CHAPTER 6

Discussing Cases

In this chapter, we consider professional development experiences based on the "case study method." Here teachers analyze and discuss "cases" that are written narratives or video excerpts of events that are used as catalysts for raising and discussing important issues regarding school mathematics reform.

Because several cases currently being used in professional development programs show students working on mathematical tasks, there is some overlap between this category of professional development experiences and analyzing students' thinking, the category discussed in the previous chapter. However, cases can be used to focus on other educational issues besides students' mathematical thinking. Furthermore, case discussions more generally have a long tradition in a number of professions besides education. These combined reasons led us to the decision of examining the use of cases in professional development as a separate category.

Theoretical rationale and empirical support

While using cases to develop professional knowledge in education has not been widespread, there is a strong tradition of using cases in other fields, such as law and business. Engaging mathematics teachers in the analysis of practice is certainly consistent with the principle of focusing professional development on the concrete activities of teaching and learning rather than abstractions and generalities. Appropriately selected cases can also be the starting point for all the six teacher learning cycles identified by Simon (1994), as reported in Chapter 3. The guided discussion of examples of practice can indeed provide the stimulus for new constructions of meaning by evoking cognitive dissonance, especially when the cases show a problematic situation. Furthermore, discussing such concrete examples offers teachers an ideal context for reflection and for hearing alternative viewpoints.

Indeed, Barnett (1998) has argued that the public scrutiny of ideas that takes place during a case discussion often leads teachers to new knowledge about mathematics, pedagogy and student thinking. Such knowledge is co-generated by the group in a way that significantly enhances what individuals could have come up with on their own.

Proponents of using cases in professional development have also pointed out that this kind of experience can potentially develop teachers' habits of inquiry into practice (Barnett, 1998; Schifter, Bastable & Russell, 1997). Empirical evidence in support of using cases comes from research studies evaluating the effects of the Mathematics Case Methods project, a program based entirely on case discussions (Barnett, 1991; Barnett & Ramirez, 1996; Barnett & Tyson, 1993 a&b; Gordon & Heller, 1995; Gordon & Tyson, 1995; Tyson, Barnett & Gordon, 1995). Barnett & Friedman (1997) write that these studies show the following:

> Teachers involved in case discussions move towards a more student-centered approach, learn to adapt and choose materials and methods that reveal student thinking, and anticipate and assume rationality in students' misunderstandings. Moreover, it appears that without being exposed to these ideas in research literature, teachers naturally move towards constructivist views of learning and develop a complex knowledge of students' thinking processes and underlying mathematical concepts. (p. 389)

While these findings could be attributed to the particular focus for the case discussions that the Mathematics Case Methods project employed (where all cases show classroom vignettes of students grappling with ideas about rational numbers), similar outcomes were found in field-testing the *Developing Mathematical Ideas* (*DMI*) program, which also uses cases (personal communication with Keith Cochran, 2001).

Schifter, Bastable and Russell (1997) have also pointed out the value of teachers creating their own cases, not just discussing ready-made ones. In their project, Teaching for the Big Ideas, a number of teachers successfully created cases.

Illustration 5: A case discussion about rational numbers

The vignette we present in this section illustrates a typical case discussion in the Mathematics Case Methods project (Barnett, Goldenstein & Jackson, 1994 a&b). The 2-hour session featured here occurred in a training session for experienced teachers who, although they had not had

previous experience with case discussions, expressed interest in this approach and in the possibility of eventually becoming case discussion facilitators. This case discussion was the first for these teachers. The case, called "Beans, Rulers and Algorithms," is the first in a series of cases about rational numbers that the Mathematics Case Methods project (Barnett, Goldenstein & Jackson, 1994a) developed.

The session began with a brief ice-breaker activity in which participants introduced themselves by saying their name and giving an adjective to describe their personality. Then, participants worked independently on the following problem designed to engage them personally with the key mathematical ideas in the case:

> Think about what might be difficult or confusing for a child. Use beans to solve this problem: 1/3 + 3/12.

Teachers then read the case silently. It is a two-page narrative reporting a teacher's experience in a combined fifth/sixth-grade class working on fractions (Barnett, Goldenstein, & Jackson, 1994a). The students in this class had already worked with equivalent fractions, addition and subtraction of fractions with the same denominator, improper fractions and mixed numbers. They had done so with success, using both manipulatives and pencil-and-paper tasks. The class had then moved to adding fractions with different denominators. The teacher introduced this new situation by providing the students with 12 beans, asking them what part of this whole would correspond to 1/2, 1/3, 1/4, 1/6 and 1/12. She also showed them a ruler, pointing out how each inch is divided in 16 parts and asked students to locate various fractions on the ruler. Using this information and the two tools (i.e., the beans and the ruler), the teacher asked the students to add several fractions, including such problems as 1/3 + 3/12 and 1/2 + 5/16. Once again, the students seemed to understand and had no difficulty with these problems, at least as long as they worked with the manipulatives. However, when the teacher moved to adding fractions on paper a few weeks later, the students seemed suddenly to "switch from understanding the concepts to memorizing a formula," and mistakes such as 1/6 + 2/7 = 3/13, or 1/6 + 2/7 = 7/42 + 6/42 = 13/42 surfaced. These outcomes puzzled the teacher, and she questioned what the students really understood about adding fractions. She wondered what she should do in the next lessons to help them.

When almost everyone had finished reading this case, the facilitator asked the participants what they thought were the important *facts* about

this case. As participants offered suggestions, the facilitator recorded them on newsprint without comment. A list of about a dozen items was quickly generated, including such information as "it was a fifth/sixth-grade class," "students already knew how to add fractions with common denominators," "they had been working on this for some time (but not clear how long)," "they were using manipulatives," and "they did not understand the process."

The facilitator then asked participants to work in pairs to generate issues for discussion about the case, requesting that each issue be expressed in the form of a question. She pointed out that issues could be about the mathematics involved in the case, the children's thinking, aspects of the instructional practice, the materials used or even the language used. She noted that based on past case discussions, some kinds of questions generated more interesting discussions than others. Therefore, she suggested teachers avoid yes/no answer questions, such as "Did the …?" and try instead to express their questions in a more open-ended way, such as the following:

> "Why might a student . . . ?"
> "What might happen if . . .?"
> "What does . . . mean?"
> "What if the problem/manipulatives were . . .?"
> "What are the benefits/limitations of . . . ?"

She elicited a few examples of each kind of question from participants to serve as models before the group broke into pairs to work on the task.

After about 10 minutes, the group reconvened and each pair shared some of the questions it had generated. Once again, the facilitator recorded all these questions on newsprint with minimal comment, making sure that every pair had an equal chance to contribute and that every voice was heard. The list contained about 15 items that addressed a variety of elements in the case, all using the format for questions suggested by the facilitator. They included very specific questions, such as "What might have happened if they had used fraction bars or paper folding instead of beans and rulers?" to more general ones, such as "How do you make the connection from the manipulatives to the paper-and-pencil process?" and "What does 'basically understand' mean?" While the majority of questions were about the teacher's instructional choices and alternative possibilities, some

questions looked more at the children's thinking, such as: "Why might students not understand the concept using beans?" and "Why would they add numerators and denominators?" Other questions focused on the mathematics, for example: "What do the beans represent?"

The group then picked one question for further discussion: "What does 'basically understand' mean?" In the remainder of the session, teachers discussed just this one question, although several other questions on the list were also addressed in the process.

The discussion began with several teachers trying to articulate what "understanding addition of fraction," or even "understanding fractions," meant for them. To help clarify their position, the facilitator occasionally invited them to come to the board and illustrate the point they were trying to make with an example. These examples usually made the discussion more concrete and raised some interesting mathematical questions about fractions and their representations. For example, participants generated new insights about the complexity of using beans to represent fractions. They noted that, depending on the number of beans chosen as the "unit," one single bean might represent a different fraction. For example, if the unit is 12 beans, 1 bean represents 1/12, but if the unit is 8 beans, one bean (the *same* bean!) represents 1/8. This suggested to another participant a possible explanation for why students might have added numerators and denominators in the problem 1/4+1/3 when using the beans, as shown in Figure 10.

Figure 10
A participant's graphical explanation of the mistake 1/4+1/3=2/7

1/4	+ 1/3	= 2/7
0000	000	0000000
one out of four	one out of three	two out of seven

This discussion led several teachers to appreciate the importance of clearly specifying what the *unit* is whenever using discrete representations for fractions. It also revealed that students might reasonably be puzzled by the fact that the teacher chose different sets of beans as the unit depending on the problem. It also suggested the value of making the reasons behind that choice explicit for students.

Throughout the discussion, the facilitator tried not to drive the conversation in a specific direction although she was not neutral either. Rather, she tried to deepen the participants' analysis and challenge their thinking through a combination of "pulling probes:"

"Can you show us what you mean?"
"What do other people think about that?"
"What are benefits/drawbacks of this position/idea? Why do you think so?"

She also used "pushing probes:"

"What about [counterexample]?"
"Is that always true?"
"What might be the impact on students?"
"What new ideas can you envision for this situation?"

The facilitator also had to interrupt the discussion before the group could reach closure on the original question. She explained that while it is always hard to interrupt a good discussion, it is almost impossible to reach closure on this or any case. However frustrating this may feel at first, it also has the advantage that participants can continue to think on their own about the issues raised in the true spirit of inquiry.

The session concluded with a brief round-robin closure activity in which each teacher identified something he/she was thinking about differently as a result of the experience. Participants also gave feedback on the process by filling a process check form; the facilitator quickly reviewed the results of this feedback before the end of the session so that the group could think about how the process could be improved the next time around.

Illustration 6: Examining an example of teaching mathematics through inquiry

We took the next illustration from the Leadership Seminar in the Making Mathematics Reform a Reality (MMRR) project that we described in Chapter 2. At the beginning of this project, one of the main goals of the Leadership Seminar was to develop a common understanding among lead

teachers of what it means to teach mathematics through inquiry and what it takes to put such an approach into practice.

To these ends, the facilitators devoted a 1 1/2-hour session to discussing a vignette of an inquiry lesson. The participants first read a four-page account of a lesson on constructing a congruent triangle given a side and two angles, where the students used creatively what they already knew about triangles and constructions to accomplish this novel task (Borasi, 1995, pp. 44-48).

The facilitators then carefully framed the discussion of this teaching episode. They asked the teachers to refrain from commenting on the quality of the lesson or the suitability of the example for teaching mathematics through inquiry. Instead, they should identify the elements of teaching mathematics through inquiry that were illustrated in the vignette.

As individual teachers shared the elements they had identified, facilitators asked them to explain why they had reached their conclusions and encouraged other participants to challenge these conclusions and ask for further explanation if it seemed necessary. A facilitator then recorded on newsprint the elements of inquiry-based instruction that the group agreed upon.

This exercise produced an extensive list of elements that characterize teaching mathematics through inquiry. It represented the group's shared understanding of this instructional approach at this point in time. This list was later reproduced for all participants, and they referred to it frequently in later sessions as the group continued to refine its understanding of inquiry-based mathematics as a vehicle for mathematics reform.

Main elements and variations

The two illustrations we offer in this chapter only begin to illustrate the variety of interpretations about what constitutes a case and how cases can be used in mathematics teacher education. However, a number of elements are common to all these interpretations and are thus worth highlighting as characteristic of this kind of professional experience, despite its many variations:

- ***Teachers engage in the in-depth analysis of a shared example of practice.*** The concreteness of the case enables participants to ground their reflection and discussion of more abstract ideas about school mathematics reform.

- ***Each case is carefully selected to stimulate debate on specific issues.*** A case is not simply a story, but rather a story with a "point" – although case discussions may sometimes surprise the facilitator by developing in unexpected directions!
- ***Facilitators elicit and explore multiple perspectives and opinions about the cases.*** One of the main benefits of case discussions is that teachers can benefit from the group interaction to construct meaning and knowledge that goes beyond what they, as individual participants, could have achieved. However, this requires careful facilitation of the discussion.

Within these guidelines, case discussions may differ considerably with respect to both the *nature of the case* used as a starting point and the *nature of the discussion* that is orchestrated around the case. Cases may differ along the following important dimensions:

- ***The content of the case.*** While most cases used in teacher education deal directly with classroom instruction, some feature other aspects of teachers' and/or students' practice. For example, there are cases that portray teachers' interactions with colleagues, teachers' experiences in professional development settings or even students' learning as it occurs outside of the classroom.
- ***The format in which the case is presented.*** The vignette may be presented as a story, in narrative form, or conveyed through a video. Each of these media has unique advantages and disadvantages. Most notably, while videos can allow the direct observation of non-verbal as well as verbal behaviors, they are less flexible than a narrative format and less able to convey background information about the event.
- ***Whether the case is a "stand-alone" or part of a collection.*** While almost any case can be used in isolation, programs that rely on case discussions as their primary vehicle tend to use carefully sequenced collections of cases, designed to provide teachers with multiple opportunities to examine a complex concept in different contexts. Multiple cases examined in a sequence make it possible to highlight different aspects of a topic each time, allowing for meanings to be constructed and revised over time.

- ***The extent to which the case illustrates exemplary practice.*** While most cases make no assumptions about the quality of the practice they portray (as, for example, the case on rational numbers used in Illustration 5), some are created specifically to illustrate exemplary – although never perfect! – practice (as exemplified by the case used in Illustration 6).

- ***How "real" the case is.*** The cases currently available in the literature cover the entire spectrum from faithful representations of real-life events to fictitious situations. Most cases, however, are composites of several real-life events that have been created for the purpose of illustrating specific issues.

- ***How "pointed" the case is.*** A case is usually selected or constructed to illustrate specific points. This is especially true in collections of cases designed to help teachers grapple with different topics, such as elementary students' developing conceptions of numbers and operations. However, Illustration 6 shows that almost any account of practice can become a case if it is appropriately framed for participants.

The other major area of variation depends on how the facilitator organizes the discussion about the case. Important variations can occur along any of the following dimensions:

- ***How the case discussion is framed.*** Facilitators may determine the specific goals and foci for the discussion in advance and communicate this to the participants upfront, or be more open-ended and willing to set goals together with the participants.

- ***How the case discussion is facilitated.*** As mentioned earlier, all facilitators should ensure that participants feel free to express their opinions and show respect for others' ideas. Facilitators should also try to elicit multiple opinions, encourage debate, and invite further articulation of ideas among the participants. However, there are various ways to achieve these goals. Some programs, such as the Mathematics Case Method featured in Illustration 5, expect facilitators to follow a carefully articulated set of practices, while others are less prescriptive about what the facilitator should do.

- ***What activities may accompany the case discussion.*** While case discussions may occur in isolation, most often they are accompanied by other activities intended to strengthen or extend the outcomes of the discussion. For example, teachers in the rational numbers case discussion (Illustration 5) engaged first as learners in the same mathematical tasks discussed in the case. In this way, they gained a personal understanding of the mathematics involved and began to think about alternative ways to approach these tasks. In other implementations, teachers have been invited to further pursue issues raised in the discussion through follow-up readings, or even mini action research projects in their own classrooms.

Cases can be used in a great variety of professional development formats – including summer institutes, courses, workshops and study groups.

Case discussion facilitators may require different kinds of expertise depending on the content and focus of the case. Whenever the case involves mathematics, a good understanding of the mathematical topic involved is critical to be able to direct the discussion in productive ways. However, cases focusing on leadership and school reform issues more generally may not require any mathematical expertise in the facilitator. Regardless of the content of the case, facilitators can greatly benefit from specific training in conducting case discussions, to learn strategies to set a conducive learning environment and to ask questions that can move the conversation in productive directions without dominating it.

Teacher learning needs addressed

Cases are indeed a flexible professional development tool that can address most of the teacher learning needs we identified in Chapter 1. The extent to which this potential can be met for each specific need, however, depends on both the content of the case and the nature of the discussion about it.

- ***Developing a vision and commitment to school mathematics reform.*** Barnett (1998) argues that cases are a non-threatening way to expose teachers to innovative pedagogical practices and to help them develop pedagogical content knowledge even before they have made any commitment to reform. This exposure may in turn

engender an interest in teachers toward changing their instructional practices and in becoming a part of reform efforts.

Cases that portray learning experiences and/or teaching practices consistent with school mathematics reform, such as the congruent triangle case reported in Illustration 6, can contribute to teachers' images of what reform looks like. When developing a vision for school mathematics reform is one of the main goals, however, cases should be chosen to represent exemplary practice.

Cases that capture the conflicts and challenges that reform teachers may encounter contribute an additional dimension to understanding the demands of school mathematics reform. Thus, they help teachers develop realistic expectations before committing to reform.

- ***Strengthening one's knowledge of mathematics.*** Although it may seem surprising at first, developing teachers' mathematical knowledge is a stated goal of some professional development programs that use cases extensively, such as the Developing Mathematical Ideas project and the Mathematics Cases Method. To achieve this goal, a sequence of cases is carefully constructed around a key mathematical concept. Before they read the case, teachers work the same mathematical tasks featured in it. In this way, they engage personally with the mathematical concept before they examine other learners' approaches to the same task and speculate on their thinking processes, as shown in Illustration 5.

  Misconceptions and errors often play an important role in these cases because teachers may uncover some important mathematical ideas while trying to explain the origin of the errors. When developing mathematical understanding is a focus, the facilitator needs to pay special attention to eliciting alternative mathematical ideas from the participants and helping them see the significance and connections between ideas.

- ***Understanding the pedagogical theories that underlie school mathematics reform.*** Cases that focus on classroom instruction are likely to stimulate observations and analyses that challenge teachers' taken-for-granted beliefs about teaching and learning. These situations, in turn, may be used to motivate additional inquiry into the

learning theories that distinguish school mathematics reform from traditional mathematics instruction, through readings, presentations and further discussions. Interestingly, Barnett and Friedman (1997) report gains in teachers' understanding of the basic tenets of constructivist learning theories even from just experiencing the workshop, without additional readings or presentations about the research that supports those theories.

- ***Understanding students' mathematical thinking.*** Many cases currently available in the literature have the analysis of student thinking at their very core. These cases all include students' mathematical activities as a central part of the vignette. To facilitate teachers' understanding of students' mathematical thinking most effectively, the discussion of these cases should focus, at least in part, on making sense of the thinking behind the activities. Barnett (1998) also suggests that, prior to reading the case, participants should engage as learners in the same mathematical tasks featured in the vignette and speculate about how their students would see and approach the same task – in other words, try to see the task through their students' eyes.

  The goal of understanding student mathematical thinking can be furthered if teachers test the insights generated in the case discussion with students in their own classes.

- ***Learning to use effective teaching and assessment strategies.*** Many cases showing instructional episodes can, at the very least, expose teachers to effective instructional practices. Whether the case features traditional mathematics instruction or shows practices promoted by school mathematics reform, teachers may benefit from critically examining these practices from various perspectives. For example, participants can explore the assumptions about student and teacher roles, examine the students' responses these roles elicit, and discuss their effectiveness in promoting student learning. Results from the Mathematics Case Method project suggest that these experiences often make teachers more willing to experiment with new practices and then reflect on these experiences. In addition, cases that portray exemplary practices have the added benefit of providing teachers with an image and a model of reform practices

that they can refer to as they begin to experiment with instructional changes.

- ***Becoming familiar with exemplary instructional materials and resources.*** Cases that portray instructional episodes in which exemplary materials are featured help teachers get to know such materials and use them effectively. Although only a few such cases are currently available in the literature, teachers who are using the materials could choose to create such cases themselves and learn even more about the materials by doing so! Cases of this kind certainly help teachers anticipate students' responses to the non-traditional tasks at the core of the exemplary materials, examine the nature of the mathematical learning that results and grapple themselves with the mathematics involved. They can also provide an image of the kind of teaching practices that such materials support and help teachers begin to identify what it takes to implement such practices effectively. Even "non-exemplary" vignettes featuring exemplary materials can be helpful because they can make teachers aware of potential pitfalls in using the materials.

- ***Understanding equity issues and their implications for the classroom***. Cases featuring inclusive classrooms, or even just classrooms with diverse students, can generate worthwhile discussions about equity issues and their implications for teaching mathematics. Especially when the case is presented as a video excerpt with little interpretation of the events, participants can observe teacher-student interactions and draw their own conclusions about possible biases at work. An in-depth analysis of the interactions observed may indeed bring prejudices to the surface that participants may not know they have. Because discussions about issues of equity are often accompanied by strong feelings, they must be facilitated sensitively.

- ***Coping with the emotional aspects of engaging in reform.*** Cases can also feature the struggles and emotional challenges experienced by teachers engaged in reform. This element is often present in cases designed to support participants' inquiry about school reform (Miller & Kantrov, 1998). However, participation in any case discussion, regardless of its focus, is likely to address some of

the teachers' emotional needs because it breaks their isolation and offers them opportunities to share and discuss their concerns with other colleagues.

- ***Developing an attitude of inquiry toward one's practice.*** Many proponents of the use of cases (e.g., Barnett, 1998; Shulman, 1992) state that a central goal of this kind of professional development experience is to help teachers develop an attitude of inquiry toward their practice. By definition, the discussion of a case, regardless of its specific content or focus, engages participants in a critical reflection on practice. As importantly, these reflections can benefit from the guidance of an expert and the generation of ideas with other practitioners. Barnett and Friedman (1997) also suggest that avoiding closure on case discussions may contribute to developing habits of inquiry. Leaving issues unresolved may motivate teachers to pursue them on their own.

The discussion of a case, regardless of its specific content or focus, engages participants in a critical reflection on practice.

Summary

Cases discussions have a multitude of possible uses in professional development. Depending on the content of the case and the focus of the discussion, this type of activity can address all the teacher learning needs we identified in Chapter 1. The extreme flexibility in using cases is one of its greatest strengths as a professional development tool. At the same time, because cases can vary so much, it is more difficult to evaluate their effectiveness without context-specific information.

Suggested follow-up resources

If you are interested in learning more about how to use cases for a variety of professional development goals, we recommend the following resources, in addition to the *Developing Mathematics Ideas* (*DMI*) materials already mentioned in Chapter 5:

Barnett, C., Goldenstein, D., and Jackson, B. (Eds.) (1994b). *Fractions, decimals, ratios, and percents: Hard to teach and hard to learn?* (casebook and facilitator's guide) Portsmouth, NH: Heinemann.

This set of 29 teacher-written cases illustrates recurring dilemmas and problems in teaching and learning fractions, decimals, ratios and percents. The editors primarily intend these cases for mathematics teachers in grades 4 – 8; however, we find them to be beneficial for teachers from kindergarten through grade 12. The facilitator's guide identifies the central mathematical and pedagogical issues addressed by each case, offers suggestions for facilitating the discussions, and identifies some of the common misconceptions that can emerge during the discussions.

Stein, M. K., Smith, M. S., Henningsen, M. A., and Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers College Press.

This book, intended for teacher educators and teachers, is more than a set of cases. The authors introduce their *mathematical task framework* and describe the typical pedagogical patterns teachers use in implementing tasks, uncovered as the result of their research of middle school mathematics classrooms. The explicit description of the task framework and the pedagogical patterns helps teachers become aware of the cognitive demands of a mathematical task and of the issues involved in maintaining the cognitive level of a task. The cases, inspired by real classroom experiences, provide opportunities for teachers to practice identifying the cognitive demands of a particular mathematical task, to see firsthand how pedagogical practices impact the task, and to grapple with the issues raised by the example. Each case includes a section in which the featured teacher discusses her class and a section describing her implementation. In addition, each case is accompanied by a set of discussion questions and notes to support the case discussion.

Harvard Mathematics Case Development Project (in press). *Cases in secondary mathematics classrooms*. New York: Teachers College Press.

This book includes several cases at each level of high school mathematics (i.e., Pre-Algebra and Algebra, geometry, Algebra II and Trigonometry, Probability and Statistics, Pre-Calculus and Calculus). Before presenting the cases, the authors outline in detail their theoretical framework for constructing them. They examine the mathematical, pedagogical, student assessment and contextual issues they believe teachers need in order to promote learning at high levels. The book also includes a guide for case facilitators and for participants in case discussions, and it lists the major mathematical and pedagogical issues raised by each case. Each case is supplemented by notes for the facilitator that include a prediscussion activity, tips for teaching the case, suggested discussion questions, possible extensions and annotated references for further reading on either the mathematics content or the pedagogy.

Miller, B., and Kantrov, I. (1998). *Casebook on school reform*. Portsmouth, NH: Heinemann.

This book includes six ready-to-use cases that describe teachers' reform efforts in mathematics or science. The introduction of the book provides a rationale for using cases, an explanation of why and how the cases were developed and some suggestions for how to use them. The cases, developed to highlight issues raised when educators engage in school reform, are intended to stimulate thinking and discussions from multiple perspectives. Each case is accompanied by a facilitator's guide that suggests ways to elicit discussion about the "big ideas" underlying the case.