Appendix B

ANNEX

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ANNEX 1 – ISDA-LIBA-TBMA COUNTERPARTY RISK MARKET SURVEY 2003

ANNEX 1

ISDA-LIBA-TBMA
COUNTERPARTY RISK MARKET SURVEY 2003

A- INTRODUCTION AND BACKGROUND

ISDA initiated a dialogue with the Models Task Force on a possible review of the counterparty risk treatment of OTC derivative transactions in 2001. The Capital Accord reform seemed the perfect opportunity to undertake this review, and ISDA presented concrete proposals to this end in its commentary on CP2, published in May 2001. The Models Task Force unfortunately chose to carve out derivatives counterparty risk from the scope of the New Accord, a decision contrasting with the steps simultaneously taken by the Capital Group to amend the counterparty risk treatment of repurchase agreements and stock lending. Although the industry welcomes the recognition of portfolio VaR modelling for repo-style transactions, the Associations are concerned that the current divergence of treatment between derivatives and repos may preclude the "regulatory" netting of future exposure between these products, despite the many common economic features between them (repos can be represented as forwards). At worst, industry endeavours to achieve a higher degree of cross product netting may be frustrated at source.

We remain hopeful however, that the Models Task Force and the Capital Group will jointly review the capital treatment of repos and OTC derivatives very shortly after the publication of the New Accord, with a view to implementing any necessary changes at the same time as the Accord.

In a spirit of co-operation, ISDA, TBMA and LIBA have designed a survey aimed at providing the Models Task Force with further information on (i) the modelling of future exposure arising from OTC derivative and securities financing transactions by member firms –Question One- ; (ii) collateral management practices for OTC derivatives – Question Two- ; (iii) the reasonableness of the weak independence assumption underpinning the May 2001 ISDA proposal on counterparty risk – Question Three-.

The contents of this survey were discussed with a subset of the Models Task Force before the survey itself was published.

We are reporting below on the survey findings, with a view to providing as detailed information as possible.

For confidentiality reasons it has not been possible to append a list of respondents, but some useful information on the type of firm and principal place of business of respondents is included immediately below.

B- SURVEY RESPONDENTS

Fifteen firms responded to the survey including three investment firms, and twelve internationally active banks. Most of these firms are major players in the securities financing and OTC derivatives markets. A geographical breakdown is provided in Table 1 below.

Table 1

Breakdown by region of survey respondents

	Europe	North America	Asia
Geography [%]	47%	40%	13%
Geography [number]	7	6	2

Most firms (14 out of 15) responded to Question One. The response rate was comparable for Question Two. However, only 40% of firms were able to produce the graphs requested in Question Three. Of the remaining 60%, most quoted technical hindrances and cost as primary motivations for not contributing.

Table 2

f the Survey Completed

stion One

	lobally	Europe	North Am.	Asia
pleted	94%	47%	40%	7%

stion Two

	lobally	Europe	North Am.	Asia
pleted	87%	47%	33%	7%

stion Three

	lobally	Europe	North Am.	Asia
pleted	40%	20%	13%	7%

C- SURVEY FINDINGS

The survey consisted of three questions. Question One focused on the measures of future exposure used by firms to set credit limits and allocate capital internally, distinguishing between OTC derivatives and securities financing. Question Two aimed to provide a broad description of industry practices in the field of OTC derivatives collateral management. Question Three was designed to measure how well the weak independence assumption underpinning ISDA's response to CP2 was supported by firms' own exposure correlation data.

Question One
"Please detail the measures of counterparty credit exposure (e.g. : expected exposure, PFE, EPE) your firm uses for the specified purposes, distinguishing between collateralised and uncollateralised exposures."

Firms were asked to describe the measures of future exposure (FE) they use for managing counterparty risk in OTC derivatives and securities financing portfolios, as well as the methodology employed to obtain these measures. A distinction was drawn between FE measures used (i) for setting credit limits and (ii) for calculating counterparty risk economic capital. 14 out of the 15 firms included in the survey responded, showing uniformity of practice:

1- Strong consistency exists among respondents on the approach to setting counterparty risk limits in portfolios of OTC derivatives and securities financing transactions. The measure of exposure used in both cases is generally **peak exposure**, evaluated at a percentile comprised between 90% and 99%.

The exposure profile is measured on a full time to maturity basis for unsecured trades. The time horizon employed for collateralised positions generally reflects any applicable margining agreement. Where daily margining applies, the liquidation period retained varies between 1 and 15 days, and is shorter for securities financing (typically between 1 and 5 days) than for collateralised derivative trades (generally approximately 10 days).

2- For allocating economic capital, industry practice increasingly converges towards an EPE based standard. In OTC derivative portfolios, 9/15 of respondents use Expected Positive Exposure (EPE) or loan equivalent exposure for measuring FE. The latter metric is generally presented as conceptually consistent with EPE, albeit more conservative : some firms set it equal to EPE plus an upward adjustment capturing counterparty credit quality and/or concentration risk. 3/15 of respondents use peak exposure.

For securities financing transactions, only four firms filled in the questionnaire. ¾ use adjusted EPE as a basis for calculating counterparty risk capital.

Going forward, respondents aspire to achieving a higher degree of netting. Legal agreements allowing cross product netting, such as the Cross Product Master Agreement [CPMA], promoted by TBMA and ISDA, or the ISDA Agreement Bridge, were designed to enable this further level of offsetting. Beyond the netting of current market values, firms are increasingly focusing on the netting of future exposure. This ideally requires to use a common measure of future exposure across the array of products embedded under cross product netting agreements. In the light of counterparty risk market practice, it would be sensible to base this common measure upon EPE.

3 -Firms generally **apply the same exposure modelling methodology for setting credit limits and for calculating economic capital**. **Monte Carlo simulation is the approach of choice for portfolios of OTC derivatives.** Where it is not, or only partially used, firms often mention their intention to move to full Monte Carlo in the near term future.

Less elaborate modelling is employed for securities financing, including historical simulation, variance-covariance and MTM+add-ons.

The level at which aggregation of future exposure occurs generally reflects the sophistication of modelling. Most firms using an add-on based measure do not, or only partly, net FE at counterparty level. By contrast, all firms using Monte Carlo simulation do so.

The detail of the responses is provided in the tables below.

Table 3

OTC derivatives - Measures of future exposure used for setting counterparty risk limits

Firm	Measure of exposure used	How is exposure profile produced	How are transactions aggregated	Time horizon
1	97.5% peak exposure	Monte Carlo simulation	At counterparty level (*)	Time to maturity
2	95% peak exposure	MTM + internal add-on	Counterparty level	Time to maturity
3	97.5% peak exposure	Monte Carlo simulation	Counterparty level	Time to maturity for uncollateralised 10 days for collateralised
4	98% peak exposure	Variance Covariance	Counterparty level	Time to maturity for uncollateralised
5	95% peak exposure Limit schedule is imposed on PFE profile. Lower limits are imposed on longer term exposures.	Monte Carlo simulation	Counterparty level	Time to maturity for uncollateralised 10 days for collateralised
6	Peak exposure	MTM + add-on Monte Carlo to be used in future	Counterparty level.	Time to maturity
7	97.5% peak exposure	Monte Carlo simulation	Counterparty level	Time to maturity
8	99% peak exposure	Analytical approximation to Monte Carlo simulation	Counterparty level	Time to maturity
9	95% peak exposure	Revaluing in three scenario, 2 extre-me, one intermediate	Counterparty level Netting of trades at risk factor level	Time to maturity for uncollateralised 15 days for collateralised
10	97.7% peak exposure	Mainly Monte Carlo simulation For exposures not covered by Monte Carlo, MTM+add-on	Counterparty level No netting of add-ons for counterparties falling outside the scope of Monte Carlo simulation	Time to maturity for all transactions
11	95% peak exposure	Monte Carlo simulation for large customers, otherwise MTM + add-on For some trade, a constant future exposure is used	Counterparty level. A discount is applied to the sum of add-ons to reflect netting	Time to maturity
12	Peak exposure 90%	Monte Carlo simulation	Counterparty level	Time to maturity
13	Peak exposure 95%	Delta-gamma approximation with variance covariance Full Monte Carlo for more exotic products	Counterparty level	For unsecured, time to maturity For collateralised, liquidation period for daily margining
14	Risk equivalent exposure (EPE based)	Monte Carlo simulation [both derivative exposure and counterparty credit risk]	Counterparty level	10 day interval at each point in time in the simulation
15				

(*) Counterparty level means that netting is applied within netting groups, according to available documentation. Positive exposures are generally summed across netting groups at counterparty level.

Table 4

OTC derivatives - Measures of future exposure used for allocating capital internally for counterparty risk

Firm	Measure of exposure used	How is exposure profile produced	How are transactions aggregated	Time horizon
1	Expected credit exposure	Monte Carlo simulation	At counterparty level	Time to maturity
2	Uncollateralised : MTM+add-on Collateralised : 95% peak	MTM + add-on Collateralised : variance covariance In future : Monte Carlo	Counterparty level	Uncollateralised : time to maturity Collateralised : 10 days
3	No internal economic capital allocation			
4	98% peak exposure	Variance Covariance	Counterparty level	1 year for uncollateralised
5	Loan equivalent exposures Currently building model allowing for cross risk factor diversification	Monte Carlo simulation	Counterparty level	Time to maturity for uncollateralised 10 days for collateralised
6	Loan equivalent exposure	MTM + add-on Monte Carlo to be used in future	By transaction type	Time to maturity
7	Loan equivalent exposure	Monte Carlo simulation	Counterparty level	Time to maturity
8	Loan equivalent exposure	Analytical approximation to Monte Carlo simulation	Counterparty level	Both 1 year and time to maturity
9	Currently no economic capital calculation is performed. EPE was used in the past	Same scenarii used as for setting credit risk limits	Trades with one counterparty are partitioned by risk factor. Summation is used across risk factors	1 year for uncollateralised 15 days for collateralised
10	Peak of expected exposure profile	Mainly Monte Carlo simulation For exposures not covered by Monte Carlo, MTM+add-on	Counterparty level No netting of add-ons for counterparties falling outside the scope of Monte Carlo simulation	Peak expected exposure calculated on time to maturity basis, then used in a capital model of horizon equal to 1 year
11	EPE	Monte Carlo simulation for large customers, otherwise MTM + add-on Add-on conservatively set at 95% worst case	Counterparty level. A discount is applied to the sum of add-ons to reflect netting	1 year
12	EPE, with upward adjustment reflecting wrong way risk, counterparty rating and concentration risk	Monte Carlo simulation	Counterparty level	1 year
13	EPE, with adjustment reflecting concentration risk	Delta-gamma approximation for simple products, full Monte Carlo for exotic products.	Counterparty level	1 year (less where margining applies)
14	Risk equivalent exposure (EPE based)	Monte Carlo simulation [both derivative exposure and counterparty credit risk]	Counterparty level	10 day interval at each point in time in the simulation
15				

Table 5

Repos - Measures of future exposure used for allocating internal economic capital

Firm	Measure of exposure used	How is exposure profile produced	How are transactions aggregated	Time horizon
1				
2	Current value	MTM In future : Monte Carlo	Counterparty level	NA
3				
4				
5				
6				
7				
8	Loan Equivalent exposure	Analytical approximation to Monte Carlo simulation	Counterparty level	Both 1 year and time to maturity
9				
10				
11				
12	EPE with adjustment	Monte Carlo simulation	Counterparty level	Min (1 year, maturity)
13	EPE, with adjustment reflecting concentration risk	Historical simulation	Counterparty level	1 year (less where margining applies)
14				
15				

Table 6

Repos - Measures of future exposure used for setting counterparty risk limits

Firm	Measure of exposure used	How is exposure profile produced	How are transactions aggregated	Time horizon
1	Peak exposure	Model based on maximum daily changes in risk factors	At counterparty level	1 day
2	Peak exposure	MTM + add-on (based on actual market volatilities)	Counterparty level	3 days
3	Peak exposure 97.50%	Variance Covariance Moving to historical simulation	Counterparty level	10 days
4	98% peak exposure	Variance Covariance	Transaction level	3 days for suitably documented trades Otherwise time to maturity
5	Peak exposure	MTM+ add-on Moving to full fledged VaR models	Transaction level	Between 3 and 10 days
6	Peak exposure	MTM + add-on	Transaction level	Time to maturity
7	97.5% peak exposure	Monte Carlo simulation	Counterparty level	Time to maturity
8	99% peak exposure	Analytical approximation to Monte Carlo simulation	Counterparty level	Time to maturity
9	95% peak exposure	Revaluing transaction assuming 95th worst case move in underlying govt yield curve + spread curve	Counterparty level Individual transactions are aggregated allowing for consistency in yield/spread moves	5 days
10	97.7% peak exposure		Counterparty level	Time to maturity for all transactions
11	95% peak exposure	MTM + add-on	Counterparty level. A discount is applied to the sum of add-ons to reflect netting	Time to maturity
12	Peak exposure 97.50%	Repo : Variance covariance Others : MTM + add-on	Counterparty level	Time to maturity
13	Peak exposure 95%	Historical simulation	Counterparty level	Time to maturity, down to liquidation period if daily margining applies
14	99% peak exposure	Monte Carlo simulation [both derivative exposure and counterparty credit risk]	Counterparty level	10 day interval at each point in time in the simulation
15				

Question Two:
Question Two aimed to gather information on firms' collateral management practices in OTC derivatives portfolios.

13 out of 15 firms responded.

Question 2.1 : Percentage of derivatives portfolio collateralised

Respondents indicated that between a third and two thirds of their transactions had collateral attached to them [1]. The percentage of total exposure collateralised was fairly stable across firms, at around 33%. These results are consistent with the 2002 ISDA Margin Survey (available on www.isda.org, see section 4.4. on page 12). The Margin Survey further analyses collateralisation practice by type of derivative, and shows that collateral taking is more developed on fixed income and FX derivatives at large firms.

Use of collateral is constantly growing : respondents to the 2002 Margin Survey report over 28,000 collateral agreements in place, compared with 16,000 in the 2001 Survey and 11,000 in the 2000 Survey. This trend is unlikely to abate, in view of the increasing number of corporate downgrades, and the recognition of a wider range of collateral under the New Capital Accord.

Only two respondents attempted to measure the impact of collateral use on their firm's economic capital. They respectively estimated their capital savings at 6.4% and around 25%.

[1] What is measured here is the number of collateralised transactions divided by the total number of transactions.

Question 2.2 : Types of collateral used

Responses received appear in the table below.

Table 7

Types of collateral used				
Firm No.	Cash	AAA gov't debt	Exposures to other investment grade counterparites	Exposures to non-investment grade counterparties
1	100%	0%	0%	0%
2	70%	25%	4%	1%
3	70%	30%	0%	0%
4	82.30%	15.50%	2.20%	0%
5	great majority		0%	0%
6	90%	10%	0%	0%
7	67%	21%	12%	0%
8	59%	15%	18%	8%
9	75%	17%	5%	3%
10	cash principally			
11	64%	36%	0%	0%
12	17.60%	1.00%	1.40%	0%
13	N/A	N/A	N/A	N/A
14	N/A	N/A	N/A	N/A
15	N/A	N/A	N/A	N/A

US dollar and Euro cash remain the most commonly used collateral assets, followed by government securities. 6 out of 12 respondents accept investment grade corporate bonds as collateral, whilst only 3 firms accept non investment grade bonds. These results again echo the 2002 ISDA Margin Survey : among the large firms, the percentage of those accepting and delivering corporate bonds as collateral has nearly doubled from 2001, jumping from 25% to 46% (see section 3.2., page 8).

Question 2.3 : Frequency of margining

Margining occurs daily with the vast majority of counterparties, as evident from table 8 below.

Table 8

How often is margining applied?				
Firm no.	Daily	Weekly	Monthly	Other
1	100%	0%	0%	0%
2	76%	14%	8%	2%
3	100%	0%	0%	0%
4	76.2%	16%	0%	7.8%
5	Majority	0%	0%	0%
6	Banks	0%	Corporates	0%
7	89.8%	3.7%	5.5%	1%
8	0%	Majority	0%	0%
9	87%	0%	13%	0%
10	N/A	N/A	N/A	N/A
11	82.2%	8.6%	7.2%	2%
12	59%	17%	17%	7%
13	89%	3%	6%	2%
14	N/A	N/A	N/A	N/A
15	N/A	N/A	N/A	N/A

Question 2.4 Collateral thresholds and minimum transfer amounts (MTAs)

13 out of 15 firms commented on this question.
Practice varies substantially from one respondent to the next, reflecting diversity of internal policy. A number of salient features are however worth reporting:

- For all respondents, collateral thresholds and minimum transfer amounts are set with regard to the credit quality of the counterparty. 7 respondents explicitly link threshold amounts to the external rating attributed to the counterparty. These firms also tend to vary minimum transfer amounts by rating.
- Depending on firms, between 25% and 50% of collateral agreements include thresholds, the majority of which (between 50% and 60%) are set below EUR/US$ 10MM. Thresholds of above EUR/US$ 50MM represent less than 10% of the total and are typically reflective of the very high (AAA/AA rating equivalent) credit quality of the counterparty.
- A high percentage (typically between 75% and 100%) of collateral agreements include minimum transfer amounts. MTAs for high quality counterparties tend to exceed EUR/US$ 500,000, whereas for low quality counterparties, they average EUR/US$ 100,000.

Question Three
Firms were asked to graph the distribution of pair-wise correlation and covariances of exposures in their OTC derivative and repo portfolios.

The intended purpose was to measure the degree of dependence between pairs of counterparties in firms' OTC derivatives and repo portfolios. Correlations or covariances clustering around zero for a substantial percentage of counterparties are an indicator of weak independence. For background, weak independence is the main assumption behind ISDA's original recommendation to the Basel Committee in favour of Expected Positive Exposure[2]. In portfolios where weak independence is not found, EPE is too lenient a measure of future exposure.

Six firms were able to produce the graphs. Only two in the sample analysed both OTC derivatives and securities financing portfolios. The others focused purely on derivatives.

The graphs confirm the existence of clustering around zero for both securities financing and OTC derivatives counterparties, at all firms but one. The exception seems, contrary to other contributors, to be an end-user of derivatives rather than a dealer. This would result in over-sensitivity to a very small number of market risk factors and the predominance of one-way exposures in their portfolio, resulting in clustering around correlations of –1 and +1.

The graphs, and where available, assumptions used by respondents, are reproduced below.

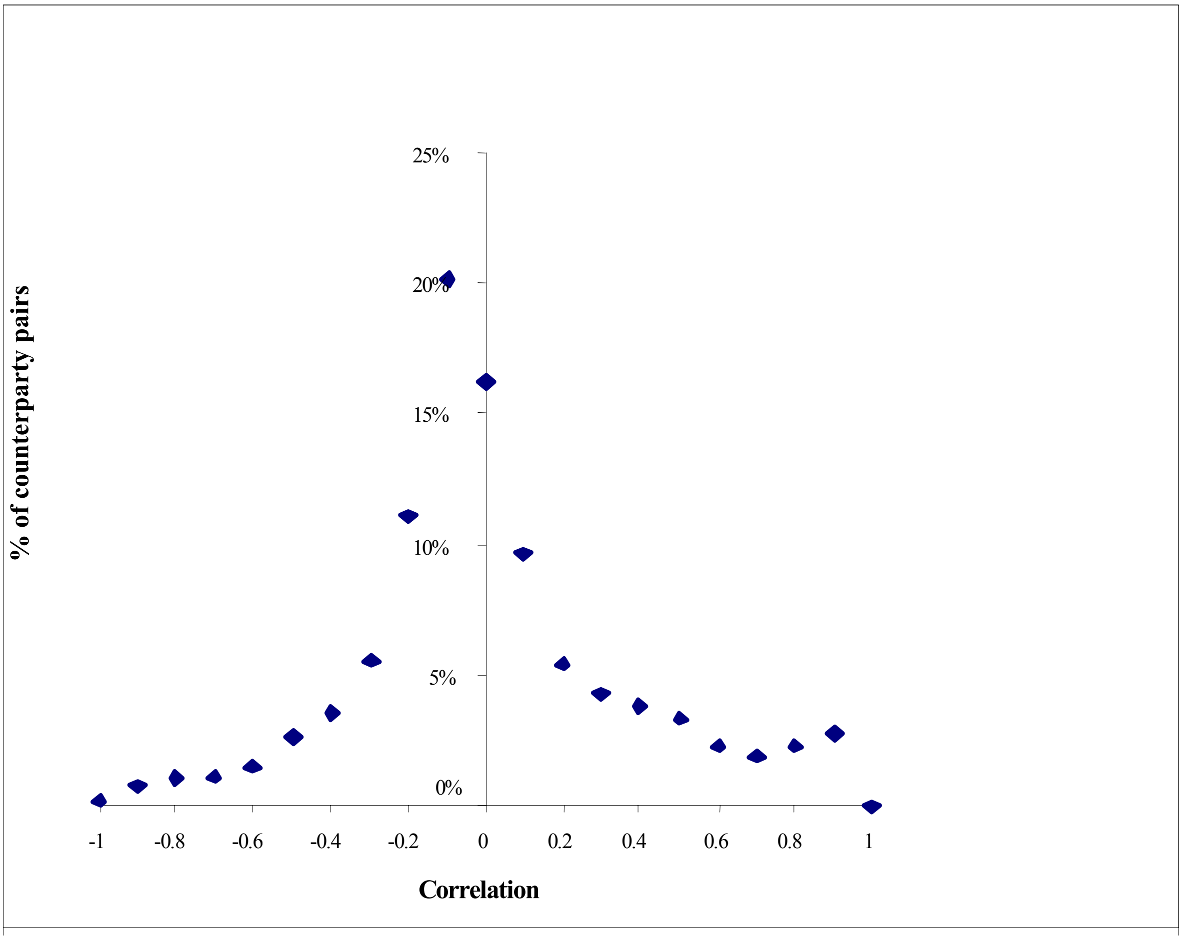
FIRM SEVEN

Assumptions:
Correlation between counterparty exposures is estimated using exposures simulated at the one-year point in the Monte Carlo simulation.
The exposures reflect the impact of netting and collateral.
Key points:

- Correlation is estimated between counterparty exposures
- Correlation is estimated at one future point in the Monte Carlo simulation, i.e., the one-year point.
- Collateralized and uncollateralized counterparties are not separated. The impact of collateral is reflected in the exposures.

[2] See ISDA's response to CP2, Annex 1, published May 2001

FIRM SEVEN



25%
20%
15%
10%
5%
0%
% of counterparty pairs
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Correlation

FIRM ELEVEN

The analysis is based on the fixed income derivatives outstanding with the 50 largest customers. Modelling proceeds as follows:

1. Generating 10,000 market rate scenarios starting today to 1 month in the future (only changes in interest and FX-rates were taken into account; volatilities and correlations were estimated using 4 years of historical data)

2. Calculating today's exposure (MTM value of portfolio) taking account of netting contracts. Collateralisation was ignored. This exposure number is fixed (non-stochastic) for each customer.

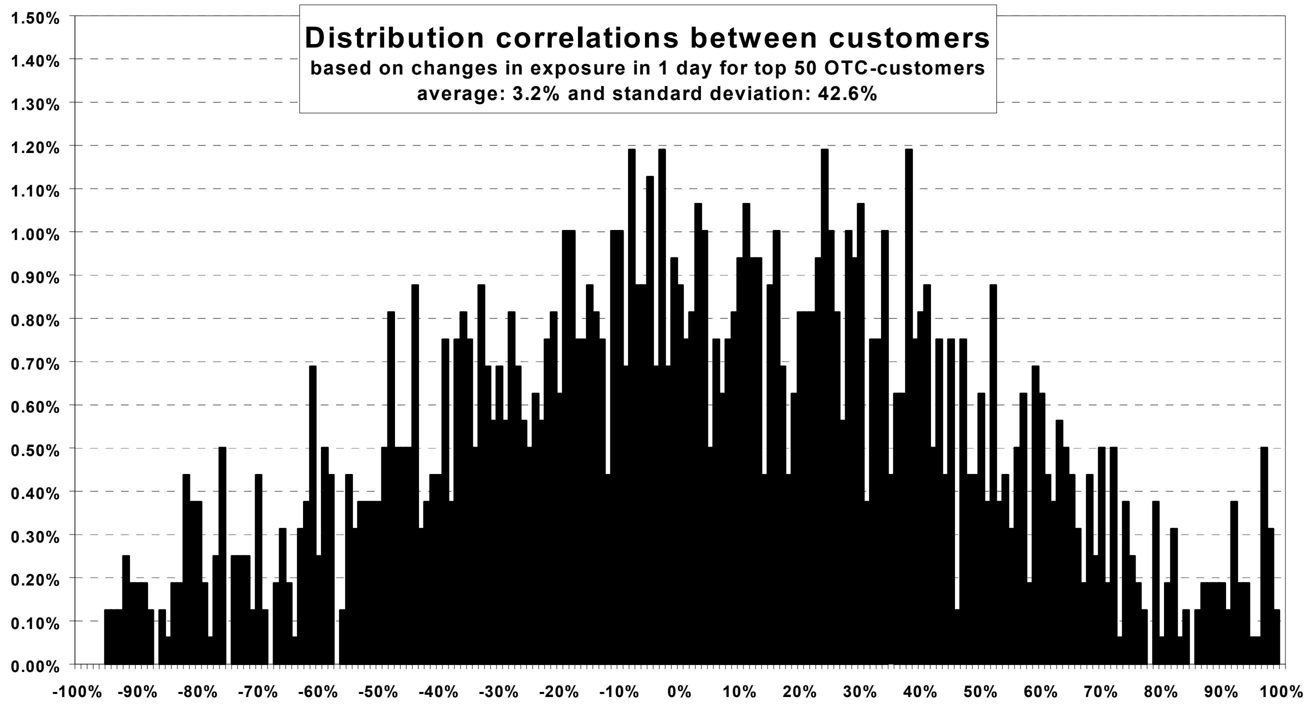
3. Calculating tomorrow's exposures and exposures in 1 month from today for each customer in each market scenario.

4. Calculating the change in exposure between today and tomorrow (as input for 1-day correlation) and the change in exposure between today and 1 month in the future (as input for 1-month correlation), for each customer and in each market scenario.

5. Calculating for each pair of customers the 1-day and the 1-month correlation based on the change in exposure over 1 day or 1 month. In this analysis, the changes in exposures for different customers are compared and based on the same market scenario.

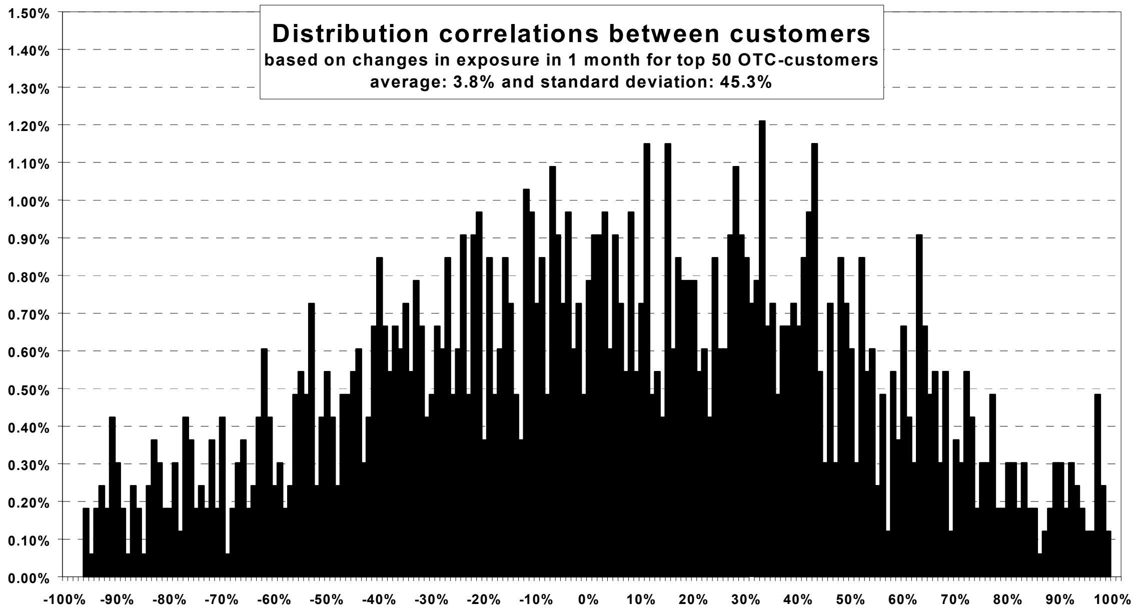
Please find below the histograms obtained following this analysis. The difference between the histogram of 1-day and 1-month correlations is limited. The distribution of both correlations is quite close to a uniform distribution, which supports the weak independence assumption.

FIRM ELEVEN



Distribution correlations between customers
based on changes in exposure in 1 day for top 50 OTC-customers
average: 3.2% and standard deviation: 42.6%
1.50%
1.40%
1.30%
1.20%
1.10%
1.00%
0.90%
0.80%
0.70%
0.60%
0.50%
0.40%
0.30%
0.20%
0.10%
0.00%
-100%
-90%
-80%
-70%
-60%
-50%
-40%
-30%
-20%
-10%
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%

FIRM ELEVEN


Distribution correlations between customers
based on changes in exposure in 1 month for top 50 OTC-customers
average: 3.8% and standard deviation: 45.3%
1.50%
1.40%
1.30%
1.20%
1.10%
1.00%
0.90%
0.80%
0.70%
0.60%
0.50%
0.40%
0.30%
0.20%
0.10%
0.00%
-100%
-90%
-80%
-70%
-60%
-50%
-40%
-30%
-20%
-10%
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%

FIRM TWELVE

Our internal exposure simulation engine generates exposure distributions at each of 150 future time points per counterparty. The distribution *at each of these time points* consists of 1000 values. FX and IR moves for all major currencies / currency pairs are simulated to obtain these exposures (with netting agreements and collateral taken into account).

We obtained the correlation and covariance results for the non-repo OTC portfolio study using the simulation engine mentioned above. For the repo study we used the delta approach proposed by ISDA.

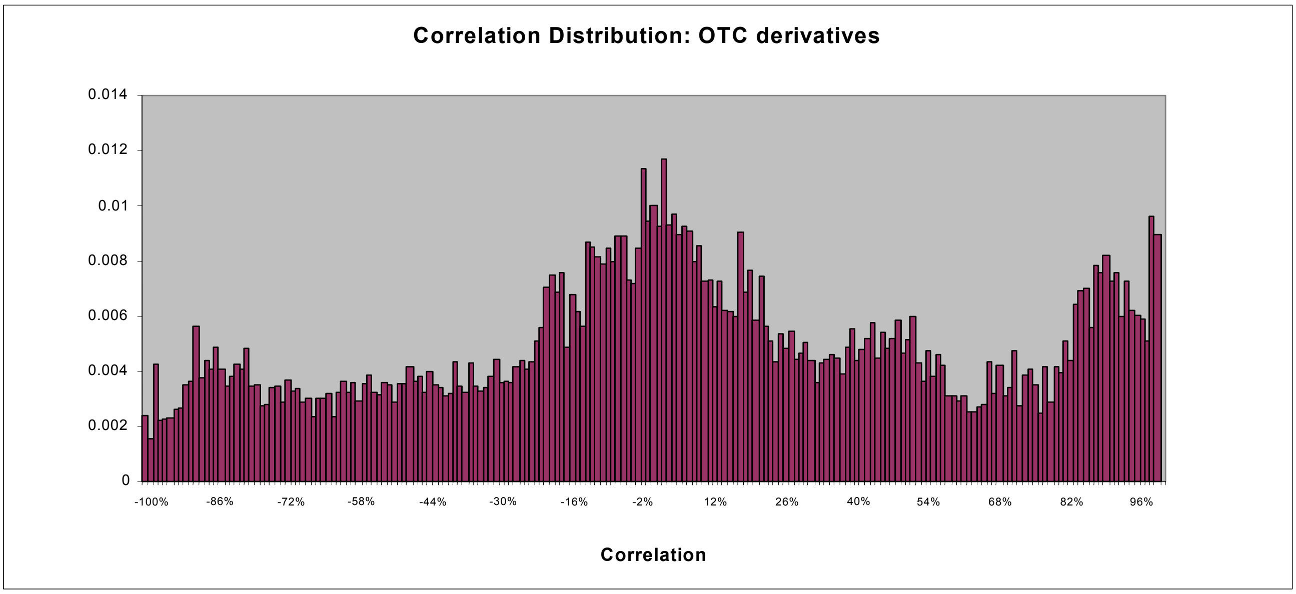
Due to the amount of processing time involved we concentrated on two hundred counterparties (representing the 'biggest risks' as measured by portfolio risk contribution) for the non-repo portfolio study. The figures and tables below summarise the results *at the one year horizon*. The data shown includes aggregated collateralised and uncollateralised OTC derivatives: we were not able to provide a finer breakdown.

The data indicates that the non-repo portfolio has an average correlation across the portfolio of 7.19%. We point out that this figure is the average across the top two hundred risk contributors (not the whole portfolio or 200 randomly selected counterparties).

The repo results provided are for the entire repo portfolio, not a subset.

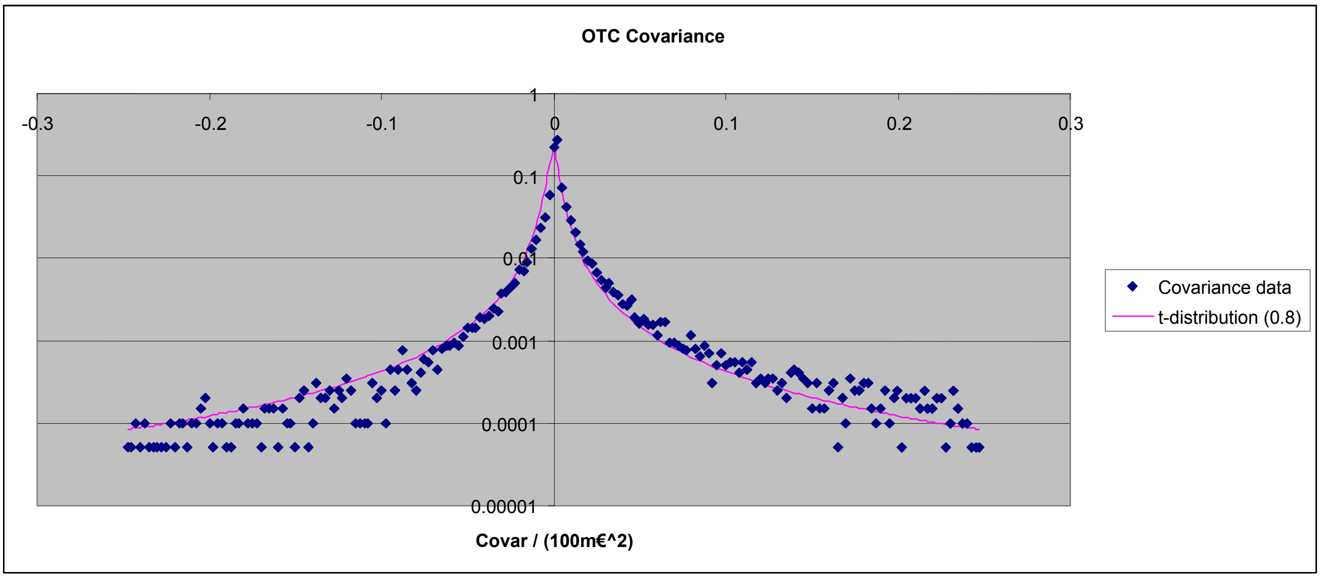
FIRM TWELVE

Frequency Distribution of OTC counterparty correlations



<u>FIRM TWELVE</u>

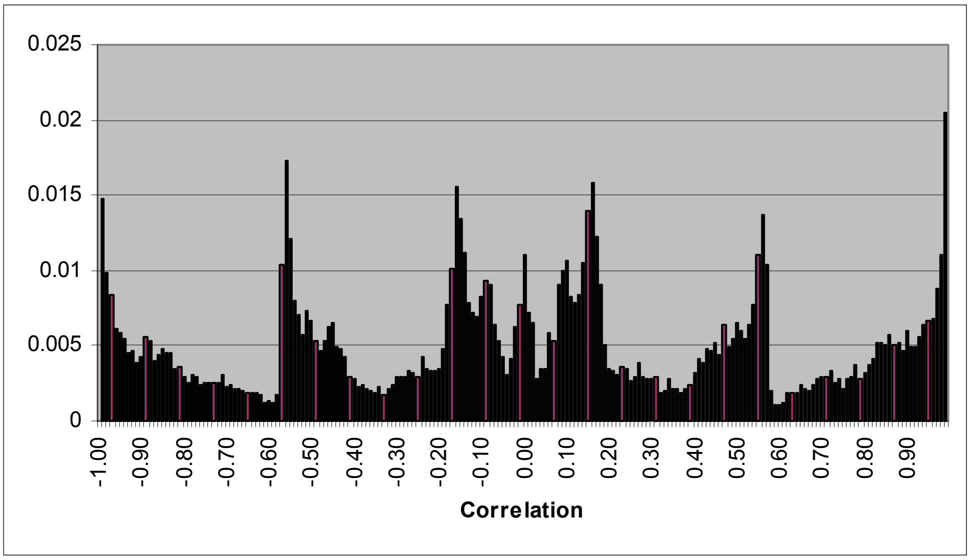
<u>Frequency distribution for OTC covariances</u>



FIRM TWELVE

Securities Financing

Probability density of correlations

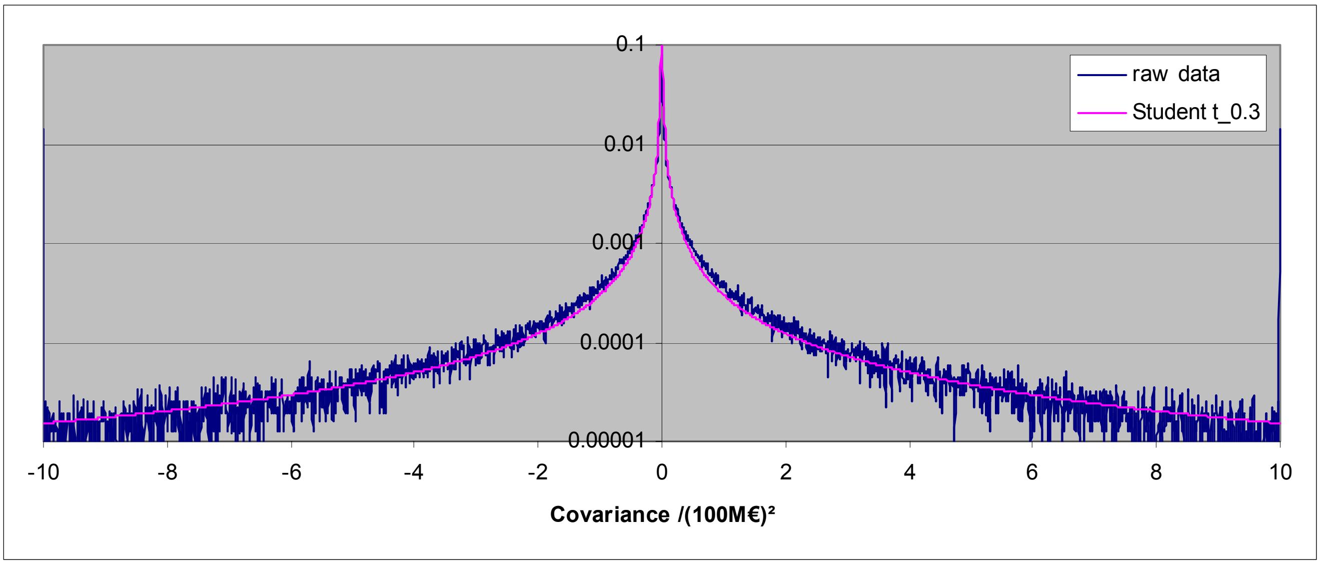


The symmetry in this graph appears to reflect the fact that risk is analysed using only the deltas.

FIRM TWELVE

Probability density of covariances

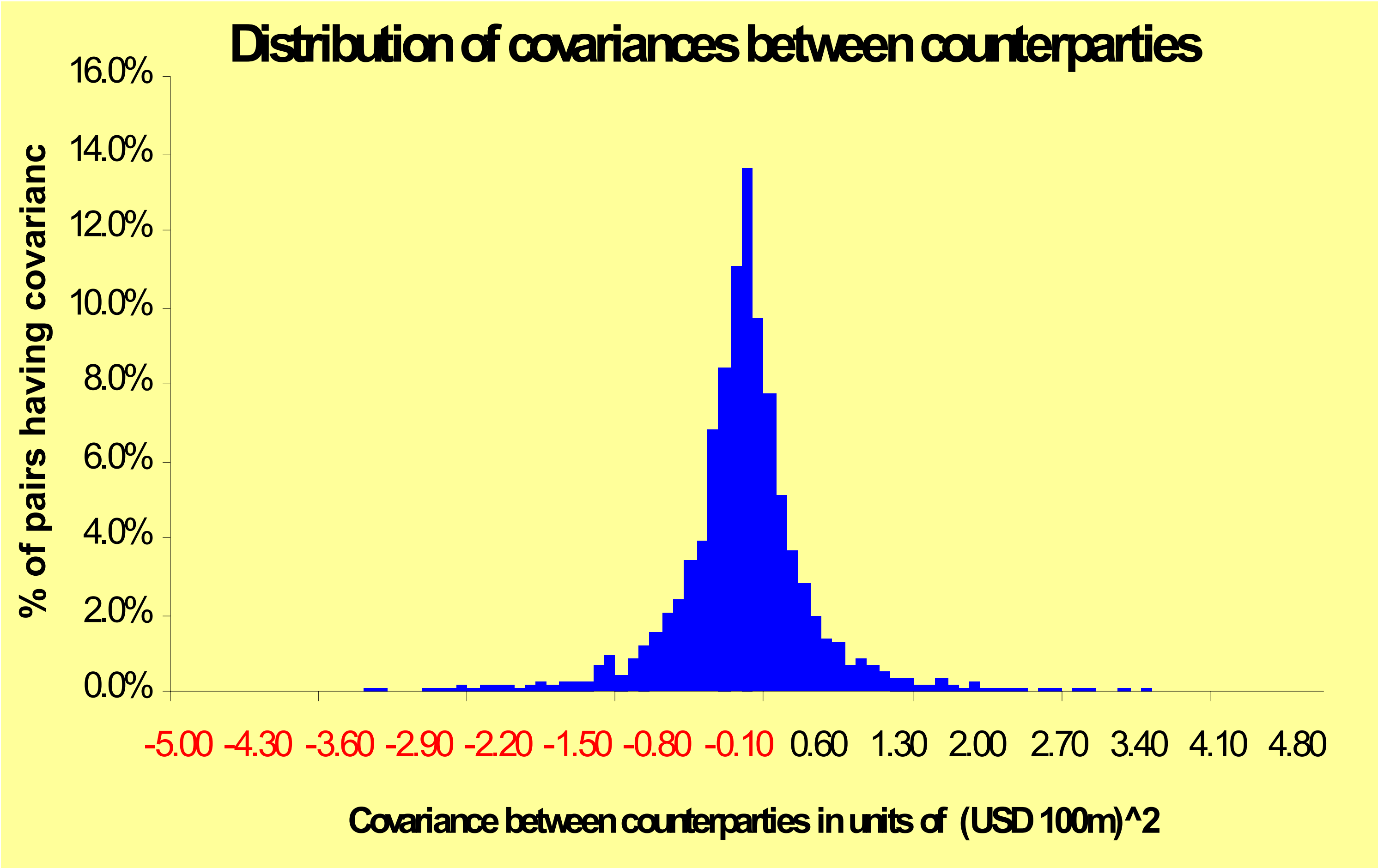
Note that the vertical scale is logarithmic.



About 2% (1% on each side) of the data points lie outside the range $\pm 10^{17}$€^{2} and hence are missing from the graph. The covariances incidentally follow very closely a Student t distribution with 0.3d.f.

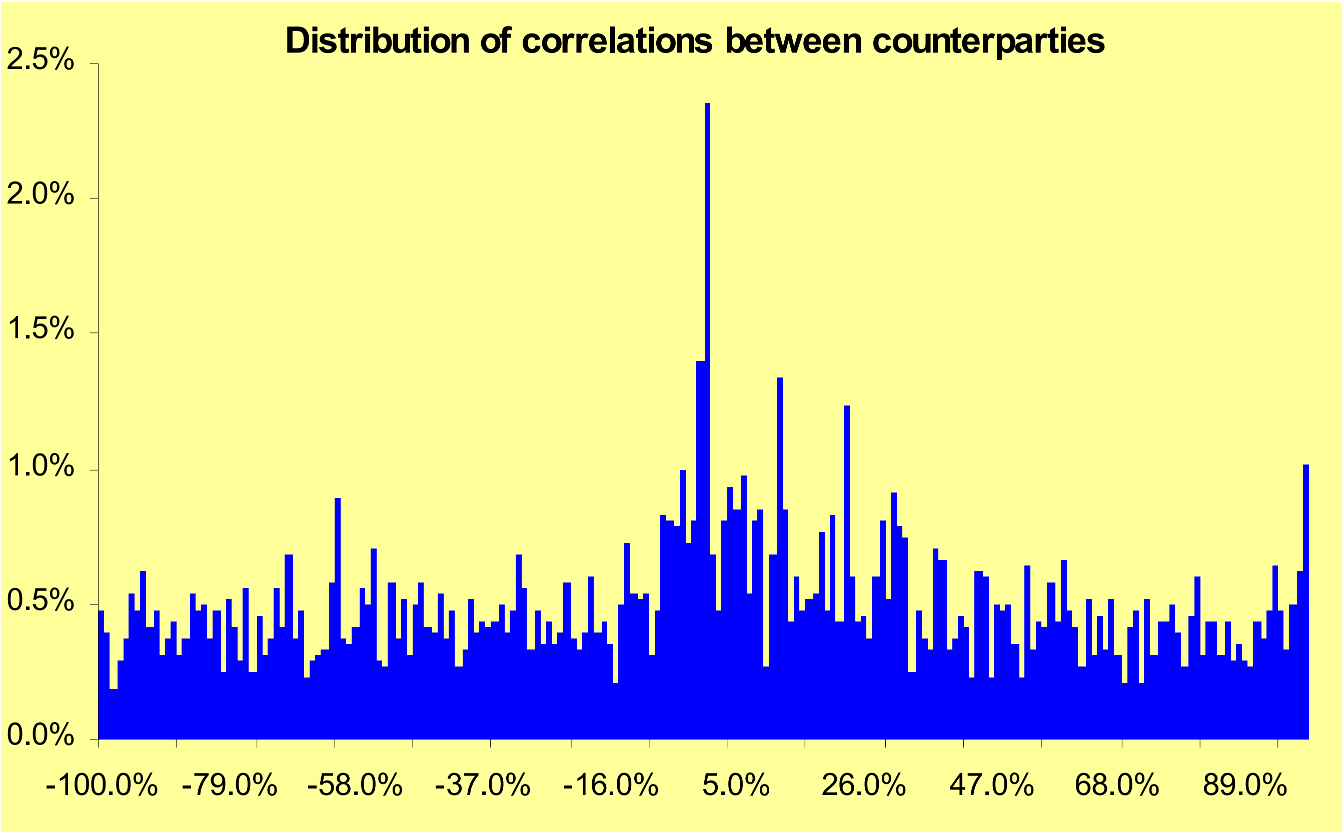
FIRM THIRTEEN

Covariance chart- OTC derivatives

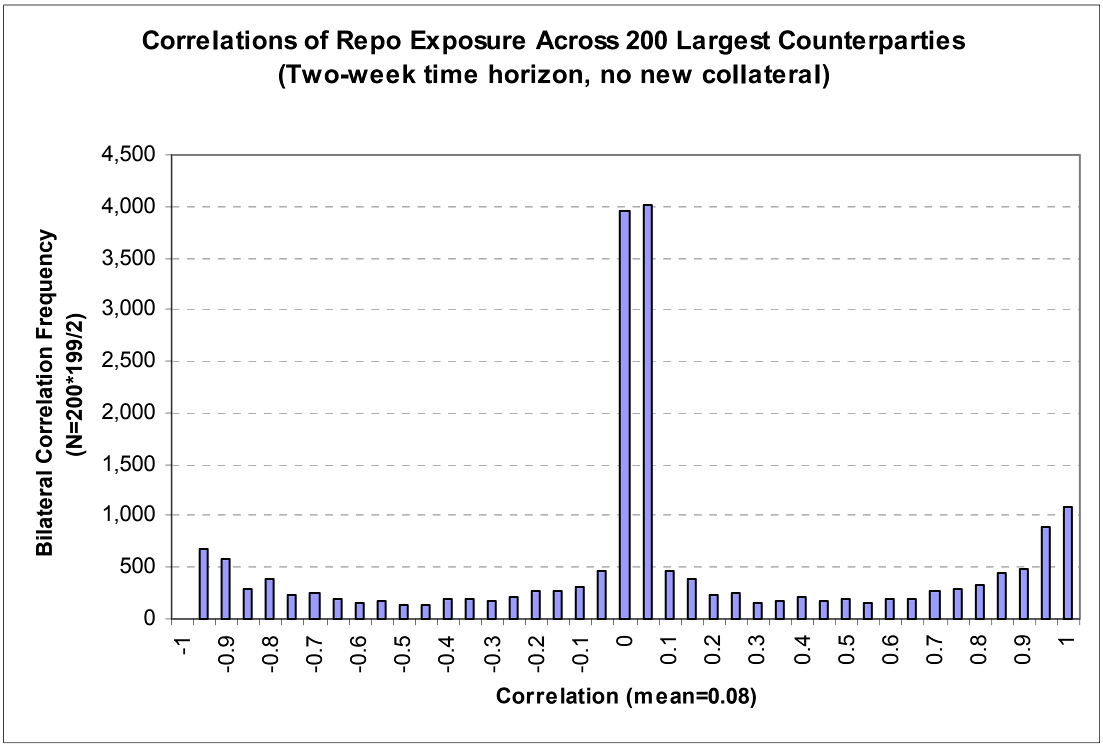


FIRM THIRTEEN

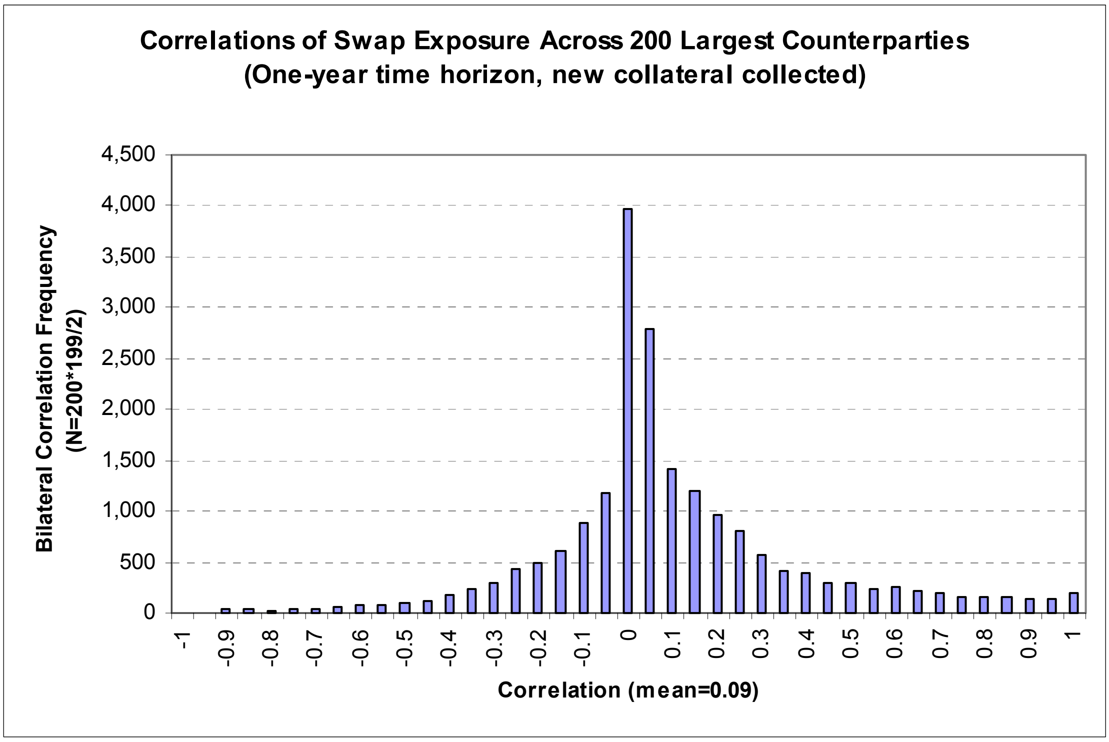
Correlation chart – OTC derivatives



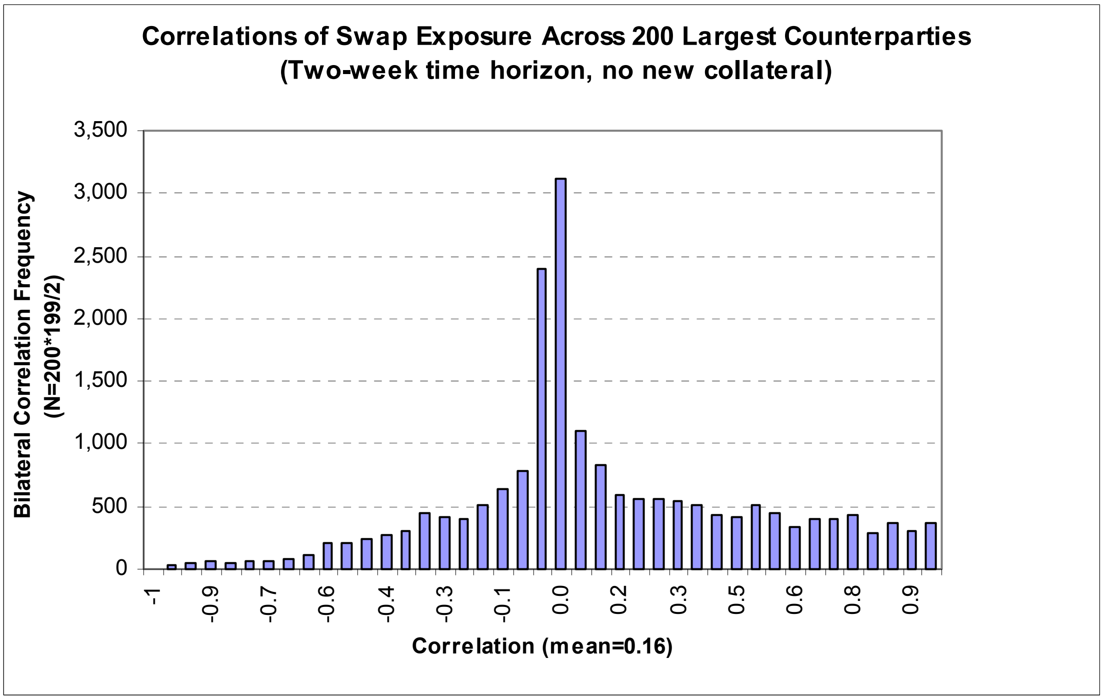
FIRM FOURTEEN



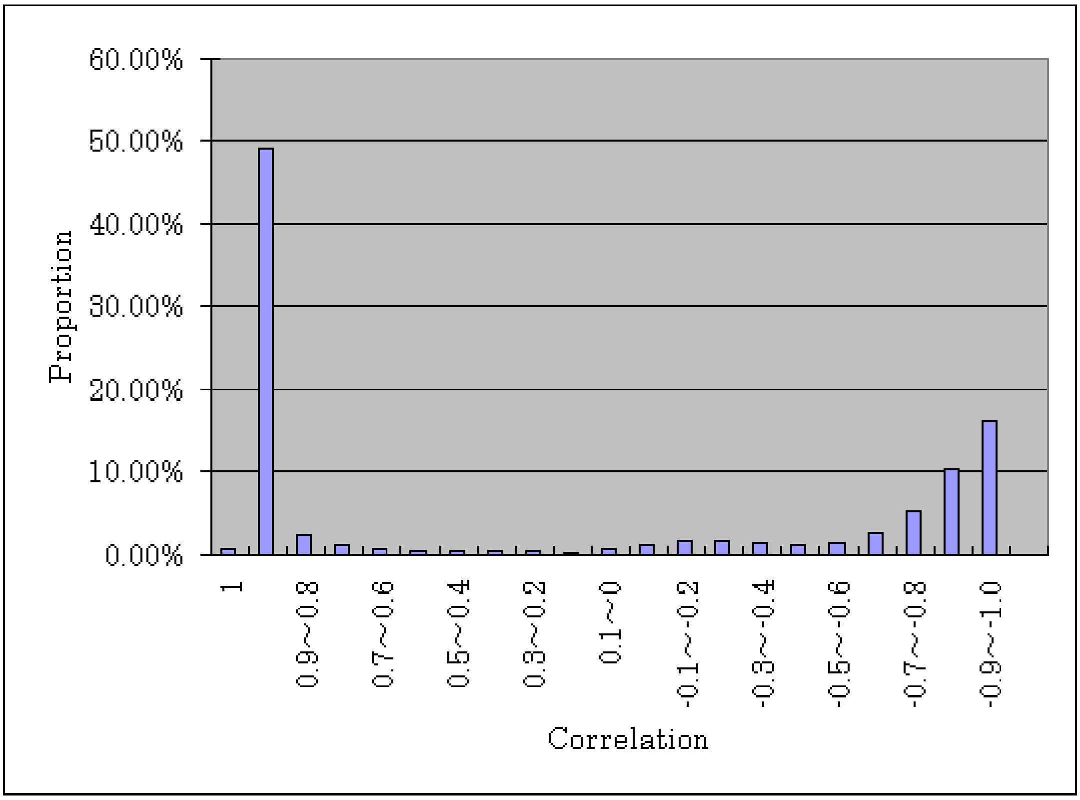
FIRM FOURTEEN



FIRM FOURTEEN



FIRM FIFTEEN



APPENDIX: COUNTERPARTY RISK MARKET PRACTICE SURVEY

ISDA	LIBA	THE BOND MARKET ASSOCIATION
International Swaps and Derivatives Association, Inc. One New Change London, EC4M 9QQ Telephone: 44 (20) 7330 3550 Facsimile: 44 (20) 7330 3555 email: isda@isda-eur.org website: www.isda.org	LONDON INVESTMENT BANKING ASSOCIATION 6 Frederick's Place London, EC2R 8BT Tel: 020 7796 3606 Fax: 020 7796 4345 e-mail: liba@liba.org.uk website: www.liba.org.uk	THE BOND MARKET ASSOCIATION 40 Broad Street New York, NY 10004-2373 Tel.: 212.440.9400 Fax: 212.440.5260 website: www.bondmarkets.com

25 February 2002

Counterparty risk market survey

Dear All,

The Models Task Force has informed ISDA that, in view of the Basel Committee's revised timetable, they will not include a review of counterparty risk for OTC derivatives in CP3. They are willing, however, to revisit the subject as soon as the New Capital Accord has been finalised; this is likely to happen in late 2003. Provided we act in a timely manner, changes in the capital treatment of counterparty risk that are developed after publication of the Accord could be implemented at the same time as the Accord itself.

With regard to the content of the survey itself, the MTF has requested a more targeted and detailed view of the proposed measures of counterparty risk than we originally suggested. We have consequently revised the survey and designed the questions to provide information regarding the following regulatory concerns:

Expected exposure - The MTF has objected to the adoption of expected exposure on the grounds that it is not a commonly used measure. ISDA has responded that firms use different measures for different purposes, that expected exposure is the appropriate measure for economic capital allocation, and that some firms have adopted more sophisticated measures that are consistent with expected exposure.

Secured financings - ISDA has argued that issues applying to OTC derivatives also apply to secured financing and that the approach taken to measuring future exposure should be consistent for both.

Margin practices – The MTF has requested information regarding the convergence of derivatives collateral management practices with securities financing management practices.

Weak independence - The MTF has questioned the reasonableness of the weak independence assumption, which underpins use of expected positive exposure.

Attached is the final version of the survey form. **Please send your completed survey form to Emmanuelle Sebton (esebton@isda-eur.org) by 31 May 2002.** Should you require additional information, please do not hesitate to contact Emmanuelle in London, or David Mengle in New York (dmengle@isda.org).

Yours sincerely,

Emmanuelle Sebton	David Mengle	Katharine Seal
ISDA	ISDA	LIBA
Head of Risk Management	Head of Research	Director

Omer Oztan
The Bond Market Association
Vice President and Assistant General Counsel

Counterparty Risk Market Survey

ISDA, LIBA and TBMA members are kindly requested to provide the following information regarding their counterparty risk management practices ***by 31 May 2002****.*

Question 1: Measures of counterparty credit exposure
Please detail the measures of counterparty credit exposure (e.g., expected exposure, PFE, EPE) your firm uses for the specified purposes. Please distinguish between treatment of collateralized and uncollateralized exposures.

A. OTC derivatives
   1. Setting counterparty risk limits
      a. What measure do you use?

      b. How do you approximate the exposure profile?

      c. How do you aggregate exposures (by transaction or on portfolio basis)?

      d. What time horizon do you use for estimation of the exposure profile?

   2. Economic capital calculation
      a. What measure do you use?

      b. How do you approximate the exposure profile?

      c. How do you aggregate exposures (by transaction or on portfolio basis)?

      d. What time horizon do you use for estimation of the exposure profile?

Other uses: Please detail in Remarks section on following page.

B. Securities financing (includes both repurchase agreements and securities lending)
   a. What measure do you use?

   b. How do you approximate the exposure profile?

   c. How do you aggregate exposures (by transaction or on portfolio basis)?

   d. What time horizon do you use for estimation of the exposure profile?

Question 1 (continued): Measures of counterparty credit exposure

Remarks: (Please specify additional detail, including other uses of counterparty exposure measures used, which might be useful to supplement the above information.)

Question 2: Collateralisation practice.

Please specify:

1. What percentage of your OTC derivatives portfolio is collateralised

2. Types of collateral used

Collateral type	**Percent of exposures covered**
Cash	
AAA government debt	
Exposures to other investment grade counterparties	
Exposures to non-investment grade counterparties	

3. How often margining is applied (daily, weekly, monthly, other)

4. What collateral thresholds or minimum transfer amounts apply? Please specify if they vary by rating.

Question 3: Weak independence.

Please graph separately the distribution of correlations and covariances in the following portfolios: (a) collateralised (margined) OTC derivatives; (b) uncollateralised OTC derivatives; and (c) repurchase agreements. The resulting graphs should show:

(i) Proportion of counterparty pairs having given correlation (in appropriate bands, e.g. bands of 1% from –100% to + 100%); and

(ii) Proportion of counterparty pairs having given covariance (in appropriate bands as for correlation).

We encourage firms to use the modelling approaches and parameterisation they believe appropriate for producing the graphs mentioned above. Please include a brief description of the key assumptions used.

If it is not feasible to use an internal model, we suggest responding firms use the following alternative methodology:

1. For each counterparty, compute the FX and IR deltas in each major currency (as a guide, an internationally active bank will typically need at least 10 currencies and therefore used 20 risk drivers to capture material risks, but it may not be necessary to include "minor" drivers such as equity and commodity risk). **Spot deltas may be used, but we encourage firms to use average risk over one year if possible.**
2. Compute the risks in appropriate units, namely FX deltas in US$ and IR deltas in US$ per basis point. Compute the covariance between each pair of counterparties A and B using the standard formula:

Cov(A, B) = sum over risk types R and S of [delta of A in R] * [delta of B in S] * Cov(R, S).

It is important to ensure covariances Cov(R, S) between market factors are expressed in the correct units, consistent with the units used for the FX and IR deltas. Covariances should cover a one year horizon.

3. Compute the correlation between A and B as:

   Corr(A, B) = Cov(A, B) / [SD(A) * SD(B)]

where SD(R) = the annualised standard deviation (annual volatility) for market factor R, and so on.

4. Graph the covariances and correlations as described above.

ANNEX 2 - New Proposal To Ascertain Credit Equivalent Amount for Counterparty Credit Risk

ANNEX 2

MEMORANDUM TO: Emmanuelle Sebton, ISDA
FROM: Evan Picoult
RE: New Proposal To Ascertain Credit Equivalent Amount for Counterparty Credit Risk.
DATE: Originally Written, September 23, 2002
Edited with minor changes, March 24, 2003

SUMMARY OF MOTIVATION FOR PROPOSAL

In 2001 ISDA proposed that the Credit Equivalent Amount (CEA) of counterparty exposure should be defined as the average of the counterparty's expected exposure over a one-year horizon.

ISDA's argument in support of this proposal was made in the context of some broad assumptions about the characteristics of a bank's total counterparty credit exposure and risk.

The proposal I am putting forward has as its main objective the measurement of the effects of different characteristics of a bank's total portfolio of counterparty exposure on the loan equivalent economic capital – i.e. on the CEA for counterparty exposure.

The type of characteristics that I propose varying are:

- The effect of the number of obligors
- The effect of the number of independent market factors
- The effect of the relative symmetry or asymmetry in exposures to a given market factor.
- The effect of margin and, more importantly, in having asymmetries in the exposures that are margined (as described below).
- The effect of the risk rating of the counterparty on the CEA.

Here are some reasons for investigating the effect of these characteristics on the CEA:

1. Not all counterparties have margin agreements. Many large corporate customers do not enter into margin agreements. In addition, in many countries around the world there does not exist an appropriate legal basis for a bank to have a sufficient degree of certainty about the legal enforceability of either netting or margin agreements in the event of default.
2. Many corporate customers have issued fixed rate debt and then swapped into floating rate exposure – i.e. they transact an interest rate swap to pay floating and receive fixed. A bank will tend to hedge this market risk by transacting offsetting interest rate swaps in the inter-bank market.
3. When 1) and 2) are combined we see that although the net market risk of a large swap portfolio may be very small, the counterparty exposure profiles generated by these swaps tend to be asymmetric in the following way - in this example, corporate obligors tend to be net receivers of the fixed rate without any margin agreements and interbank obligors tend to be net receivers of floating rates with margin agreements. This is one reason no conclusions about exposure over time can be inferred from an analysis of factor sensitivities at t =0.

4. Although the total counterparty exposure may depend on thousands or tens of thousands of market factors, changes in the total exposure to many large obligors tends to be dominated by a relatively few market factors (e.g. LIBOR yield curves of several major currencies; spot FX rates of a few major currency pairs such as US$/Euro, US$/Pound and US$/Yen; several major equity indices). A consequence of this is that even if counterparty risk could be characterized as being generated by many counterparties, each with a small exposure (which is not the actual situation), the exposure to most counterparties is dominated by a relatively small number of market factors. Consequently even if counterparty defaults were for the most part independent, large changes in the exposure across counterparties tend to be correlated.
5. Margin agreements are themselves not uniform. Some derivative margin agreements have zero threshold. Other agreements have material thresholds, or thresholds that vary with the counterparties risk rating. Some margin agreements require daily margining, others have varying margin intervals (e.g. weekly or longer).

One needs to identify the consequence of these factors on the CEA of real portfolios.

ESSENCE OF NEW PROPOSAL

The essence of this proposal is to create test portfolios with different characteristics in order to systematically measure the effect of these characteristics on α. α is the ratio of the Economic Capital calculated with full simulation to the Economic Capital calculated using the Expected Positive Exposure Profile (EPP) of each obligor. For a full explanation of these concepts see the Appendix, particularly the section entitled "LOAN EQUIVALENT PROFILES AND SCALE FACTOR α".

α is the scale factor needed to transform the expected positive exposure profile of a counterparty into an accurate measure of the loan equivalent of economic capital – i.e. to transform the expected positive exposure profile into a CEA for Basel 2.

α can be defined broadly, for all obligors, or more narrowly, as a function of characteristics of each obligor (such as risk rating or whatever) or other features of the obligor's portfolio.

As explained in the Appendix, the difference between Economic Capital calculated with full simulation versus Economic Capital calculated using the Expected Positive Exposure Profile of each counterparty is as follows:

- Full simulation means simulating the potential loss distribution of the portfolio of counterparties by coherently simulating the potential exposure of each counterparty over time as a consequence of the path market factors.
  - The sequence of steps in full simulation is: first generate a path of market factors; then measure the corresponding exposure profile of each counterparty for that path; then simulate thousands of scenarios of defaults and recoveries, for the exposure profiles of all counterparties, for that path. Finally repeat the sequence by looping over thousands of simulated paths of market factors. For more details, see the Appendix.
- Simulation with fixed exposure profiles means simulating the potential loss distribution by assuming that the potential exposure of each counterparty can be represented by a fixed exposure profile, specific to that counterparty, that is independent of any particular path market factors might takes over time. For example, the fixed exposure profile of a counterparty could be defined to be proportional to its Expected Positive Exposure Profile.
  - The sequence of steps in fixed exposure profile simulation is: Simulate the potential loss distribution by simulating thousands of potential scenarios of default and recovery, using only the fixed exposure profiles of each counterparty.

At first glance, the primary difference between full simulation and simulation with fixed exposure profiles is that the former entails looping over thousands of potential paths of market factors over time. Whereas the latter does not directly require the simulation of any path of market rates – the exposure at each future date is defined by the fixed profile per counterparty.

If the fixed exposure profile of each counterparty is defined to be proportional to the counterparty's Expected Positive Exposure Profile, we can specify an additional important difference between the two methods for simulating economic capital.

First note that when one calculates Economic Capital using each counterparty's Expected Positive Exposure Profile, the sequence of steps is: The Expected Positive Exposure Profile of each counterparty is first calculated by looping over thousands of potential paths of market factors. This simulation over potential market paths is done for each counterparty independently. One then simulates the effect of default and recovery for the portfolio of counterparties, by representing the exposure of each counterparty by the counterparty's Expected Positive Exposure Profile.

The difference between the two methods of simulation of Economic Capital is:

- Full simulation entails the coherent simulation of changes in market factors and captures the volatility of each counterparty's exposure and the correlation of exposure across counterparties. To emphasize this point, full simulation captures
  1. The volatility of the potential exposure of each counterparty and the effect of this volatility on Economic Capital. All else held constant; a variable exposure increases the amount of Economic Capital relative to a fixed exposure.

2. The correlation of the potential exposure of each counterparty corresponding to each potential path of market factors. The fact that some counterparties have offsetting exposure profiles to a particular path of market factors (e.g. one counterparty is net paying fixed while another counterparty is net receiving fixed) tends to reduce the Economic Capital, all else held constant.

- Simulation by means of the Expected Positive Exposure Profiles ignores the potential volatility of each counterparty's exposure and the correlations of exposures between counterparties because each counterparty's exposure profile is independently and separately calculated by the simulation of paths of market factors over time.

If the effect of the volatility of potential exposure and the effects of the correlation of potential exposure were exactly offsetting, for all tenors, then it should be obvious that the economic capital calculated with full simulation would be identical to the economic capital calculated using each counterparty's expected positive exposure profile – i.e. α would equal 1.0

The effect of varying the characteristics of the portfolio of obligors on the value of α needs to be measured.

The characteristics that need to be varied include:
- N, the number of obligors. As N increases does α asymptotically approach a constant and, if so, which constant?
- M, the number of independent market factors. All else held constant, is α dependent on the number of independent market factors? In what way? What is the implication if changes in most exposure profiles are dominated by a relatively few market factors?

CEAS FOR THE COUNTERPARTY CREDIT EXPOSURE OF FX AND DERIVATIVES; REPO AND REVERSE REPOS; AND SECURITY BORROWING AND LENDING.

Repos and Reverse Repos are almost always transacted with daily margin and zero threshold. The same is true for stock borrowing and lending. As a consequence the exposure profiles of these forms of security finance are identical to that of the equivalent derivative transactions with daily bilateral margin agreements and zero threshold.

As a consequence, the CEA for security finance transactions should be treated the same way as the counterparty credit exposure of FX and derivatives.

Note one **very important difference** from the VAR-type calculation proposed in Basel 2:
- The Basel proposed VAR-type calculation is for a static portfolio over a very short window (e.g. five or ten days).
- In contrast an exposure profile, even for transactions with daily margin, needs to be calculated over the lifetime of the portfolio or the appropriate CEA time horizon (e.g. one year or three years), whichever is shorter.

To illustrate this point, consider an extreme example. Assume a counterparty had entered into only one derivative transaction, a ten year interest rate swap with daily margin and zero threshold. The daily margin agreement would materially reduce the magnitude of the potential exposure. The potential exposure would equal the amount the swap could increase in value over a five or ten day margin period of risk, for each such forward period over the remaining life of the swap. This is because there potentially could be some exposure over the full ten years of the swap. A calculation of the life time credit risk of the swap would need to take into account the default probability over the full ten years as well as the potential exposure over that period of time.

Consequently under this proposal, in order to have a consistent method for the calculation of the CEA for FX and derivative counterparty risk, repos and reverse repos and security borrowing and lending, an exposure profile would need to be calculated for all these transactions over the life of the credit exposure or the appropriate time horizon, whichever was shorter.

APPENDIX - DEFINITIONS OF KEY CONCEPTS AND TERMS

SIMULATION OF COUNTERPARTY EXPOSURE PROFILE

The steps in calculating a **counterparty's exposure profile** are:

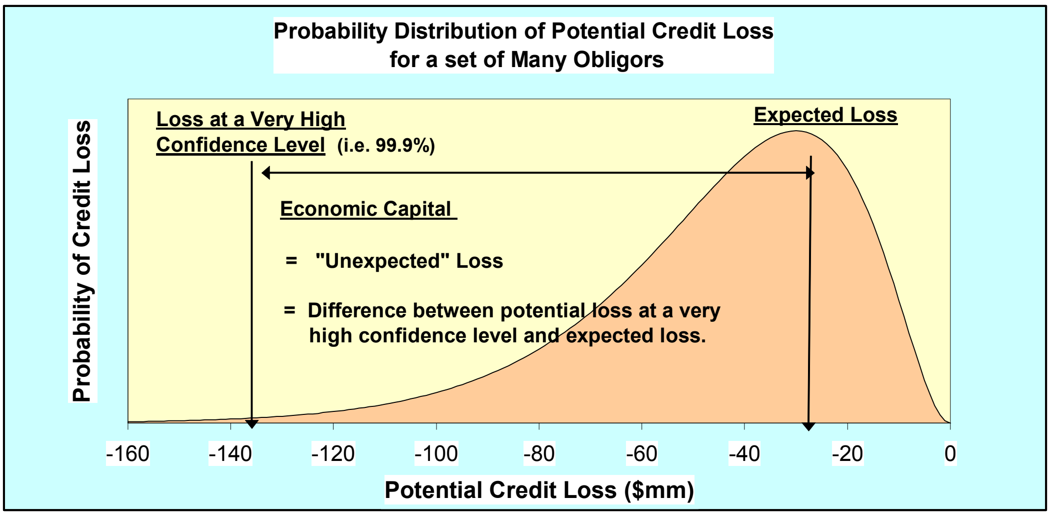
1. Simulate many **paths of the state of market factors** into the future, extending out many years over the full life of the transactions in the counterparty's portfolio.
2. Measure the **simulated market value of each transaction** of the counterparty at many future dates along each path, by means of full revaluation of each transaction, given the transaction's terms and conditions and the simulated state of the market.
3. Calculate the **simulated exposure of the counterparty** at a set of future dates along each simulated path, by aggregating the simulated future market value of each transaction of the counterparty, at a future date, in the context of legally enforceable risk mitigant agreements (e.g. netting, margin, option to early termination).
4. Calculate the **potential exposure to the counterparty at some specified confidence level**, given the distribution of simulated exposures at each future time.

A counterparty's exposure profile is a statistical picture of a firm's potential exposure to the counterparty over time; i.e. over the lifetime of the remaining transactions with the counterparty. It can be defined at any confidence level. For example, one can calculate:

- The **exposure profile of the counterparty at a high confidence level** (e.g. 99% CL).
- The **expected positive exposure profile - EPP(t)**, whose value at time t is defined as the expected value of all positive exposures (the obligor owes our firm), with negative exposures (our firm owes the obligor) set to zero.
- The negative exposure profile at any specified confidence level (useful for measuring liquidity risk).

FULL SIMULATION OF ECONOMIC CAPITAL FOR COUNTERPARTY RISK

The calculation of economic capital for the counterparty risk of a portfolio of many obligors will be based on the potential loss distribution, as below.



The potential loss distribution critically depends on:

- **The definition of potential loss**:
  - Loss only due to potential default and potential recovery,
  - Loss due to potential fall of economic value: This includes not only the loss due to potential default and recovery, but also the loss due to an increase in the market value adjustment (a.k.a. "credit value adjustment" at some firms) for counterparty credit risk. The market value adjustment is the adjustment to the risk free valuation that takes into account each counterparty's risk rating and general market spreads.
- The **time horizon** over which the loss is calculated (e.g. one year, three years, lifetime).

Steps For Full Simulation Of Potential Loss Distribution Due To Default:

1. Simulate many **paths of the state of market factors** into the future. Each path specifies the value of all the market rates (needed to value all contracts in the portfolio) at a set of future dates
2. For **each simulated path** calculate the **simulated exposure of each counterparty** at a set of future dates:
   a. Measure the **simulated market value** of each transaction at many future dates along the path, as above.
   b. Calculate the **simulated exposure of each counterparty** at many future dates along the path, by aggregating the potential market value of each transaction in accordance with all legally enforceable risk mitigant agreements (e.g. netting, margin, option to early termination), as above[3].
3. For **each simulated path**, calculate the **potential loss** for all of the counterparties in the portfolio. In other words, for each simulated path, at a set of forward intervals, simulate thousands of scenarios of obligor default and recovery. Each scenarios will differ by how many and by which obligors default in any future interval and by the simulated recovery, given default.
4. From the **set of simulated paths**, calculate an overall potential loss distribution. **Economic capital** is the difference between the potential loss at a high confidence level and the expected loss.

If we knew the future state of the market with certainty we would only need one path to describe the future state of the market. The simulated exposure profile of each counterparty (step 2) would be the loan equivalent for that path and we would stop our calculation with step 4 without the need to loop over many simulated paths of the market. In reality we do not know the future state of the market so we must loop over steps 1 to 3 for thousands of simulated paths. All things being equal, having variable rather than fixed exposures increases the width of the loss distribution and increases the amount of economic capital needed.

[3] Note: Step 2 is equivalent to calculating a path specific loan equivalent exposure of each counterparty. That is, for a given path, each counterparty will have a specific exposure at each future date. More generally, for a specific path the credit exposure (positive or negative at any future date) can be replicated by a portfolio of spot and forward loans from and to the counterparty (i.e. loans from when our firm owes the cntprty, loans to when cntprty owes us).

MEASURING SCALE FACTOR α AND DEFINING THE LOAN EQUIVALENT EXPOSURE FOR ECONOMIC CAPITAL

Let us define two measures of the economic capital of a portfolio P:

$EC(\mathcal{P}; CL, T)_{Full\_Sim\_Default}$ = Economic Capital calculated by Full Simulation, default only

$EC(\mathcal{P}; CL, T)_{Fixed\_EPP\_Sim\_Default}$ = Economic Capital calculated by assuming the exposure profile of each counterparty can be represented by a fixed exposure profile equal to its Expected Positive Profile.

$\mathcal{P}$ = The portfolio P, composed of N counterparties, each with many transactions.

CL = Confidence level at which EC is measured.

T = Time horizon over which EC is measured.

Therefore, the scaling factor, α, is defined as[4]:

$$\boxed{\alpha(\mathcal{P}; CL, T) \quad = \quad EC(\mathcal{P}; CL, T)_{Full\_Sim\_Default} / EC(\mathcal{P}; CL, T)_{Fixed\_EPP\_Sim\_Default}}$$

The difference between calculating Economic Capital with full simulation and with a simulation using the Expected Positive Profile is described and discussed in the main text

LOAN EQUIVALENT PROFILES AND SCALE FACTOR α

From the above description of the calculation of the potential loss distribution due to default we can readily derive the appropriate definition of **a loan equivalent of counterparty risk for economic capital.** It is the fixed exposure profile that under simulation of defaults and recoveries generates the same economic capital as generated by full simulation.

It is not necessary for the potential loss distribution calculated by full simulation and the potential loss distribution calculated by fixed exposure profiles to be identical at each confidence level. It is only necessary that the two loss distributions share one aspect in common: the difference between the loss at the confidence level used for economic capital and the expected loss should be identical for the two distributions.

Note that just as the loan equivalent of counterparty risk for economic capital will depend on the confidence level and the characteristics of the portfolio, it may also

[4] I have made use of the fact that the Economic Capital of the set of fixed exposure profiles $\{\alpha^*EPP_k(t)\}$ is equal to α times the Economic Capital of the set of fixed exposure profiles $\{EPP_k(t)\}$.

depend on whether Economic Capital is defined from a default only perspective (as above) or a loss of economic value perspective (as described above).

Scale Factor α

We thus see another way of expressing α, defined above as the ratio of Economic Capital due to default and recoveries calculated with full simulation to the Economic Capital calculated with Expected Positive Profiles:

Let us define:

$LEP_k(\mathcal{P}; CL, T)$	=	Loan Equivalent Exposure Profile of counterparty k, calculated <u>under full simulation</u> – i.e. the Fixed Profile for each counterparty that results in the same economic capital due to default and recovery as derived by full simulation.
EPP_k	=	Expected Positive Exposure Profile of counterparty k (see definition above)

Then

$$\boxed{\mathbf{LEP_k}(\mathcal{P}; CL, T) = \alpha(\mathcal{P}; CL, T) * \mathbf{EPP_k}}$$

Where α, $\mathcal{P}$, CL and T are defined as above.

The scale factor, α, will depend on the composition of the portfolio as well as both the confidence level and the time horizon over which Economic Capital is defined.

A bank can specify α for all obligors with counterparty exposure, based on the general characteristics of the total portfolio of all obligors. This would be appropriate for measuring total regulatory risk weighted assets for counterparty risk. In principal, a bank could also specify α as a function of some characteristic of a sub-group of counterparties, such as their risk rating. The difficulty with the latter is that since α depends on the characteristics of the portfolio for which it is measured, including the degree of portfolio diversification, it may be difficult to consistently define α for subsets of counterparties.

ANNEX 3 - CALCULATION OF ECONOMIC CAPITAL BASED ON EPE

CALCULATION OF ECONOMIC CAPITAL BASED ON EPE

OTC derivative counterparty exposures are variable and driven by market risk factors (mainly the principal components of the most liquid interest and foreign exchange rates, commodities and equities prices). The tail of the probability distribution of potential credit losses over a certain time horizon is determined primarily by the credit and market concentrations in the portfolio of counterparty exposures.

In its response to the Basel Committee's Consultative Paper of January 2001 (CP2), ISDA has shown that, in an asymptotically fine-grained portfolio of exposures with zero market-induced correlation on average, the economic capital could be calculated based on the expected positive exposure (EPE) to each counterparty. This proposal is in contrast, for example, to an alternative where capital is based on high confidence-level counterparty potential exposures (e.g. at 95% or 99% confidence levels).

In this paper, we describe a model to represent a portfolio of OTC derivatives counterparty exposures and estimate the proper equivalence factor α to be applied to EPE for the calculation of the economic capital of a portfolio of finite and possibly correlated counterparty exposures.

The factor α has been defined by Evan Picoult (Citigroup) as the ratio **A/B** where:

A = 99.9%-confidence default-only loss based on coherent simulation of a portfolio of counterparty exposures;
B = 99.9%-confidence default-only loss based on the expected positive exposure (EPE) to each counterparty.

Our model retains the essential elements of the risk dynamics of market-driven exposures and yet it is sufficiently simple and flexible to allow for the isolation of the fundamental structural characteristics of portfolios of exposures and the measurement of the sensitivity of α with respect to each of them.

Specifically, we look into how α varies with respect to:
1. Initial level of current exposures;
2. Correlation among default drivers in a normal 1-factor asset model;
3. Number of market risk factors driving the counterparty exposures;
4. Granularity of the portfolio of counterparty exposures;
5. Number of counterparties;
6. Number of margined counterparties
7. Probability of default of the counterparties over the horizon;
8. Confidence level used to define economic capital.

Our results suggest that, for a typical portfolio held by a large derivatives dealer, α is most likely to be in the range of 1.0 to 1.25. Only in a few extreme portfolio configurations α is larger than 1.5.

I) A MODEL FOR DERIVATIVES COUNTERPARTY CREDIT RISK

2-date factor model for market-driven exposures

We assume a portfolio of i=1,…,N derivatives counterparties and a 2-date model for market changes where:

- the value of the derivatives trades with each counterparty "i" is $V_i(0)$ at t=0;
- the value of the trades with each counterparty at t=horizon is a linear function of a set of "K" orthogonal market risk factors:

$$\mathbf{V_i(horizon) = V_i(0) + m_i \cdot \left(b_{i,1} \cdot f_1 + ... + b_{i,K} \cdot f_K\right)}$$

for i = 1,…,N

$V_i(0)$ is determined by the current exposure (CE) parameter:

- $V_i(0)$ = -CE, for "i" equal to an odd number (i=1,3,5,…);
- $V_i(0)$ = +CE, for "i" equal to an even number (i=2,4,6,…).

According with this specification, half of the counterparties start with mark-to-market value equal to +CE and half of the counterparties with mark-to-market value equal to –CE.

When examining the sensitivity of α to granularity, we assume that the $\log(m_i)$'s are independent (across counterparties i's) normal random variables with mean $-G^2/2$ and variance G^2. "G" defines the amount of dispersion of the standard deviations of V_i(horizon), i=1,…,N. Such dispersion breaks the homogeneity of the portfolio and makes some counterparty exposures more volatile ("larger") than others. Larger "G's" imply more granular portfolios of market-driven counterparty exposures.

Factor sensitivities

All factor sensitivities are independent, randomly generated by the following scheme:

1) for each counterparty "i", we generate $b_{i,k} = 2(U_{i,k}-0.5)$ for k=1,…,K; "U" is a random variable uniformly distributed on [0,1]; all $U_{i,k}$'s are independent;
2) we then normalize the vector ($b_{i,1}$,…, $b_{i,K}$) of counterparty's sensitivities by dividing each $b_{i,k}$ by $\sqrt{\sum_k \mathbf{b}_{i,k}^2}$.

After normalization, each counterparty's vector of market factor sensitivities has norm equal to one. Moreover, the expected value of the sum (across counterparties) of sensitivities to each market risk factor is zero (balanced book, on average).

Margined counterparties

Margined counterparties could induce concentration of exposures by creating a "one-sided exposure book". This would be the case, for example, if a dealer predominantly payed fixed in interest-rate swaps with customers (unmargined) and hedged its market risk by receiving floating in offsetting swaps with other dealers (margined). In that case, the exposures to counterparties would be concentrated in scenarios where interest rates go up. When exposures are generated by "many" market risk factors, the one-side-book effect tends to be reduced by diversification.

We examine the effect of margining in the context of our model by allowing different degrees of one-sided book.

Let "m" be the fraction of margined counterparties on one side of the book. "m" is defined as the ratio of the number of margined counterparties by the number of counterparties on one side of the book.

If m=0, the exposure book is balanced; i.e. all counterparties in the book are unmargined.

If m=1, the exposure book is fully one-sided; i.e. half of counterparties, which are on the same side of the book, are margined.

We define "the same side of the book" based on the sign of the cosine of the hyper-angle between the vectors of sensitivities b_i's. Counterparty "i" is on the same side of the book as counterparty "1" if the cosine of the hyper-angle between b_1 and b_i is positive.

Once we have defined the two "sides" of the book, we randomly choose a fraction "m" of the counterparties on one side of the book to be margined with zero initial and variation margin thresholds. The exposures (current and potential) to margined counterparties are identically equal to zero.

Simulation of market-driven exposures-at-default

We simulate j=1,…,2000 market scenarios. Each market scenario corresponds to a realization of the "K" independent market risk factors $f_{1,j},\ldots,f_{K,j}$. Each market risk factor has a standard normal distribution with mean equal to zero and variance equal to one. The values of the positions with each and all counterparties (V_i(horizon)) are computed by the linear model above. All V_i(horizon), i=1,…,N, have normal distributions with mean equal to $V_i(0)$ and variance equal to m_i. When G=0, all m_i's are equal to one. The exposure-at-default $E_{i,j}$ to each counterparty "i" in market scenario "j" is:

$$E_{i,j} = \max\left(V_i(\text{horizon}), 0\right)$$

After the market simulation is concluded, we have a matrix (2000 x N) of N counterparty exposures in each of 2000 market scenarios. Each row of the matrix corresponds to a set of coherent counterparty exposures, i.e., exposures generated by the same market scenario.

Based on this matrix we compute the expected positive exposure (EPE) to each counterparty:

$$\mathbf{EPE_i} = \sum_{j} \mathbf{E_{i,j}} \; / \; \mathbf{2000}$$

When "G" and "CE" are equal to zero, the expected exposures to all counterparties are the same (within the random error of the simulation) and equal to 0.3989.

When CE $\neq$ 0, there are two values of expected exposures: one for the "i" odd counterparties and another for the "i" even counterparties. The effect of CE $\neq$ 0 is important because it creates volatility in the exposure to defaulted counterparties even when capital is calculated via the EPE-based simulation. That is, the total exposure to defaulted counterparties in each credit scenario depends on the specific set of counterparties that defaulted. In the limit, when CE is large, the volatility created by the binary EPE_i's dominates the market-induced volatility and α converges to one.

In typical dealers' portfolios of counterparty exposures, the EPEs can vary quite a lot across counterparties. The variability of EPEs can be large when compared to market-induced variability of exposures. This tends to reduce α toward one. Our model captures some, but not all, of the attenuating effect of CE on α. Thus, we expect the α's produced by our model to be higher than the ones of real portfolios with more heterogeneity of EPEs across counterparties.

Simulation of default events

The probability of default "PD" over the horizon is the same for all counterparties. The recovery rates are also the same and are assumed to be zero for all counterparties. Those assumptions are consistent with the definition of a homogeneous portfolio.

We simulate h=1,…,200,000 credit scenarios each consisting of a random set of counterparties that default.

Default scenarios are independent of market scenarios. That is, we assume away right-way and wrong-way exposures.

When examining the sensitivity of α to the correlation among counterparty defaults, we use the following one-factor asset-based default model:

Let "A_i" be the default driver for counterparty "i".

$$\mathbf{A_i} = \sqrt{\mathbf{R}} \cdot \mathbf{Z_f} + \sqrt{\mathbf{1 - R}} \cdot \mathbf{Z_i}$$

where

"R" is the constant pairwise correlation among default drivers;
"Z_f" is a N(0,1) systematic risk factor driving defaults;
"Z_i" are independent N(0,1) idiosyncratic random drivers (i.e. specific default driver for counterparty "i");

For each credit scenario "h" we simulate Z_f and Z_i, i=1,…,N. A counterparty defaults if $A_{i,h} < N^{-1}(PD)$.

Computation of "full simulation" economic capital

For each credit scenario, we randomly select one single market scenario and compute the portfolio default losses by adding up the exposures to the counterparties that have defaulted in the credit scenario. Since we have 200,000 default scenarios and 2,000 market scenarios, we expect that each market scenario "j" will be randomly selected about 100 times.

Observe that the portfolio loss in each credit scenario "h" is subject to two sources of randomness:

a) the number of default events in the credit scenario (default volatility);
b) the sum of the exposures to the defaulted counterparties in the credit/market scenario (exposure volatility).

Computation of EPE economic capital

For each credit scenario generated for the "full simulation", we compute the total default losses by adding up the expected exposures to the counterparties that have defaulted.

Observe that the portfolio loss in each credit scenario "h" is subject to the same two sources of randomness mentioned above. In the case of CE=0, there is no exposure volatility; i.e., all EPE_i's are the same.

II) SENSITIVITY ANALYSIS

Base case

We define a base case that we consider representative of a large dealer:

- number of "effective" market risk factors = 3
- number of "effective" counterparties = 200
- PD = 0.0030
- "homogeneous" portfolio: $[V_i(t\text{=horizon})-V_i(t\text{=}0)]$ is N(0,1) for i=1,…,N
- 1-factor credit model with asset correlations 0.22;
- CE = 1.36 which implies, in our model, that the ratio of the maximum potential (95%, over 1 year) and current exposures of the portfolio is 1.30;
- Economic capital defined at 99.9% statistical confidence level

α is equal to 1.09 in the base case

In the sensitivity analysis to follow, the model parameters are kept as specified in the base case unless explicitly modified.

1) Sensitivity of α to the pairwise correlation between default drivers

R	**stdev(n_h)**	**n_h(99.9%)**	**α**	
0.00	0.77	4	1.43	
0.12	1.11	9	1.21	
0.22	**1.51**	**15**	**1.09**	**(base case)**
0.24	1.60	17	1.08	
0.50	3.20	44	1.02	

"R" is the pairwise correlation between default drivers;

"stdev(n_h)" is the standard deviation of the number of default events that occur in each credit scenario "h";

"n_h(99.9%)" is the 99.9-percentile of the distribution of the number of defaults (n_h) in each credit scenario. It is a metric of the extension of the tail the distribution of the number of defaults.

The correlation among defaults is a key determinant of α. The stronger the correlation is, the higher the variability of the number of defaults across credit scenarios. α converges to one: its numerator and denominator are driven by the variability of the number of defaults and the variability of exposures becomes less relevant.

2) Sensitivity of α to the level of current exposures

CE	**MPE/CE**	**avg(EE_i)**	**α**	
0	--	0.401	1.35	
1	1.60	0.584	1.14	
1.36	**1.30**	**0.720**	**1.09**	**(base case)**
2	1.09	1.009	1.05	
3	1.05	1.501	1.03	

"CE" is the initial level of mark-to-market value as outlined in the model specification above;

"MPE/CE" is the ratio of the maximum potential (95%, over 1 year) and current exposures of the portfolio;

"avg(EE_i)" is the average expected exposure across counterparties. Observe that in the extreme case of CE=3, avg(EE_i)=1.5 as expected.

The higher the CE is, the higher the variability of the sum of EPE_i's to the counterparties that default. α converges to one because its numerator and denominator are dominated by the variability of EPE_i's.

Example: when CE=3, half of the counterparties have EPE equal to 3 and half of the counterparties have EPE equal to zero. Suppose that there are 15 default events in a tail credit scenario; the variance of the total exposure conditional on 15 default events is: 15 x 0.5 x 0.5 x $(3\text{-}0)^2$ = 33.75 and its standard deviation is 5.81. Compare that number with the market-induced variance of exposures conditional on 15 defaults: 7.5 x 1^2 = 7.5. Conclusion: the variability of the sum of EPE_is is much larger than the additional variability introduced by the market risk factors.

3) Sensitivity of α to the number of market risk factors (K)

K	avgCorr	α	
1	0.164	1.10	
3	**0.048**	**1.09**	**(base case)**
5	0.029	1.08	
10	0.015	1.08	
50	0.002	1.08	

"K" is the number of orthogonal market risk factors;

"avgCorr" is the average pairwise correlation of counterparty exposures induced by the finite (and possibly small) number of market risk factors.

The number of orthogonal (i.e. uncorrelated) market risk factors determines the average level of pairwise correlations between market-driven counterparty exposures. A large number of orthogonal market risk factors reduce the average pairwise correlation and the variability of the sum of market-driven exposures. Consequently, α decreases.

4) Sensitivity of α to the granularity of the portfolio of counterparty exposures (<u>heterogeneous portfolio</u>)

G	1/H	MPE/CE	max/min	α	
0.0	**200**	**1.30**	**1**	**1.09**	**(base case)**
0.5	157	1.36	8	1.10	
1.0	86	1.46	69	1.21	
1.5	33	1.56	577	1.34	

"G" is a measure of the dispersion of the sensitivities of the counterparty positions to the market risk factors. The larger "G" is, the more heterogeneous is the portfolio of counterparty exposures in terms of their responses to changes in market risk factors;

"H" is the *Herfindahl concentration index* of counterparty sensitivities to markets and "1/H" can be interpreted as the <u>effective number of counterparties</u> in the portfolio, i.e., the number of counterparties in a homogeneous portfolio that would have the same H as the granular portfolio;

"MPE/CE" is the ratio of the maximum potential (95%, over 1 year) and current exposures of the portfolio;

“max/min” is the ratio of the 99-percentile over the 1-percentile of the probability distribution of m_i’s. When G=1, the 99% largest exposure is 104 times larger than the 1% smallest.

Granularity in the “deltas” of counterparty exposures to market risk factors is an important determinant of α. The variability of the market-driven exposures increases because of the higher heterogeneity in the magnitudes of the market-driven potential exposures.

5) Sensitivity of α to the number counterparties (N)

N	**α**	
20	1.26	
50	1.22	
100	1.10	
200	**1.09**	**(base case)**
500	1.04	

A higher number of counterparties causes a higher number of defaults n_h per credit scenario “h”. A higher number of defaults causes the variability of the sum of weakly correlated exposures to counterparties to decrease relative to the sum of the EPE_i’s.

6) Sensitivity of α to the fraction of margined counterparties (m) in one side of the book

m	**MPE/CE**	**α**	
0.00	**1.30**	**1.09**	**(base case)**
0.25	1.30	1.10	
0.50	1.36	1.11	
0.75	1.54	1.18	
1.00	1.83	1.24	

A higher fraction of margined counterparties on one side of the book induces concentration of exposures. The concentration is mitigated by the diversification across K=3 independent market risk factors.
With K=1 and m=1 (most extreme case), α is 1.42.

7) Sensitivity of α to the probability of default (PD)

PD	**α**	
0.001	1.17	
0.003	**1.09**	**(base case)**
0.005	1.07	
0.01	1.06	
0.05	1.05	

A higher probability of default “PD” causes a higher expected number of defaults n_h per credit scenario “h”. The higher number of defaults causes the variability of the total exposures to defaulted counterparties to decrease relative to the sum of the EPE_i’s.

8) Sensitivity of α to the confidence level defining economic capital

confidence level	**α**
99.0%	1.07
99.5%	1.10
99.9%	**1.09 (base case)**

The non-monotonic behavior of α with respect to the level of confidence used to define economic capital stems primarily from the shape of the tail of the loss distribution in the EPE-based calculation. The tail of the EPE-based loss distribution displays “discontinuities” corresponding to the discreteness of the probability distribution of the number of defaults per credit scenario.

ANNEX 4 - ANALYTIC α CALCULATIONS

ANALYTIC α CALCULATIONS

1. Summary

As explained at III, main text, work has been done by both members of the ISDA counterparty risk working group and independently by Michael Gibson at the Federal Reserve Board[5], to assess the additional capital required within ISDA's EPE framework for risks not covered in the original response[6]. To discuss this work we refer to a quantity α defined as the ratio A/ B where:

- A: = 99.9% loss with correlated market positions and stochastic exposures.
- B: = 99.9% loss for a corresponding portfolio with fixed exposures equal to EPE.

Michael Gibson's work is presented in terms of the understatement *U* of the standard deviation of the loss distribution, which is used to assess approximately the understatement of risk at the 99.9th percentile. For comparison with our work, we redefine U here as the equivalent direct concept, the understatement of the 99.9% confidence point. Then *U* is related to α by

$$\alpha = 1 + U$$

In this way, Gibson's and ISDA's results are made directly comparable. ISDA's work has been both numerical (see Annex 3) and theoretical (as presented below) with good agreement between the methods. Furthermore, except for its use of the granularity adjustment technique instead of scaling by variances to obtain exact limiting values for α, ISDA's theoretical work is conceptually very similar to Gibson's and the whole therefore appears to represent a conceptual consensus.

In this Annex,

- We present values of α obtained using ISDA's theoretical method and compare these to the simulation results set out in Annex 3. Agreement between theory and simulation is close. See Attachment 1.
- We provide further values of α using Michael Gibson's formulae, which essentially corresponds to the case of an infinitely granular portfolio. See Attachment 2. These values of α are smaller than those obtained by ISDA, due to the reference to an infinite portfolio.
- The methodology for ISDA's analytic results is presented and compared with Michael Gibson's formulae.

2. Analytic calculations for ISDA's α simulations

Eduardo Canabarro's simulations (Annex 3) determine α with various parameter combinations, using a model in which default rates are driven by a single factor conceptually consistent with both the IRB approach and the framework underlying ISDA's original proposals on EPE.

[5] Michael Gibson, "Regulatory Capital for counterparty credit risk: A response to ISDA's proposal" Federal Reserve Board, transmitted 15 November, 2002. Throughout, references to Gibson are to this article.

[6] We are not referring to wrong way risk which is discussed separately at II in the main body of this document.

We give an analytic version of most of these simulations using the granularity adjustment method. Results are close to and support Eduardo's results.

Set-up

The set-up for Eduardo's work is as follows:

- N counterparties in a Vasicek one factor model conceptually consistent with IRB.
- We consider a one period model i.e. values are taken at the horizon. Exposure is $E(1) = \max(0, V(1))$ where V is market value at one year.
- We work to a standardized σ = *1* and opening values are expressed as multiples of σ. In the set up current exposure $V(0) = CE = \pm u$ where u = 1.36 in the "base case".

Results

See Attachment 1.

- Results agree closely to Eduardo's.
- We have not performed the set of results for non zero G (Set 4) as this involves extra difficulties.

Note on validity of results and calculations

Results are first order approximations in (1 / N) and in c respectively, where N is the number of obligors and c the average covariance between their market values. The slopes with respect to these parameters are exact, but for $1/N, c \neq 0$ these are not exact andaccordingly should be seen as supplementary to the simulation results presented in Annex 3, providing an alternative point of view on the "ingredients" affecting α.

Calculation approach

α is defined as a ratio A/ B where:

A: = 99.9% loss using full simulation with correlated market positions and stochastic exposures

B: = 99.9% loss for a corresponding portfolio with fixed exposures equal to EPE.

The approach here is to calculate both these percentiles using the granularity adjustment approach

$$t_{99.9\%} = \mu(x_{99.9\%}) + \beta(x_{99.9\%})$$

where the summands are the systematic risk and granularity adjustment, evaluated at the 99.9% value of the systematic variable X The quantity α is then the ratio of percentiles.

The systematic risk is the same in each case, since as was shown in the ISDA response it is given by EPE. The relevant calculations of the granularity adjustment for the Vasicek model have already been done so nothing essentially new is needed here – see Attachment 4. The difficulty lies in calculating the conditional variance of the portfolio due to exposure covariance in Case A, when exposures are correlated. Note that unlike ISDA's response to CP2, Annex 1, we need to work here in the simpler one period model i.e. we only consider values at time t = 1, rather than over the interval $0 \le t \le 1$. This one period approach displays all the main features of t he more complex continuous time approach and is consistent with the simulation work described in Annex 3. Applying the results from ISDA's response to CP2, Annex 1, to the one period case we have conditional on the value of the systematic factor X

$$\mu(x) = \sum_A E_A P_A \text{ and } \sigma^2(x) = \sum_A F_A^2 P_A(1-P_A) + \sum_{A,B} P_A P_B \operatorname{cov}(E_A(1), E_B(1)) \quad (1)$$

where E_A, F_A are the expected positive exposure and RMS exposure respectively[7] We write P_A for $P_A(x)$, the default probability conditional on the systematic factor. As remarked μ is the same in both case A and case B.

In a portfolio of *N* assets and homogeneous with respect to credit quality, $P_A \equiv P$ is independent of A and (1) becomes

$$\mu(x) = N < E > P(x) \quad (2)$$

and

$$\sigma^2(x) = N < F^2 > P - N < F^2 > P^2 + N(< F^2 > - < E^2 >)P^2 + N(N-1)P^2 c$$

which simplifies to

$$\sigma^2(x) = N < F^2 > P - N < E^2 > P^2 + N(N-1)P^2 c \quad (3)$$

where the brackets denote average values over the portfolio, e.g. $< F_A^2 >= \frac{1}{N}\sum_A F_A^2$ and,

(Michael Gibson's notation from equation A.10 of his paper),

$$c := < Cov(E_A(1), E_B(1)) > \quad (4)$$

is the average covariance between distinct exposures (we have eliminated the diagonal term using the relationship $Var(E_A(1)) = F_A^2 - E_A^2$ which holds in the one period model).

We need to calculate all the terms of σ^2. The most difficult is the exposure covariance term *c* which we deal with next.

Exposure covariance

Non zero average covariance between exposures arises due to scattering of the correlations between pairs of assets around zero, because exposure covariance is a convex function of market covariance. In addition, scenarios have been considered in Annex 3 in which counterparties with different positions have differential tendency to be margined. This gives rise to direct exposure covariance, the effect of which can also be calculated using the methods presented here, but we have not included these calculations in the below.

Let the market values for distinct obligors *A* and *B* at time *t* be $V_{A,B}(t)$. We consider a one period model with t = 0, 1 and write *x*, *y* for the changes in market values over the period. Let these have correlation ρ (i.e. ρ is the market value correlation). Then (in the one period setting):

$$Cov(E_A(1), E_B(1)) = \iint \max(0, V_A(0)+x)\max(0, V_B(0)+y)n(x,y,\rho)dxdy - E_A E_B \quad (5)$$

where $n(x,y,\rho)$ is the bivariate standard normal density. We evaluate this as a power series in ρ using the tetrachoric expansion (see Abramowitz and Stegun, §26.3.29). After integrating term-wise, using integration by parts, this gives:

$$c_{AB} = Cov(E_A(1), E_B(1)) = \sum_{m=0}^{\infty} \frac{\rho^{m+1}}{(m+1)!} N^{(m)}(V_A(0)) N^{(m)}(V_B(0)) \quad (6)$$

[7] Throughout, exposures without time arguments, i.e. E_A and later E_+, E_- refer to *EPE* and likewise *F* refers to RMSE, while $E_A(1)$ etc means actual exposure at time $t = 1$.

where N is the standard normal cumulative density. The linear term will contribute nothing on taking expectations. Working to "first order" we will only take the quadratic term, arriving at:

$$Cov(E_A(1), E_B(1)) = \frac{\rho^2}{2} n(V_A(0)) n(V_B(0)) \qquad (7)$$

where n is the standard normal density.

We now take expectations. Here we assume that correlations are independent of current exposure levels (Eduardo's simulations specify the current exposure levels as +/- u independent of positions, so this assumption is correct, and indeed it is generally a reasonable assumption). Then to first order:

$$c = \frac{1}{2} < n(V_A(0)) >< n(V_B(0)) >< \rho^2 > \qquad (8)$$

where as above, the brackets refer to averaging across the portfolio of obligors.

Relationship to the number of market factors

In Annex 3 it is assumed that, independent of the number N of obligors, there is a fixed number K of market factors in which obligors take positions at random[8].

Thus suppose A and B have exposures depending on K orthogonalised market factors represented by normalised independent r.v's θ_{Ak}. Thus for all A:

$$V_A(1) = V_A(0) + \sum_{i=1}^{K} \theta_{Ak} X_k \text{ where } \sum_{i=1}^{K} \theta_{Ak}^2 = 1 \tag{9}$$

Then the correlation ρ between V_A and V_B is

$$\rho_{AB} = \sum_{i=1}^{K} \theta_{Ak} \theta_{Bk} \tag{10}$$

We need the mean square of this correlation across the portfolio (denoted by brackets as before):

$$< \rho^2 >=< (\sum_{i=1}^{K} \theta_{Ak} \theta_{Bk})^2 >= \sum_{i=1}^{K} < \theta_{Ak}^2 >< \theta_{Bk}^2 > = \frac{K}{K^2} = \frac{1}{K} \tag{11}$$

where we have used $\sum_{i=1}^{K} \theta_{Ak}^2 = 1$ to derive $< \theta_{Ak}^2 >= 1/K$, given that the positions are chosen at random. Substituting in (8) gives

$$c = \frac{< n(V_A(0)) >< n(V_B(0)) >}{2K} \tag{12}$$

for $A \neq B$. Finally, in the test portfolios (Annex 3) we have $V_A(0) = \pm u$ for a specified u, and we arrive at last at a simple formula relating average covariance to the number of underlying factors:

$$c = \frac{n(u)^2}{2K} \tag{13}$$

Other terms

The following auxiliary calculations are essentially in the original ISDA document on counterparty risk.. In the simulations each current exposure is one of $\pm u$ and we use this to simplify the calculations, as above. We write subscript +/**-** to distinguish the two cases. We have:

Expected exposures (EPE)

$$E_+ = uN(u) + n(u) \text{ and } E_- = -uN(-u) + n(u) \tag{14}$$

RMS exposures

$$F_+^2 = (u^2 + 1)N(u) + un(u) \text{ and } F_-^2 = (u^2 + 1)N(-u) - un(u) \tag{15}$$

We shall need:

$$F_+^2 + F_-^2 = u^2 + 1 \tag{16}$$

Conditional mean and variance

We can now put these together to write down the conditional mean and variance of the loss distribution given the value of the systematic variable X. By (2) – (3):

$$\mu(x) = N < E > P(x) \tag{17}$$

$$\sigma^2(x) = N < F^2 > P(x) - N < E^2 > P(x)^2 + N(N-1)P(x)^2 c \tag{18}$$

[8] This approach avoids the difficulty noted by Gibson (A-1) of specifying the expanded correlation structure as N is increased. This is similar to the underlying approach for one factor modelling, where by viewing correlation as arising via coupling to a systematic variable one can specify that new obligors are identical to the old ones, rather than worrying about creating new correlations for them.

We now write these out explicitly in Eduardo's cases:

Case (A): Portfolio considered as consisting of fixed exposures equal to EPE.
Then

$$\mu(x) = \frac{N}{2}(E_+ + E_-)P \tag{19}$$

$$\sigma^2(x) = \frac{N}{2}(E_+^2 + E_-^2)P(1-P) \tag{20}$$

Case (B): Actual (correlated stochastic) exposures

We have from (2): $\mu(x) = \frac{N}{2}(E_+ + E_-)P$ as before (21)

Using (3), (13) and (16), we also have

$$\sigma^2(x) = \frac{N}{2}(u^2+1)P + [-\frac{N}{2}(E_+^2 + E_-^2) + N(N-1)\frac{n(u)^2}{2K}]P^2 \tag{22}$$

Obtaining the percentiles, and hence α

In both cases A and B the systematic risk is given by

$$\mu(x_{99.9\%}) = \frac{N}{2}(E_+ + E_-)P(x_{99.9\%}) \tag{23}$$

For the unsystematic risk element we use the granularity adjustment (Wilde, "Probing Granularity", RISK, August 2001, equation 4[9]).

$$\beta = \frac{-1}{2f_X}\frac{d}{dx}\left(\frac{f_X\sigma^2}{d\mu/dx}\right) \tag{24}$$

Where

$$\mu(x) = \varepsilon P(x) \text{ with } \varepsilon = \frac{N}{2}(E_+ + E_-) \tag{25}$$

which is fixed depending on the portfolio. Referring to the formulae above we have, both cases, the general form:

$$\sigma^2(x) = aP(x) + bP(x)^2 \tag{26}$$

Where *a* and *b* are coefficients depending on which case we are in. The calculation of the granularity adjustment in this case is dealt with in Attachment 3. We get coefficients β_a and β_b depending on the default probability, percentile confidence level and asset correlation but nothing else (and therefore which are the same for cases *A* and *B*) such that in each case:

$$t_{99.9\%} = \varepsilon P + \frac{a}{\varepsilon}\beta_a + \frac{b}{\varepsilon}\beta_b\bigg|_{x=x_{99.9\%}} \; ; \; \alpha = \frac{\varepsilon P + \beta_a a(B)/\varepsilon + \beta_b b(B)/\varepsilon}{\varepsilon P + \beta_a a(A)/\varepsilon + \beta_b b(A)/\varepsilon}\bigg|_{x=x_{99.9\%}} \tag{27}$$

VBA code implementing this formula and used for the results is exhibited at Attachment 2.

[9] See for example, Gordy, "A risk factor model for ratings based capital rules", Federal Reserve, October 2002, or Martin and Wilde "Unsystematic Credit Risk", RISK, November 2002.

3. Discussion of Michael Gibson's results

The results discussed here are all to be found in Section 3 of Gibson's paper.

Summary of Gibson's paper Section 3 (with translation to ISDA terminology)

Section 3 of Gibson assesses the impact of non zero covariances between market positions, paralleling Eduardo's work and the above to some extent. Gibson provides

- A formula for exposure covariance with given market correlation (page 6);
- A formula for the amount of understatement U of risk using EPE when exposure covariance is present, in the case when the portfolio is infinite.
- Gibson's U (with our interpretation of U as the direct understatment of pecentiles) is related to ISDA's α by $\alpha - 1 = U$, and so conveys the same information.

Brief conclusions on this and its relation to ISDA's work are as follows:

- The two ingredients (determination of the covariance c between exposures, and of the sensitivity of percentiles to c) are the same as the two essential ingredients in the above analysis and so ISDA's work and Gibson's are in essential accord.
- Gibson has not calculated numerical values of U to compare with ISDA's, but this can easily be done using Gibson's formulae in the ISDA scenarios – see Attachment 2. Essentially, Gibson's results give lower values of understatement than ISDA's, because Gibson works in the case $N = \infty$ while the results in Annex 3 are for a finite portfolio.
- Gibson's formula for U is based on scaling percentiles according to the impact on standard deviation. This is an approximate treatment, but often gives good results. Below, we present an exact first order formula which gives higher but not very different values for U as a function of c. **The most material difference between Gibson and ISDA is therefore not to do with *c* but arises in respect of assumptions about *N*, the number of obligors.**

Attachment 2 shows results in the Gibson case obtained using respectively Gibson's formula (32) and the exact first derivative formula (39) for the coefficient α_{Gib}.

Gibson's formula for covariance between exposures

For all the calculations presented below, the covariance c is calculated using equation (7) which is valid for arbitrary spot exposures. Gibson also gives a closed formula for covariance between exposures when both spot values are zero, which we digress briefly to consider. In our notation, and in terms of covariance, Gibson's formula[10] Page 6 is:

$$c = \rho(\frac{1}{4} + \frac{1}{2\pi}\sin^{-1}\rho) + \frac{\sqrt{1-\rho^2} - 1}{2\pi} \tag{28}$$

Gibson presents his formula as holding between averages over time $\frac{1}{T}\int E_A dt$ and $\frac{1}{T}\int E_B dt$, but seems to assume that $V_A(t) = \sigma_A \sqrt{t} X_A$ for a (single) random variable X_A, and similarly for B. This assumption is not very realistic for the time development of value, which might more reasonably be assumed to follow Brownian motion. Nevertheless the equation above holds good as a one period equation, and all our

[10] Michael Gibson, page 6.

results have in any case been in the one period case. In that case, in fact (28) is the sum of the tetrachoric series (6) when both exposures are zero.

Note however that, under the more realistic assumption that the values V_A and V_B follow correlated Brownian motion, i.e.

$$dV_{A,B}(t) = \sigma_{A,B} d\omega_{A,B} \,;\; d\omega_A d\omega_B = \rho dt$$

the formula for covariance between time averages is as follows[11]:

$$c = \frac{\rho}{12} - \frac{32}{9.16\pi} + \frac{\rho(10\rho^2 - 1)\sqrt{1-\rho^2} + (8\rho^4 + 1)\sin^{-1}\rho}{3.16\pi\rho^3} \tag{29}$$

Incidentally, when $A = B$ and $\rho = 1$, this formula gives rise to variance of the time averaged exposure of

$$c = \frac{51\pi - 64}{288\pi} \approx 0.1 \tag{30}$$

This formula appears at footnote 5 in ISDA's 2001 response.

Gibson's formula for understatement (U) in terms of exposure covariance

We turn to the calculation of understatement $U = \alpha - 1$. In our notation, Gibson estimates the understatement U of risk arising from neglecting counterparty exposure correlation as

$$U \cong \alpha_{Gib} \frac{c}{< E >^2} \tag{31}$$

where$<E>$ is the average EPE in the portfolio and c is the average covariance between exposures as defined earlier[12]. The coefficient is called α by Gibson, so to avoid confusion with ISDA's $\alpha = 1 + U$ we refer to this coefficient as "Gibson's α" denoted α_{Gib}. As mentioned, Gibson uses standard deviation scaling to estimate this coefficient and derives the result:

$$\alpha_{Gib} = \frac{1}{2} \frac{\mathsf{E}(q(X)^2)}{\sigma^2(q(X))} \tag{32}$$

Gibson remarks that we typically have $0.6 \le \alpha_{Gib} \le 0.8$ approximately, and this is borne out by the examples in Annex 3 when formula (32) is used. The exact formula for understatement of percentiles (see below) gives similar but somewhat higher values, which unlike equation (32) depend on the confidence level.

We may alternatively write (32) as:

$$\alpha_{Gib} = \frac{1}{2} \frac{\sigma^2(q(X)) + \mu^2(q(X))}{\sigma^2(q(X))} = \frac{1}{2}(1 + \frac{1}{\omega^2}) \tag{33}$$

where ω is the so-called default rate volatility (an explicit input into the CreditRisk+ model, or a function of asset correlation in the Vasicek model). Typical values of ω in the Vasicek model are 150% – 300% for investment grade assets, which correspond to values of α_{Gib} (calculated by this formula) in the range 0.55 – 0.72, consistent with Gibson's suggested range of 0.6 – 0.8.

The true value of α_{Gib} for small c

[11] Tom Wilde, calculations communicated to the CRWG, March 2002.

[12] Michael Gibson, equation (1), page 5 or equation (A.29). Take care to note that (in our notation), we want $<E>^2$, not $<E^2>$ which appeared earlier.

Gibson's derivation of the coefficient α_{Gib} is based on comparing standard deviations of the distributions of loss with and without covariance. Our value, based directly on the understatement of percentiles, depends on the confidence level, which equation (32) does not. The method we have already used for Eduardo's calculation gives this value, as follows:

By definition (referring to the direct assessment of percentile understatement):

$$\alpha_{Gib} = \frac{1}{<E>^2} \frac{dt}{dc}\bigg|_{c=0}$$

where t is the percentile of the loss distribution at given confidence. This derivative is given exactly by the granularity adjustment (24). The exact value of the α_{Gib} coefficient is therefore given by taking $N = \infty$ in formula (27). In more detail, scaling (2) and (3) to a constant total notional of 1 unit, we have:

$$\mu(x) = <E> P(x) \tag{34}$$

$$\sigma^2(x) = (<F^2> P - <E^2> P^2)/N + (1 - 1/N)P(x)^2 c \tag{35}$$

Letting $N \to \infty$ gives $\sigma^2(x) = P(x)^2 c$. By (27) left hand formula,

$$t_q(c) = <E> P(x_q) + \frac{c}{<E>} \beta_b(x_q) \tag{36}$$

Finally

$$U = \frac{t(c)}{t(0)} - 1 = \frac{\beta_b(x)}{<E>^2 P(x)} c\bigg|_{x=x_q} \tag{37}$$

This will be in the same form as Gibson's formula A.29 (repeated at 31 above), if we put

$$\alpha_{Gib} = \frac{\beta_b}{P} \tag{38}$$

in place of Gibson's formula (Gibson A.25) for the coefficient. Using $\beta_b = P(\beta_a - \frac{1}{2})$ this becomes:

$$\alpha_{Gib} = \beta_a(x_q) - \tfrac{1}{2} \tag{39}$$

Note this is the *exact* value $\alpha_{Gib} = (1/<E>^2) dt/dc\big|_{c=0}$ holding in the limit of small c, although it is not necessarily more valid that Gibson's value when c is large since in that case the first order approximation will not be valid, while it is generally found that variances do give a reasonable guide to the percentiles of distributions in most cases. α_{Gib} computed this way lies in the range 0.8 – 1.3, above but by no means essentially different from values obtained using (32).

Attachment 1: Agreement between α simulations (Annex 3) and analytic results

The scenarios are as detailed in Eduardo's notes. The right hand column shows Eduardo's values for α and the red column labelled "α" shows results using the analysis presented here. Scenario 4 corresponding to the heterogenous portfolio is not performed.

Asset ρ	CE+/-	K = No factors	G	No of cpties	PD	Conf level	α	α Monte Carlo
Base case								
22%	1.36	3	0	200	0.30%	99.9%	**1.08**	**1.09**
1. Sensitivity to pairwise correlation								
0%	1.36	3	0	200	0.3%	99.9%	**1.46**	**1.43**
12%	1.36	3	0	200	0.3%	99.9%	**1.15**	**1.21**
24%	1.36	3	0	200	0.3%	99.9%	**1.07**	**1.08**
50%	1.36	3	0	200	0.3%	99.9%	**1.02**	**1.02**
2. Sensitivity to level of current exposures								
22%	**0**	3	0	200	0.3%	99.9%	**1.33**	**1.35**
22%	**1**	3	0	200	0.3%	99.9%	**1.12**	**1.14**
22%	**2**	3	0	200	0.3%	99.9%	**1.04**	**1.05**
22%	**3**	3	0	200	0.3%	99.9%	**1.02**	**1.03**
3. Sensitivity to the number of market risk factors								
22%	1.36	**1**	0	200	0.3%	99.9%	**1.09**	**1.1**
22%	1.36	**5**	0	200	0.3%	99.9%	**1.08**	**1.08**
22%	1.36	**10**	0	200	0.3%	99.9%	**1.07**	**1.08**
22%	1.36	**50**	0	200	0.3%	99.9%	**1.07**	**1.08**
4. Sensitivity to granularity of exposures (heterogenous portfolio)								
22%	1.36	3	**0.5**	200	0.3%	99.9%		**1.08**
22%	1.36	3	**1.0**	200	0.3%	99.9%		**1.21**
22%	1.36	3	**1.5**	200	0.3%	99.9%		**1.34**
22%	1.36	3	**2.0**	200	0.3%	99.9%		**1.21**
5. Sensitivity to number of counterparties								
22%	1.36	3	0	**20**	0.3%	99.9%	**1.31**	**1.26**
22%	1.36	3	0	**50**	0.3%	99.9%	**1.20**	**1.22**
22%	1.36	3	0	**100**	0.3%	99.9%	**1.13**	**1.1**
22%	1.36	3	0	**500**	0.3%	99.9%	**1.04**	**1.04**
6. Sensitivity to probability of default								
22%	1.36	3	0	200	**0.1%**	99.9%	**1.12**	**1.17**
22%	1.36	3	0	200	**0.5%**	99.9%	**1.06**	**1.07**
22%	1.36	3	0	200	**1.0%**	99.9%	**1.05**	**1.06**
22%	1.36	3	0	200	**5.0%**	99.9%	**1.04**	**1.05**
7. Sensitivity to confidence level								
22%	1.36	3	0	200	0.3%	**99.0%**	**1.10**	**1.07**
22%	1.36	3	0	200	0.3%	**99.5%**	**1.09**	**1.10**

Note – see also Attachment 2

Attachment 2 compares these results (which, on account of the general good agreement between simulation and analytic approximation should be regarded as one set of results, the "ISDA results"), with rather different results obtained for the case $N = \infty$ analysed by Michael Gibson.

Attachment 2. Results in the case N = ∞ (Michael Gibson's case).

The table shows the scenarios and results presented at Attachment 1, together with Michael Gibson's corresponding results in the right hand columns. The key difference is that Gibson's results are for $N = \infty$. Results presented are using Gibson's approximate formula (32), and to the left, results using the exact derivative with respect to covariance from (39).

Gibson's method of approximating by variances gives results that are about 2/3 of the exact asymptotic values (0.6 vs 0.9 for the "Gibson α" coefficient). But the results for α are much smaller than the Canabarro α's because of the omission of N as a driver, as discussed in the text.

Asset ρ	CE+/-	K = No factors	G	No of cpties	PD	Conf level	Comparison to monte carlo (see Annex 1)		Comparison with Gibson: α with N = ∞ in all cases			
							Wilde	Canabarro	Wilde (GA)		Gibson (Variance)	
							α	α	"Gibson α"	α	'Gibson α"	α
Base case									[See note]		[See note]	
22%	1.36	3	0	200	0.3%	99.9%	1.26	1.09	0.90	1.0073	0.608	1.0049
1. Sensitivity to pairwise correlation												
0%	1.36	3	0	200	0.3%	99.9%	1.53	1.43	∞	∞	∞	∞
12%	1.36	3	0	200	0.3%	99.9%	1.37	1.21	1.32	1.0106	0.78	1.0063
24%	1.36	3	0	200	0.3%	99.9%	1.25	1.08	0.85	1.0069	0.59	1.0048
50%	1.36	3	0	200	0.3%	99.9%	1.12	1.02	0.42	1.0034	0.52	1.0042
2. Sensitivity to level of current exposures												
22%	**0**	3	0	200	0.3%	99.9%	1.98	1.35	0.90	1.1508	0.61	1.1014
22%	**1**	3	0	200	0.3%	99.9%	1.41	1.14	0.90	1.0259	0.61	1.0174
22%	**2**	3	0	200	0.3%	99.9%	1.13	1.05	0.90	1.0004	0.61	1.0003
22%	**3**	3	0	200	0.3%	99.9%	1.06	1.03	0.90	1.0000	0.61	1.0000
3. Sensitivity to the number of market risk factors												
22%	1.36	**1**	0	200	0.3%	99.9%	1.27	1.10	0.90	1.0218	0.61	1.0147
22%	1.36	**5**	0	200	0.3%	99.9%	1.26	1.08	0.90	1.0044	0.61	1.0029
22%	1.36	**10**	0	200	0.3%	99.9%	1.26	1.08	0.90	1.0022	0.61	1.0015
22%	1.36	**50**	0	200	0.3%	99.9%	1.26	1.08	0.90	1.0004	0.61	1.0003
4. Sensitivity to granularity of exposures (heterogenous portfolio)												
22%	1.36	3	**0.5**	200	0.3%	99.9%	Not performed	1.08	Not defined in N = ∞ case			
22%	1.36	3	**1.0**	200	0.3%	99.9%		1.21				
22%	1.36	3	**1.5**	200	0.3%	99.9%		1.34				
22%	1.36	3	**2.0**	200	0.3%	99.9%		1.21				
5. Sensitivity to number of counterparties												
22%	1.36	3	0	**20**	0.3%	99.9%	1.38	1.26	All same as base case as N = ∞ here			
22%	1.36	3	0	**50**	0.3%	99.9%	1.33	1.22				
22%	1.36	3	0	**100**	0.3%	99.9%	1.29	1.10				
22%	1.36	3	0	**500**	0.3%	99.9%	1.24	1.04				
6. Sensitivity to probability of default												
22%	1.36	3	0	200	**0.1%**	99.9%	1.25	1.17	0.71	1.0057	0.57	1.0046
22%	1.36	3	0	200	**0.5%**	99.9%	1.28	1.07	1.03	1.0083	0.63	1.0051
22%	1.36	3	0	200	**1.0%**	99.9%	1.32	1.06	1.24	1.0100	0.68	1.0055
22%	1.36	3	0	200	**5.0%**	99.9%	1.55	1.05	2.20	1.0177	0.88	1.0071
7. Sensitivity to confidence level												
22%	1.36	3	0	200	0.3%	**99.0%**	1.17	1.07	0.38	1.0031	0.61	1.0049
22%	1.36	3	0	200	0.3%	**99.5%**	1.20	1.10	0.53	1.0043	0.61	1.0049

Note: "Gibson α". This is the coefficient defined by Gibson (definition above equation (34)) for which he uses the letter α .Gibson notes that this α typically lies in the range 0.6 - 0.8 (pA-4) which is borne out by the above. The quantity that we have called α is more or less the Gibson α times the average exposure correlation, and is equivalent to 1 + U where Gibson defines U at A.29.

Attachment 3: VBA code for the function α used for Attachment 1.

Values of α shown in the table at Attachment 1 are obtained from the parameters using the following VBA function which implements the analysis in these notes. The input parameters are the columns of the table from left to right (except "*G*" which is not an input).

```
----------------------------------------------------------
Function alpha(rho As Double, u As Double, K As Double, N As Double, p As
Double, PC As Double) As Double

Dim x, q, b1, b2 As Double

x = Application.NormSInv(PC)
q = Application.NormSDist((Application.NormSInv(p) + rho ^ 0.5 * x) / (1 -
rho) ^ 0.5)

b1 = -0.5 * (1 - q * (x * (1 - 2 * rho) - rho ^ 0.5 * Application.NormSInv(p)) / _
((rho * (1 - rho)) ^ 0.5 * Application.NormDist(((Application.NormSInv(p) +
rho ^ 0.5 * x) / (1 - rho) ^ 0.5), 0, 1, 0)))
b2 = (b1 - 0.5) * q

Dim Epos As Double
Dim Eneg As Double

Epos = u * Application.NormSDist(u) + Application.NormDist(u, 0, 1, 0)
Eneg = Epos - u

Dim mu, VA1, VA2, VB1, VB2 As Double

mu = N / 2 * (Epos + Eneg)

VA1 = N / 2 * (Epos ^ 2 + Eneg ^ 2)
VA2 = -N / 2 * (Epos ^ 2 + Eneg ^ 2)

VB1 = N / 2 * (u ^ 2 + 1)
VB2 = VA2 + N * (N - 1) * Application.NormDist(u, 0, 1, 0) ^ 2 / (2 * K)

alpha = (mu * q + VB1 / mu * b1 + VB2 / mu * b2) / (mu * q + VA1 / mu * b1
+ VA2 / mu * b2)

End Function
----------------------------------------------------------
```

Attachment 4. Calculation of the granularity adjustment coefficients β1 and β2

We are using the granularity adjustment formula for capital, equation (27)[13]

$$t_{99.9\%} = \mu(X_{99.9\%}) + \beta(X_{99.9\%}) \tag{A1}$$

where β is the "granularity adjustment" (24):

$$\beta = \frac{-1}{2f_X}\frac{d}{dx}\left(\frac{f_X\sigma^2}{d\mu/dx}\right)$$

The Vasicek dependence of default probability is

$$P(X) = N\left(\frac{N^{-1}(p)+\rho^{1/2}X}{(1-\rho)^{1/2}}\right) \tag{A2}$$

where X is the standard normal factor. We have to evaluate β in this case and for a general quadratic dependence

$$\mu(x) = \varepsilon P(X)$$

$$\sigma^2(x) = aP(X) + bP(X)^2$$

The following workings are not original – see notes at end. We have

$$\beta = \frac{-1}{2f_X}\left(\frac{d(f_X\sigma^2)/dx}{d\mu/dx} - \frac{f_X\sigma^2 d^2\mu/dx^2}{(d\mu/dx)^2}\right) \tag{A3}$$

or

$$\beta = \frac{-1}{2}\left(\frac{\sigma^2 d(\log f_X)/dx}{d\mu/dx} + \frac{d\sigma^2/dx}{d\mu/dx} - \frac{\sigma^2 d^2\mu/dx^2}{(d\mu/dx)^2}\right) \tag{A4}$$

Seeing that $d(\log f_X(x))/dx = -x$, this is

$$\beta = \frac{-1}{2}\left(-\frac{\sigma^2 x}{d\mu/dx} + \frac{d\sigma^2/dx}{d\mu/dx} - \frac{\sigma^2 d^2\mu/dx^2}{(d\mu/dx)^2}\right) \tag{A5}$$

Writing $\sigma^2(x) = aP(X) + bP(X)^2$ we have a corresponding split $\beta = a\beta_a + b\beta_b$ where

$$\beta_a = \frac{+P}{2}\left(\frac{x}{dP/dx} - \frac{1}{P} + \frac{d^2P/dx^2}{(dP/dx)^2}\right) \tag{A6}$$

and

$$\beta_b = \frac{+P}{2}\left(\frac{xP}{dP/dx} - 2 + P\frac{d^2P/dx^2}{(dP/dx)^2}\right) = P(\beta_a - \frac{1}{2}) \tag{A7}$$

On rearrangement we get

$$\beta_a = \frac{-1}{2}\left(1 - \frac{P}{dP/dx}\left(x + \frac{d^2P/dx^2}{dP/dx}\right)\right) \tag{A8}$$

On differentiating:

$$\frac{dP_A}{dx} = \frac{\rho_A^{1/2}}{(1-\rho_A)^{1/2}} n\left(\frac{N^{-1}(p_A)+\rho_A^{1/2}x}{(1-\rho_A)^{1/2}}\right) \tag{A9}$$

$$\frac{d^2P_A}{dx^2} = -\frac{\rho_A(N^{-1}(p_A)+\rho_A^{1/2}x)}{(1-\rho_A)^{3/2}} n\left(\frac{N^{-1}(p_A)+\rho_A^{1/2}x}{(1-\rho_A)^{1/2}}\right) \tag{A10}$$

[13] See for example, Gordy, "A risk factor model for ratings based capital rules", Federal Reserve, October 2002, or Martin and Wilde "Unsystematic Credit Risk", RISK, November 2002.

Hence

$$x + \frac{d^2 P_A / dx^2}{dP_A / dx} = x - \frac{\rho^{1/2}(N^{-1}(p) + \rho^{1/2} x)}{(1-\rho)} = \frac{x(1-2\rho) - \rho^{1/2} N^{-1}(p)}{(1-\rho)} \tag{A11}$$

We therefore obtain finally

$$\beta_a = \frac{-1}{2}(1 - N(\frac{N^{-1}(p) + \rho^{1/2} x}{(1-\rho)^{1/2}})\frac{x(1-2\rho) - \rho^{1/2} N^{-1}(p)}{\rho^{1/2}(1-\rho)^{1/2} n((N^{-1}(p) + \rho^{1/2} x)/(1-\rho)^{1/2})}) \tag{A12}$$

and from 4.0;

$$\beta_b = P(x)(\beta_a - \frac{1}{2}) \tag{A13}$$

These are incorporated directly into the VBA code in Attachment 2.

The relation (A8) for β_a is from Wilde, "Probing Granularity", RISK, August 2001 and (A13) for β_b in Pyhtkin and Dev "Analytic Approach to Credit Risk Modelling", RISK, March 2002.