional development manual*. Palo Alto, CA: Seymour.

This manual is intended to support teacher educators interested in replicating the 3-week summer institute developed and field-tested by the NSF-funded Teach-Stat project. This program was designed to prepare

elementary teachers to teach statistics and at its core has a carefully-designed series of experiences where the teachers themselves learn statistics in the inquiry-oriented way they are expected to encourage in their students. The manual provides valuable directions and support about how to plan and implement the summer institute.

Fonzi, J., and Borasi, R. (2000). *Orchestrating math experiences for teachers.* (videotape + facilitator's guide) (available from the authors).

This 50-minute videotape features the mathematical inquiry on area described in Illustration 1. The accompanying guide provides additional information about and a commentary on this mathematical learning experience and a rich set of questions to help teacher educators use the illustration to design similar mathematical learning experiences for teachers.

Fonzi, J., and Borasi, R. (2000). *Promoting focused reflections on learning experiences.* (videotape + facilitator's guide) (available from the authors).

This 40-minute videotape features excerpts from three reflective sessions that followed the inquiry on area featured in *Orchestrating math experiences for teachers* and another inquiry on the topic of tessellations. Taken together, the three sessions illustrate complementary ways to focus and structure follow-up reflections, a critical component of effective experiences as learners. The accompanying guide offers additional information about and a commentary on the illustrations and questions to help teacher educators analyze what it takes to successfully design and facilitate this kind of reflective session.

Borasi, R., and Fonzi, J. (in preparation). *Introducing math teachers to inquiry: A framework and supporting materials for teacher educators.* (multi-media package) (available from the authors).

This multimedia package supports mathematics teacher educators who want to implement a professional development program to begin the process of school reform. It shows teacher educators how to design experiences as learners that introduce teachers to an inquiry approach to teaching mathematics. The package contains two 2-hour-long videos, each featuring an experience-as-learners. The CD-ROM included in the package contains a detailed set of artifacts from these experiences and suggestions for implementing similar ones.