Prepared in cooperation with the

U.S. Army Corps of Engineers, Memphis District and the

Arkansas Soil and Water Conservation Commission

CONJUNCTIVE-USE OPTIMIZATION MODEL OF THE MISSISSIPPI RIVER VALLEY ALLUVIAL AQUIFER OF NORTHEASTERN ARKANSAS

Water-Resources Investigations Report 03-4230

U.S. Department of the Interior
U.S. Geological Survey

Cover*:* Center pivot irrigation system in eastern Pulaski County, Arkansas. Photograph by John B. Czarnecki, U.S. Geological Survey.

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Little Rock, Arkansas
2003

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Denver, Colorado 80225

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Multiply	By	To obtain
inch (in.)	2.54	centimeter (cm)
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
million cubic feet per day (Mft^3/d)	7.481	million gallons per day (Mgal/d)
square foot (ft^2)	0.0929	square meter (m^2)
square mile (mi^2)	2.590	square kilometer (km^2)
cubic foot (ft^3)	7.481	gallon (gal)
cubic foot per day (ft^3/d)	0.0283	cubic meter per day (m^3/d)

In this report, vertical coordinate information is referenced to the National Geodetic Vertical Datum of 1929 (NGVD of 1929). Horizontal coordinate information is referenced to North American Datum of 1927 (NAD 27).

Altitude, as used in this report, refers to distance above or below NGVD of 1929.

Conjunctive-Use Optimization Model of the Mississippi River Valley Alluvial Aquifer of Northeastern Arkansas

By John B. Czarnecki, Brian R. Clark, and T.B. Reed

ABSTRACT

The Mississippi River Valley alluvial aquifer is a water-bearing assemblage of gravels and sands that underlies about 32,000 square miles of Missouri, Kentucky, Tennessee, Mississippi, Louisiana, and Arkansas. Because of the heavy demands placed on the aquifer, several large cones of depression over 100 feet deep have formed in the potentiometric surface, resulting in lower well yields and degraded water quality in some areas. A ground-water flow model of the alluvial aquifer was previously developed for an area covering 14,104 square miles, extending northeast from the Arkansas River into the northeast corner of Arkansas and parts of southeastern Missouri. The flow model showed that continued ground-water withdrawals at rates commensurate with those of 1997 could not be sustained indefinitely without causing water levels to decline below half the original saturated thickness of the aquifer.

To develop estimates of withdrawal rates that could be sustained in compliance with the constraints of critical ground-water area designation, conjunctive-use optimization modeling was applied to the flow model of the alluvial aquifer in northeastern Arkansas. Ground-water withdrawal rates form the basis for estimates of sustainable yield from the alluvial aquifer and from rivers specified within the alluvial aquifer model. A management problem was formulated as one of maximizing the sustainable yield from all ground-water and surface-water withdrawal cells within limits imposed by plausible withdrawal rates, and within specified constraints involving hydraulic head and streamflow. Steady-state flow conditions were selected because the maximized withdrawals are intended to represent sustainable yield of the system (a rate that can be maintained indefinitely).

Within the optimization model, 11 rivers are specified. Surface-water diversion rates that occurred in 2000 were subtracted from specified overland flow at the appropriate river cells. Included in these diversions were the planned diversions of 63,339,248 ft^3/d for the Bayou Meto project area and 55,078,367 ft^3/d for the Grand Prairie project area, which factor in an additional 30 and 40 percent transmission loss, respectively. Streamflow constraints were specified at all 1,165 river cells based on average 7-day minimum flows for 10 years. Sustainable yield for all rivers ranged from 0 (Current, Little Red, and Bayou Meto Rivers) to almost 5 billion cubic feet per day for the Arkansas River. Total sustainable yield from all rivers combined was 12.8 billion cubic feet per day, which represents a substantial source for supplementing ground water to meet the total water demand.

Sustainable-yield estimates are affected by the allowable upper limit on withdrawals from wells specified in the optimization model. Ground-water withdrawal rates were allowed to vary as much as 200 percent of the withdrawal rate in 1997. As the overall upper limit on withdrawals is increased, the sustainable yield generally increases. Tests with the optimization model show that without limits on pumping, wells adjacent to sources of water would have optimized withdrawal rates that were orders of magnitude larger than rates corresponding to those of 1997. The sustainable yield from ground water for the entire study area while setting the maximum upper limit as the amount withdrawn in 1997 is 360 million cubic feet per day, which is only about 57 percent of the amount withdrawn in 1997 (635.6 million cubic feet per day). Optimal sustainable yields from within the Bayou Meto irrigation project area and within the Grand Prairie irrigation project area are 18.1 and 9.1 million cubic feet per day, respectively, assuming a maximum allowable withdrawal rate equal to 1997 rates. These values of sustainable yield represent 35 and 30 percent respectively of the amount pumped from these project areas in 1997.

Unmet demand (defined as the difference between the optimized withdrawal rate or sustainable yield, and the anticipated demand) was calculated using different demand rates based on multiples of the

1997 withdrawal rate. Assuming that demand is the 1997 withdrawal rate, and that sustainable-yield estimates are those obtained using upper limits of withdrawal rates of 100-, 150-, and 200-percent of 1997 withdrawal rates, then the resulting unmet demand for the entire model area is 275.5, 190.9, and 110 million cubic feet per day, respectively. Whereas, if the demand is specified as 100-, 150-, and 200-percent of the 1997 withdrawal rate, and the sustainable-yield estimates remain the same, then the resulting unmet demand for the entire model area is 275.5, 508.8, and 745.8 million cubic feet per day, respectively. These unmet demands for ground water could be obtained from large sustainable surface-water withdrawals.

INTRODUCTION

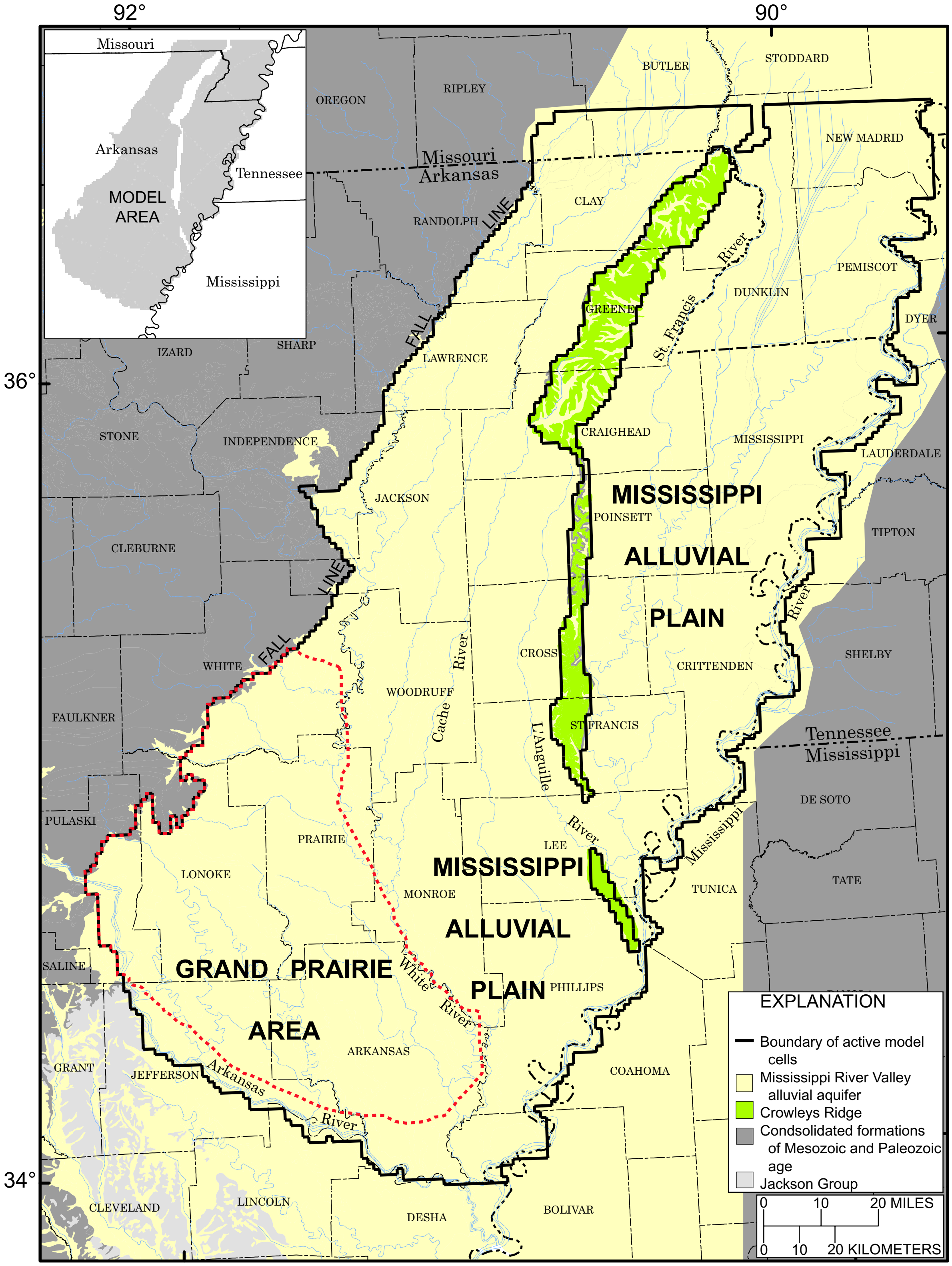
The Mississippi River Valley alluvial aquifer, often termed simply the “alluvial aquifer,” is a water-bearing assemblage of gravels and sands that underlies about 32,000 mi^2 of Missouri, Kentucky, Tennessee, Mississippi, Louisiana, and Arkansas. In Arkansas, the alluvial aquifer occurs in an area generally 50 to 125 mi wide and about 250 mi long adjacent to the Mississippi River. The alluvial aquifer is the uppermost aquifer in this area, and generally ranges in thickness between 50 and 150 ft. The alluvial aquifer is under both confined and unconfined conditions depending on location (Czarnecki and others, 2002). Withdrawal of ground water from the alluvial aquifer for agriculture started in the early 1900’s in the Grand Prairie area for irrigation of rice and, to a lesser extent, soybeans. Water-level declines in the alluvial aquifer were first documented in 1927 (Engler and others, 1963, p. 21). From 1965 to 2000, water use from the alluvial aquifer in eastern Arkansas increased 637 percent. In 1997, 635.6 million cubic feet per day (Mft^3/d) of water were pumped from the aquifer, primarily for irrigation and fish farming. In 2000, 97 percent of the ground water obtained in eastern Arkansas came from wells completed in the alluvial aquifer (Terry Holland, U.S. Geological Survey, written commun., 2002).

Because of heavy demands placed on the aquifer, several large cones of depression over 100 ft deep have formed in the potentiometric surface, resulting in lower well yields and degraded water quality in some areas. Several counties in the Grand Prairie area, which are within the extent of the alluvial aquifer, have been designated Critical Ground-Water Areas by the Arkansas Soil and Water Conservation Commission (ASWCC). One criterion associated with the designation of a Critical Ground-Water Area applies when water levels drop below half the original saturated thickness of the formation.

Ground-water flow models of the alluvial aquifer show that continued ground-water withdrawals at rates equal to those of 1997 could not be sustained indefinitely without causing water levels to decline below half the original saturated thickness of the formation (Reed, 2003). To develop estimates of withdrawal rates that could be sustained in compliance with the constraints of critical ground-water area designation, the U.S. Geological Survey (USGS), in cooperation with the ASWCC and the Memphis District of the U.S. Army Corps of Engineers (USACE) applied conjunctive-use optimization modeling to an existing model of ground-water flow for the alluvial aquifer (fig. 1). Conjunctive use involves the withdrawal of both ground water and surface water. Conjunctive-use optimization modeling is a technique that can be used to determine maximum withdrawal rates from both surface water and ground water while meeting constraints with respect to water levels and streamflow. These withdrawal rates would form the basis for estimates of sustainable yield from the alluvial aquifer and from rivers specified within the alluvial aquifer model.

Purpose and Scope

The purpose of this report is to describe the application and evaluation of a conjunctive-use optimization model (hereafter referred to as the optimization model) of the Mississippi River Valley alluvial aquifer of northeastern Arkansas. The optimization model was formulated as a linear program, and utilized a ground-water model developed for the study area by Reed (2003) as a basis for evaluation. The purpose of the optimization model was to: (1) determine maximum withdrawal rates from model cells at which ground-water withdrawals occurred in 1997 and (2) determine maximum withdrawal rates from model cells at stream locations while maintaining ground-water levels at or above specified levels and streamflow at or above specified rates. The report describes the amount of projected total water demand that can be met by the alluvial aquifer and by available surface water while



Base from U.S. Geological Survey digital data, 1:100,000 and 1:2,000,000
Digital geological data adapted from Haley and others (1976) and Bicker (1969)

Figure 1. Location of study and modeled area.

maintaining specified constraints. In this report, sustainable yield is defined as the amount of water that can be withdrawn indefinitely from ground water and from surface water without violating specified hydraulic-head or streamflow constraints. If an anticipated demand for water is known, an unmet demand may be calculated by subtracting the sustainable yield from the anticipated demand. Sustainable yield from ground water will be compared to anticipated demand for various withdrawal rates, because of concerns about water-level declines in the alluvial aquifer. The results of the optimization modeling can provide water managers and policy makers with information that can be used to assist in the management of the ground-water resources of the alluvial aquifer in northeastern Arkansas in a sustainable manner.

Previous Studies

Many investigators have described the underlying sediments of the Mississippi River Alluvial Plain. One of the earliest reports describing subsurface geology and ground-water resources in southern Arkansas and northern Louisiana was written by Veatch (1906). Ground-water resources of northeastern Arkansas were described and a detailed inventory was provided by Stephenson and Crider (1916). Fisk (1944) reported on extensive geologic investigations along the Mississippi River Valley made by the U.S. Army Corps of Engineers between 1941 and 1944. Krinitzsky and Wire (1964) expanded on the hydrogeologic work of Fisk with a comprehensive look at ground-water conditions. Cushing and others (1964) and Boswell and others (1968) provided an overview of the alluvial aquifer in their discussions of Quaternary-age aquifers on the Mississippi Embayment. Boswell and others (1968) first referred to the water-yielding sediments underlying the alluvial plain as the Mississippi River Valley alluvial aquifer.

The MODFLOW (McDonald and Harbaugh, 1988) ground-water flow model (hereafter referred to as the flow model) used in the optimization modeling of this report is based on the work of Reed (2003), who recalibrated and extended the model of Mahon and Poynter (1993) to include hydraulic-head observations for the years 1992 and 1998. Many researchers have applied conjunctive-use optimization models to the management of ground-water systems. Reichard (1995) provides a thorough review of many of these studies. Nishikawa (1998) used MODMAN 3.0 (Greenwald, 1998) (the precursor program to the one used for optimization in this report) to simulate scenarios to minimize the cost of supplying water during a design drought in Santa Barbara, California, by optimizing delivery of surface water and operation of the city’s reservoirs.

The first effort to optimize ground-water withdrawals from the alluvial aquifer was done by Peralta and others (1985) who estimated future ground-water availability in the Grand Prairie area by using a flow model coupled to an optimization routine. This analysis focused on a small subset of the area contained in this report, and did not couple the conjunctive use of ground water and surface water. Barlow and others (2003) developed conjunctive-use management models for estimating sustainable yield from surface water and ground water within an alluvial-valley stream-aquifer system in Rhode Island.

Acknowledgments

The conjunctive-use optimization routine used in this report is an adaptation of enhancements to MODMAN (Greenwald, 1998) made by Brian Wagner (U.S. Geological Survey). Wesley Danskin (U.S. Geological Survey) provided guidance on the use of MODMAN and the large-scale optimization solver MINOS (Murtaugh and Saunders, 1998). Paul Barlow (U.S. Geological Survey) provided a comprehensive and extensive review of the report. Streamflow constraints used in the optimization model were compiled by Steve Loop (Arkansas Soil and Water Conservation Commission).

Study Area

The study area (fig. 1), which is the same as the model area, is 14,104 mi^2, and includes all or part of 23 counties north of the Arkansas River in Arkansas and all or part of 5 counties in southeastern Missouri. The active cells of the model cover the area north of the Arkansas River, west of the Mississippi River, east of the surficial exposure of Paleozoic-age formations (or Fall Line), and south of about 8 mi north of the Arkansas/Missouri State line encompassing a small part of southeastern Missouri (fig. 1).

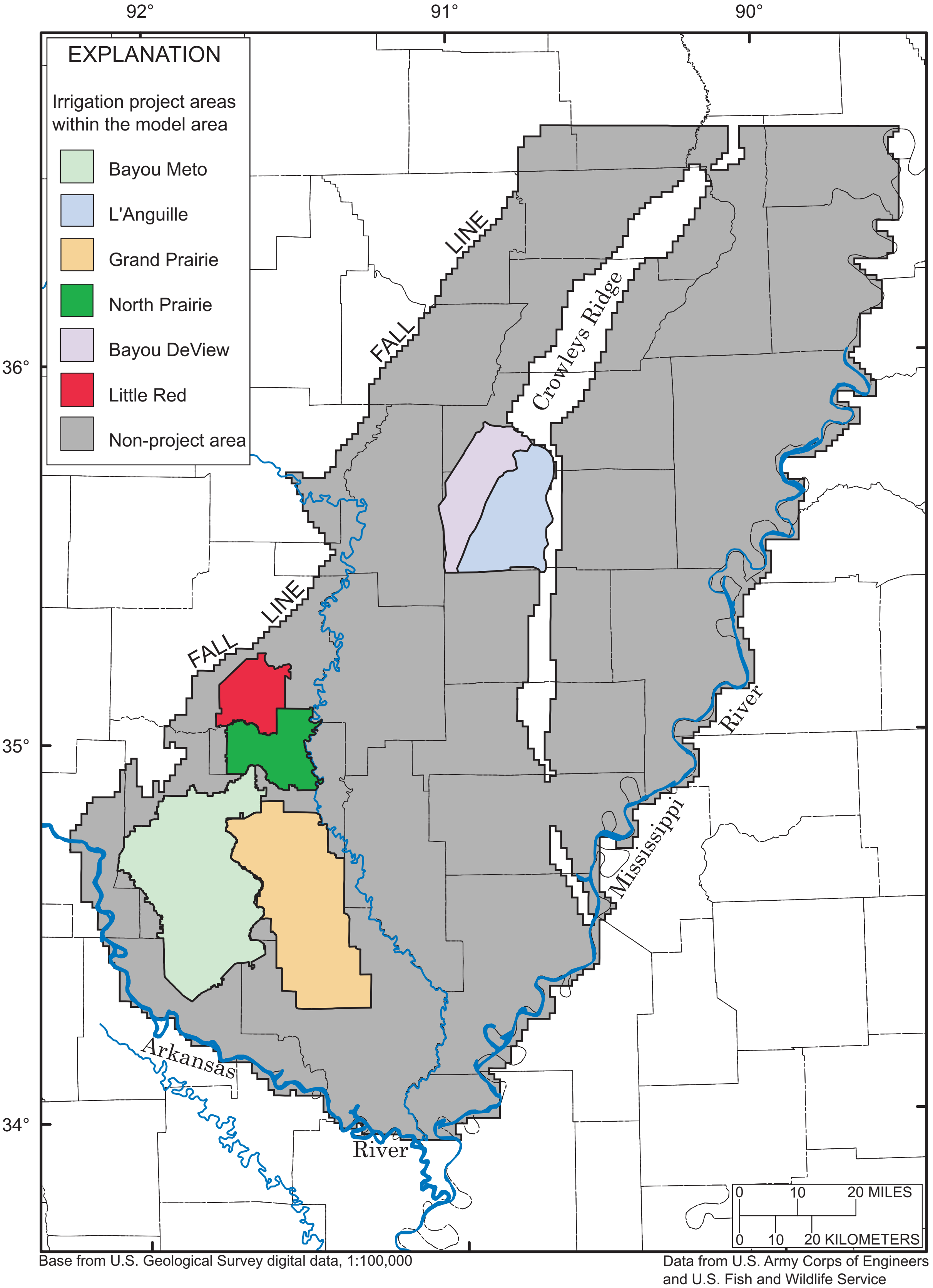


Figure 2. Irrigation project areas within the model area.

Irrigation Project Areas

Six irrigation project areas (fig. 2) have been established by various agencies for managing water resources within the alluvial aquifer (Ken Bright, U.S. Army Corps of Engineers, written commun., 2002; Jason Phillips, U.S. Fish and Wildlife Service, written commun., 2002). To meet the water needs of these areas, delivery of supplemental water from surface-water sources is being considered by agencies such as the U.S. Army Corps of Engineers and the ASWCC.

CONJUNCTIVE-USE OPTIMIZATION

The following sections describe the development and application of the conjunctive-use optimization approach applied to the model area, beginning with a review of the flow model. The optimization model is described and results evaluated.

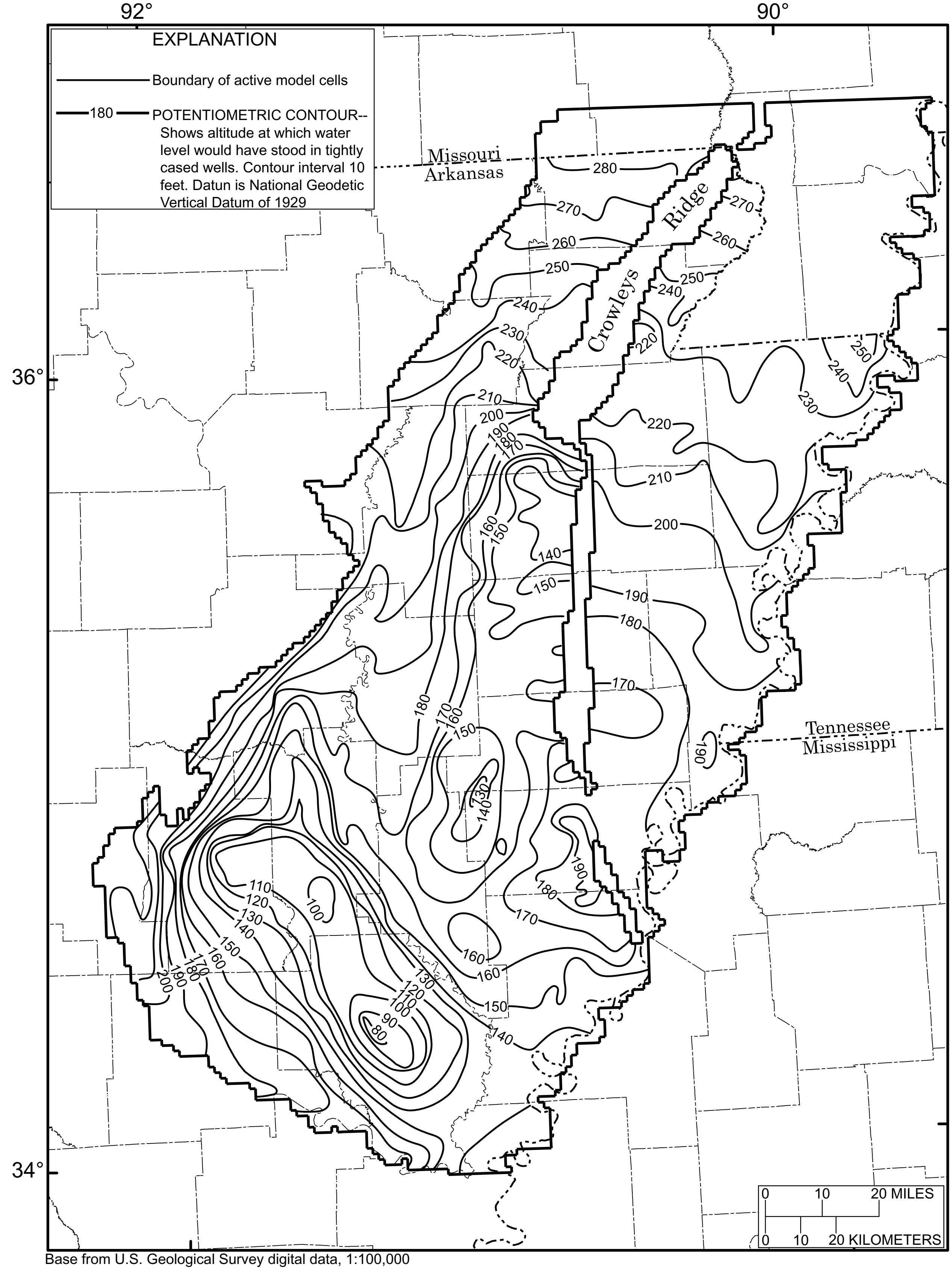
Flow Model

Although ground-water flow models had been developed previously for the Mississippi River Valley alluvial aquifer in northeastern Arkansas, these models were either at a scale that was too large to analyze the effects of projected ground-water withdrawals within the study area or were too limited in their areal extent. The flow model discussed in this report is based on the work of Reed (2003), which improved on previous models by simulating transient conditions to include observed hydraulic head from 1972 to 1998 (the previous model only included hydraulic-head observations to 1982). Characteristics for the flow model are listed in table 1. The flow model incorporates river, general head, no-flow, and areally distributed recharge boundary conditions. Flow from sources located outside the model area (such as from the underlying Sparta aquifer, or from the Interior Highlands to the west) is simulated by either general-head or areally-distributed-recharge boundary conditions. The distribution of boundary conditions specified in the model results in ground-water flow from recharge sources to areas with extensive ground-water pumping, and consequent widespread lowering of the water table. Water-level altitudes within the aquifer in spring 1998 are shown in figure 3, which shows substantial cones of depression resulting from sustained ground-water withdrawals. Both confined and unconfined conditions are simulated in the model. The model was divided into two layers of equal thickness with the lower layer having a larger hydraulic conductivity than the upper layer. This designation is consistent with observations of coarser, more transmissive sediments in the lower part of the aquifer. Model parameters were estimated in part using MODFLOW-2000 (Hill and others, 2000) to assist model calibration to observed values of hydraulic head in 1972, 1982, 1992, and 1998. Values of the mean, mean absolute, and root-mean-square difference between observed and simulated hydraulic head for all observations for all periods simulated by the model are listed in table 1. The mean difference (-0.46 ft) is a sum of the differences, both positive and negative, divided by the total number of observations; for an unbiased model, this value would be zero. The mean absolute difference (4.9 ft) is the sum of the magnitudes of the difference at each observation, divided by the total number of observations; a value close to zero is preferable. Assuming a normal distribution, approximately 67 percent of the residual values (that is, the difference between observed and simulated water-level altitudes) would lie within positive or negative values of the root-mean-square difference (that is, +/-6.4 ft). Given the difference in the range in observed hydraulic-head values (220 ft), these differences are small and indicate a good fit to the observed hydraulic-head values.

Table 1. Characteristics of the flow model (modified from Reed, 2003)

[mi^2, square mile; ft^3/d, cubic foot per day; ft/d, foot per day; ft^{-1}, inverse foot; ft, foot]

Characteristic	Value
Model area	14,104 mi^2
Cells with wells corresponding to 1997 withdrawals	9,979
Total pumpage in 1997	635,600,000 ft^3/d
Average hydraulic conductivity	230 - 480 ft/d
Specific yield	0.30
Specific storage	$1x10^{-6}$ ft^{-1}
River cells	1,165
Hydraulic-head observations	1,698
Hydraulic-head observation periods	1972, 1982, 1992, 1998
Range in observed hydraulic head values in 1998(feet above National Geodetic Vertical Datum of 1929)	78 - 298 ft
Mean difference between observed and simulated hydraulic head, all four periods	-0.46 ft
Mean absolute difference between observed and simulated hydraulic head, all four periods	4.9 ft
Root-mean-square difference between observed and simulated hydraulic head, all four periods	6.4 ft



Base from U.S. Geological Survey digital data, 1:100,000

Figure 3. Potentiometric surface within the alluvial aquifer, spring 1998.

The flow model was used to simulate ground-water flow for the period from 1918 through 2049, and to evaluate the demand for ground water from the alluvial aquifer, which has increased steadily for the last 40 years (Reed, 2003). The flow model showed that water is being withdrawn from the aquifer at rates that are much greater than what can be sustained for the long term. Based on measured water levels, the saturated thickness of the alluvial aquifer has been greatly reduced in some areas (Schrader, 2001; Czarnecki and others, 2002). This has resulted in degraded water quality, decreasing water availability, increased pumping costs, and lower well yields.

Optimization Model

For the optimization model described in this report, modifications were made to MODMAN 4.0 to: (1) incorporate stream withdrawal cells as decision variables, (2) allow specification of streamflow constraints, and (3) account for streamflow water budgeting. Modifications to the MODMAN code were initially provided by Brian Wagner (U.S. Geological Survey) in a modification to MODMAN 3.0, and adapted to MODMAN 4.0. In addition, the ability to aggregate wells within a subarea of the model and to treat an aggregate-well pumping rate as a single decision variable was added to MODMAN 4.0. However, that ability was not utilized; instead, 9,979 ground-water-withdrawal decision variables and 1,165 surface-water-withdrawal decision variables were specified.

The optimization modeling process (fig. 4) begins with the calibration and adaptation of a MODFLOW-based ground-water flow model to be compatible with the optimization modeling software (MODMAN 4.0). Adaptation entailed the conversion of the flow model from MODFLOW 2000 (Hill and others, 2000) to MODFLOW 96 (McDonald and Harbaugh, 1988; Harbaugh and McDonald, 1996), and verifying that the results were the same. Steady-state conditions were selected (as opposed to transient

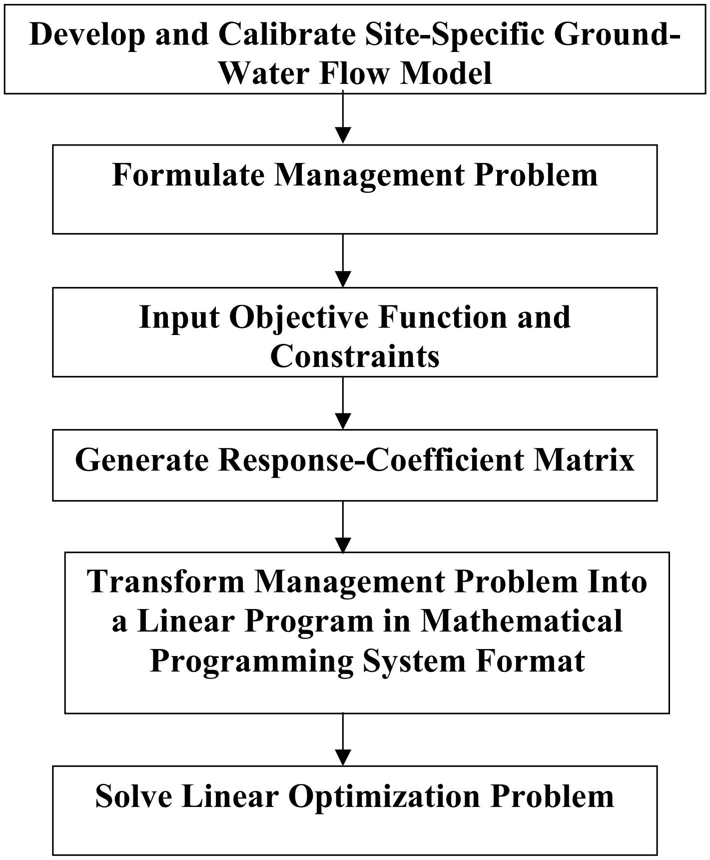


Figure 4. Flow chart of optimization modeling process.

conditions) because the maximized withdrawals are intended to represent sustainable yield of the system (a rate that can be maintained indefinitely).

A management problem is formulated to maximize a parameter, such as water production from ground water and surface water, within selected constraints, such as maintaining hydraulic heads in the aquifer above a minimum altitude or maintaining a minimum amount of streamflow. A conjunctive-use version of MODMAN 4.0 was used to generate response coefficients for each specified withdrawal cell in the model. The response-coefficient matrix consists of changes in hydraulic head or streamflow at each constraint location that occur in response to pumping at a single well or river cell at a unit rate (Greenwald, 1998; Ahlfeld and Mulligan, 2000). The unit rate was specified at 10,000 ft^3/d. To accurately represent the response of the flow model to a unit rate of pumping under unconfined conditions, selection of starting hydraulic heads should be similar to those that would result when optimal withdrawal rates are applied. Starting hydraulic head values were selected as those simulated for 1997 from the model of Reed (2003). Because some dry cells occurred in that model at that simulated point in time, hydraulic-head values at dry cells were assigned similar values as adjacent model cells that were not dry.

After all the response coefficients are calculated, they are combined to form a data-input set along with hydraulic-head and streamflow constraints, and are formulated as a linear optimization program in mathematical programming system (MPS) format. The linear program is run under MINOS. If a feasible solution exists, MINOS will provide estimates of optimal (maximum) values of ground-water and surface-water withdrawals. MINOS also identifies points in the model where hydraulic-head or streamflow constraints have been reached.

Optimal ground-water withdrawal rates calculated using the optimization model were evaluated by applying them in the flow model, to compare the resulting simulated hydraulic head against the specified hydraulic-head constraints. Non-linear flow model behavior was expected for this model because of the unconfined condition of the aquifer and head-dependent flow boundary conditions at the rivers. For this reason, starting values of hydraulic head were specified as those simulated for 1997. In a strictly linear model, such as one for a confined aquifer, ground-water flow is a function of hydraulic head through only the hydraulic-gradient term in Darcy's law:

$$Q = -K\frac{dh}{dl}A \qquad (1)$$

where Q is ground-water flow, in cubic feet per day;
K is hydraulic conductivity, in feet per day;
$\frac{dh}{dl}$ is the hydraulic gradient, dimensionless;
h is hydraulic head, in feet;
l is a distance over which the gradient is measured, in feet; and
A is the cross-sectional area through which flow occurs, in feet squared.

For unconfined conditions, A also is a function of hydraulic head. If changes in hydraulic head are small relative to the total saturated thickness, then A will remain about the same. However, if substantial change in saturated thickness occurs, A can change appreciably, because

$$A = bw \qquad (2)$$

where b is the saturated thickness, which varies with hydraulic head, in feet; and
w is the width through which flow occurs, in feet.

This is an important consideration in selecting starting values of hydraulic head for the flow model to produce a more efficient solution to the ground-water flow equation (McDonald and Harbaugh, 1988).

Problem Formulation

The optimization model was formulated as a linear programming problem with the objective of maximizing water production from wells and from streams subject to: (1) maintaining ground-water levels at or above specified levels; (2) maintaining streamflow at or above minimum specified rates; and (3) limiting ground-water withdrawals to a maximum of either 100, 150, or 200 percent of the rate pumped in 1997. Steady-state conditions were selected (rather than transient conditions) because the maximized withdrawals are intended to represent sustainable yield of the system (a rate that can be maintained indefinitely). In this model, the decision variables (a term used in optimization modeling to identify variables that can be part of a management scheme) are the withdrawal rates at 9,979 model cells corresponding to well locations and at 1,165 river cells.

Objective Function

The objective of the optimization model is to maximize water production from ground-water and surface-water sources. The objective function of the optimization model has the form:

$$\text{maximize } z = \sum q_{well} + \sum q_{river} \qquad (3)$$

where z is the total managed water withdrawal, in cubic feet per day;

$\sum q_{well}$ is the sum of ground-water withdrawal rates from all managed wells, in cubic feet per day; and

$\sum q_{river}$ is the sum of surface-water withdrawal rates from all managed river reaches, in cubic feet per day.

Hydraulic-Head Constraints

Equation 3 is computed such that the following constraints are maintained:

$$h_c \geq h_{minimum} \qquad (4)$$

where h_c is the hydraulic head (water-level altitude) at constraint location *c*, in feet; and

$h_{minimum}$ is the water-level altitude at half the thickness of the aquifer, in feet.

To accommodate the ASWCC Critical Ground-Water Area criteria that water levels within the alluvial aquifer should remain above half the original saturated thickness of the aquifer, hydraulic-head constraints were specified at 2,804 model cells. For a few cells where the original saturated thickness of the aquifer is less than 60 ft but at least 30 ft, the hydraulic head constraint was specified as 30 ft, a minimum thickness considered necessary for the aquifer to remain viable in those areas. The spatial distribution of constraint points represents approximately every fifth model cell (fig. 5). If water levels were to drop everywhere to the level of the head constraint, then the resulting saturated thickness of the alluvial aquifer would range from 30 to 100 ft, and generally be thinnest in the Grand Prairie area (fig. 5).

Streamflow Constraints

Streamflow is regulated in Arkansas by ASWCC for purposes of maintaining water quality, navigation, and species habitat. Streamflow constraints for several rivers specified in the optimization model are based on 7-day, 10-year-recurrence low-flow data (7Q10). Streamflow constraints are specified as the minimum amount of flow required at individual river cells. The equation governing the relation between streamflow constraints and flow into and out of a stream is

$$\begin{aligned} &q^R_{head} + \sum q^R_{overland} \pm \sum q^R_{ground\ water} \\ &- \sum q^R_{diversions} - \sum q^R_{river} \geq q^R_{minimum} \end{aligned} \qquad (5)$$

where q^R_{head} is the flow rate into the head of stream reach *R*, in cubic feet per day;

$\sum q^R_{overland}$ is the sum of all overland and tributary flow to stream reach *R*, in cubic feet per day;

$\sum q^R_{ground\ water}$ is the net sum of all ground-water flow to or from stream reach *R*, in cubic feet per day;

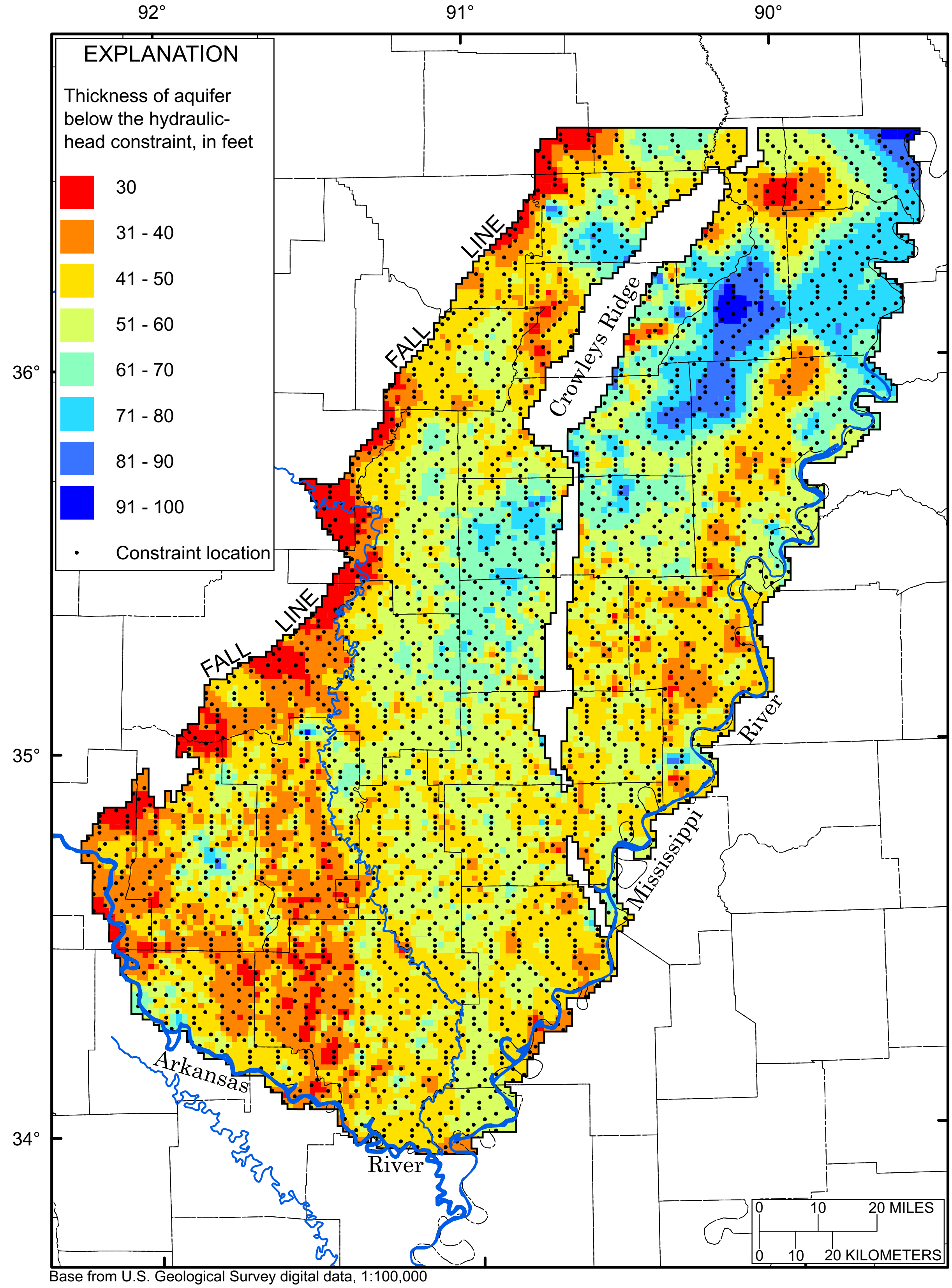
$\sum q^R_{diversions}$ is the sum of all surface-water diversions from stream reach *R*, in cubic feet per day;

$\sum q^R_{river}$ is the sum of all potential withdrawals, not including diversions, from stream reach *R*, in cubic feet per day; and

$q^R_{minimum}$ is the minimum permissible surface-water flow rate for stream reach *R*, in cubic feet per day.

Ground-Water Withdrawal Limits

The proximity of managed wells to model flow boundaries was taken into account to properly formulate the management objective. If no limit is imposed on the potential amount of water that can be pumped at each managed well, then those wells nearest model sources of water, such as rivers or general head-boundaries, will be the first to be supplied water, thus capturing flow that would otherwise reach wells further from the sources. Test simulations done with the optimization model show that without limits on pumping, wells adjacent to sources of water would have optimized withdrawal rates that were orders of magnitude larger than rates corresponding to those of 1997. Not only is it physically unlikely that individual wells could pump that much more water, but construction of sufficient additional wells in the one-square mile cells also is unlikely. The phenomenon of wells near rivers capturing induced recharge from the rivers and preventing



Base from U.S. Geological Survey digital data, 1:100,000

Figure 5. Location of hydraulic-head constraint points and thickness of aquifer below hydraulic-head constraint.

sufficient water from flowing to interior wells is, however, consistent with current conditions (Czarnecki and others, 2002).

Test simulations using 1997 withdrawal rates applied to steady-state conditions yielded large areas with dry cells in the flow model. Therefore, ground-water demand limits were specified at each cell as a multiple of the amount pumped in 1997, such that

$$0 \le q_{well_i} \le M q_{well\ 1997} \tag{6}$$

where, q_{well_i} is the optimal ground-water withdrawal for well i, in cubic feet per day;

M is a multiplier between 1 and 2; and

$q_{well\ 1997}$ is the total amount withdrawn in 1997 from all wells, in cubic feet per day.

Wells are optimized as individual wells, and therefore, have individual rates associated with each cell. For each optimization model run, the multiplier M is specified as a uniform value that applies to all 9,979 ground-water withdrawal cells.

Surface-Water Withdrawal Limits

No limits were imposed on optimized withdrawals from rivers such that the range in optimal withdrawal was between zero and the maximum amount of water available at a given point in a given river. This specification permitted analysis of where water could be produced and the maximum amount available. Withdrawals were allowed at all river cells.

Predevelopment Recharge From Non-River Sources

Optimal pumping rates are affected by the rate of recharge from non-river sources explicitly specified in the flow model because these non-river sources can be a source of water to wells instead of the rivers. Recharge from non-river sources corresponds to recharge from underlying hydrogeologic units such as the Sparta aquifer or from the Interior Highlands west and north of the modeled area. Recharge to the aquifer from non-river sources will vary with time as ground-water withdrawals reduce the hydraulic head in the alluvial aquifer (fig. 6). Initially, flow is induced from underlying and adjacent units. As the hydraulic head within the aquifer is lowered because of pumping, increased flow from underlying and adjacent units

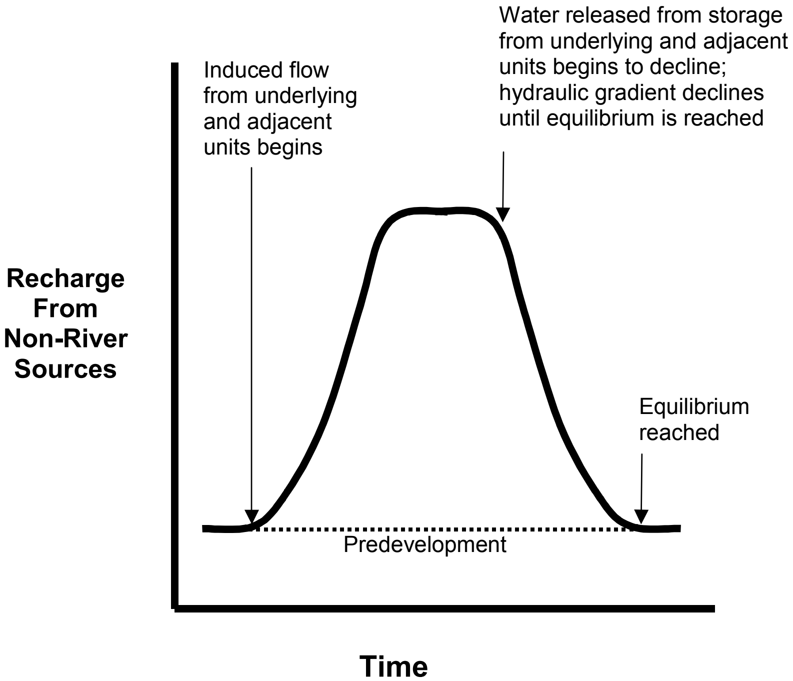


Figure 6. Hypothetical variation in recharge to the alluvial aquifer from non-river sources as a function of time.

occurs as water is released from storage. A point is reached when the gradient between the aquifer and the non-river sources equilibrates, and the change in storage is zero. At this point, recharge to the system is at predevelopment rates, which can be attributed to the long-term recharge from precipitation from distant sources. This is important because the recharge from non-river sources is specified explicitly in the model and is allowed to vary with time. Therefore, specifying recharge from non-river sources associated with predevelopment conditions for steady-state simulations is appropriate for obtaining estimates of sustainable yield.

Wells Used in Optimization Model

For optimization, 9,979 one-square mile cells were used to represent pumping from 35,043 wells in 1997. Each cell was specified as a managed well (that is, a decision variable) within MODMAN. In 1997, the annual pumping rate for all wells was 635.6 Mft^3/d. Note that in the model of Reed (2003), dry cells occurred causing pumping wells at the dry cells to become inactive, reducing total pumping to 631 Mft^3/d. For the sustainable-yield analysis, the optimized rate at each of the 9,979 cells was allowed to vary between a rate of zero to a maximum rate equal to a multiple between 1 and 2 to that which was pumped in 1997. An upper limit was specified because no limit on pumping led to unrealistic optimal withdrawal from wells adjacent to rivers.

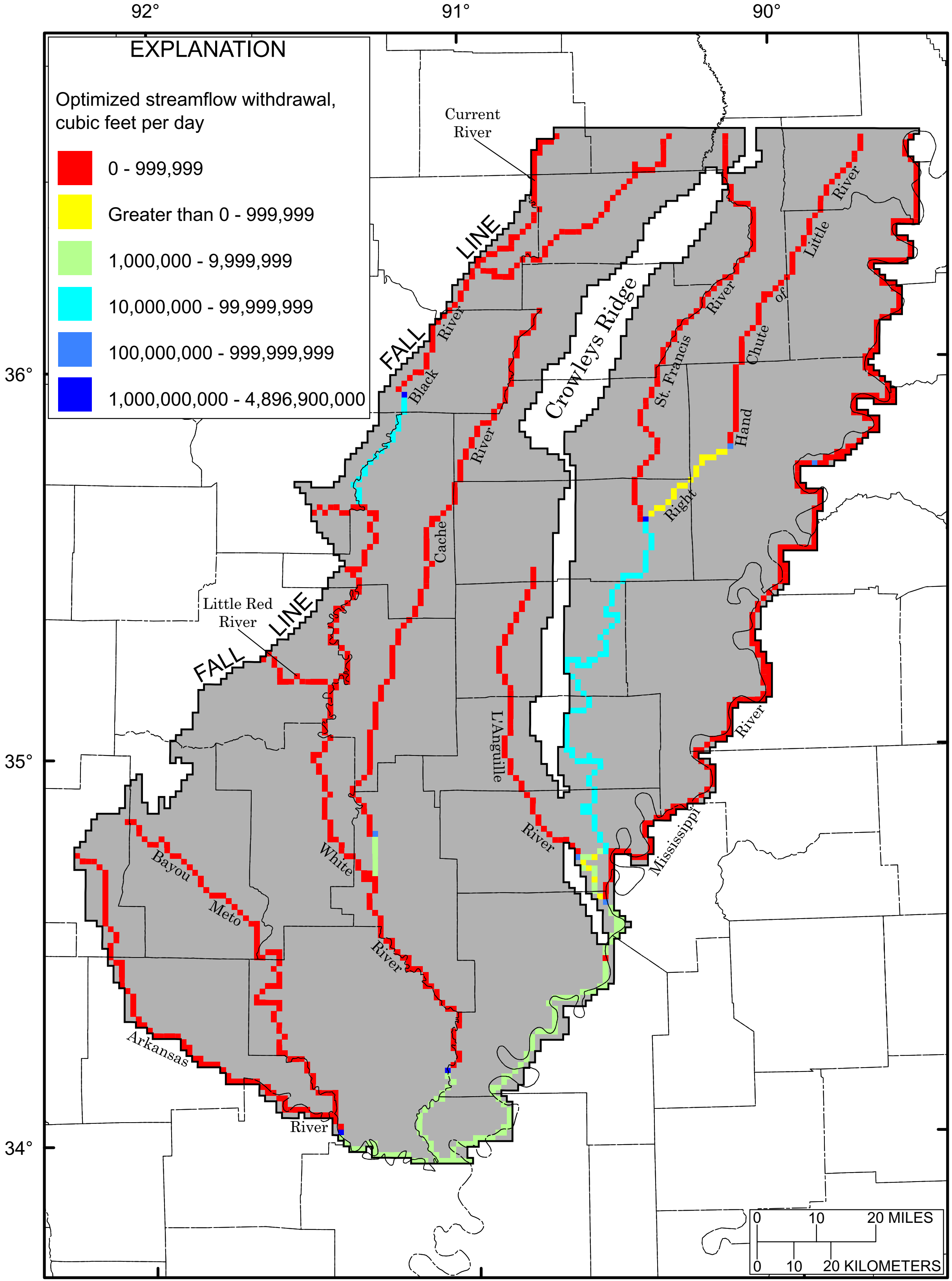
Streamflow

To allow for both the optimal conjunctive-use of surface water and ground water within the optimization model, 11 rivers were specified (table 2). Of the 11 rivers specified, 7 have streamflow constraints specified at each river cell based on 7-day, 10-year-recurrence low flows (7Q10) (Steve Loop, Arkansas Soil and Water Conservation Commission, written commun., 2001), which are derived from historical streamflow for the rivers. Where a constraint was not provided by ASWCC, an arbitrary value of zero was specified, except in the case of the Mississippi River where a value of 50 billion cubic ft per day was specified. By specifying a minimum flow constraint based on 7Q10 data, available streamflow within the optimization model would be limited all year long to an amount equal to or greater than 7Q10, although 7Q10 data reflect a statistically low flow that occurs only once every 10 years, and then for only 7 consecutive days. Streamflow constraints were specified at every river cell (1,165 total) to facilitate the calculation of optimized streamflow withdrawals (fig. 7). It should be noted that flow constraints based on 7Q10 flows are only one criterion that could be selected, the results from which reflect one specific application of the optimization model. Flow into the most upstream cell of each river contained within the model was specified based on mean annual flow, as were the cells at which tributaries connect. Because stream gages are not located at the start of the rivers simulated in the model, mean annual flow was prorated based on the drainage area up stream from that point. Overland flow (that is, surface-water runoff that would enter river cells from minor tributaries or sheet flow that were not explicitly represented in the model) was distributed equally at river cells within a river reach based on the difference in long-term average streamflow for a specific river reach, as measured between the upstream and downstream ends of the reach; or if such data were unavailable, areal estimates of runoff based on drainage areas were used (Elton Porter, U.S. Geological Survey, written commun., 2000). Surface-water diversion rates that occurred in 2000 were subtracted from specified overland flow at the appropriate river cells. Included in these diversions were the planned diversions of 63,339,248 ft^3/d for the Bayou Meto project area and 55,078,367 ft^3/d for the Grand Prairie project, which factor in an additional 30 and 40 percent transmission loss area for deliveries to the project areas, respectively.

Optimization Results

Sustainable Yield

The ultimate objective of the optimization model is to provide estimates of sustainable yield from both ground water and surface water. Sustainable yield is defined as a withdrawal rate from the aquifer or from a stream that can be maintained indefinitely (that is, to steady-state conditions) without causing violation of either hydraulic-head or streamflow constraints. For this model, ground-water levels were not allowed to drop below half the thickness of the aquifer or 30 ft above the bottom of the aquifer, whichever resulted in the higher ground-water level. Streamflow was not allowed to drop below a minimum amount specified in table 2. The optimization model was used to obtain estimates of sustainable yield at 9,979 ground-water and 1,165 streamflow withdrawal cells.



Base from U.S. Geological Survey digital data, 1:100,000

Figure 7. Location of streams within model showing cells and rates at which water could be withdrawn and still meet constraints within optimization model. The upper limit for well withdrawals was set at 100 percent of the 1997 withdrawal rate.

Table 2. Rivers, streamflows, and streamflow constraints

[ASWCC, Arkansas Soil and Water Conservation Commission; flow constraint from ASWCC based on an annual minimum 7-consecutive-day average flow with a recurrence interval of 10 years; ft^3/d, cubic foot per day]

River name	Number of model cells	Flow into uppermost river cell of model (ft^3/d)	Overland flow per river cell (ft^3/d)	Overland flow per river reach (ft^3/d)	Flow constraint (ft^3/d)	Source for value of constraint
Arkansas	97	4,903,200,000	1,000,000	97,000,000	100,224,000	ASWCC
Bayou Meto	77	17,020,800	1,000,000	77,000,000	605,000	ASWCC
Black	88	148,996,800	8,000,000	704,000,000	27,302,400	ASWCC
Cache	105	50,328,000	2,000,000	210,000,000	950,400	ASWCC
Current	31	280,886,400	2,000,000	62,000,000	0	Arbitrary
L'Anguille	54	21,556,800	2,000,000	108,000,000	3,974,400	ASWCC
Little Red	15	247,017,600	1,400,000	21,000,000	0	Arbitrary
Mississippi	305	50,185,440,000	3,000,000	915,000,000	50,000,000,000	Arbitrary
Right Hand Chute	74	244,944,000	1,000,000	74,000,000	0	Arbitrary
St. Francis	169	231,552,000	12,000,000	2,028,000,000	7,257,600	ASWCC
White	150	1,248,480,000	25,000,000	3,750,000,000	665,000,000	ASWCC
Total	1,165	57,579,422,400	--	8,046,000,000[1]	50,805,313,800	--

[1] Summation assumes that overland flow is applied at river cell.

Because sustainable yield from ground water is a function of the pumping limit specified for each managed well, multiples of the rate withdrawn in 1997 were used to set the upper limit of pumping. The distribution of optimal withdrawal rates using upper limits specified as 100-, 150-, and 200-percent multiples of the 1997 withdrawal rates are shown in figures 8 through 10. The distribution of withdrawal rates for each of these scenarios is such that most wells are either withdrawing water at a rate equal to the upper limit or at a rate of zero. This is convenient from a management standpoint because wells are generally on or off. As the withdrawal rate limit is increased, the total number of wells that can pump actually decreases (although the total amount withdrawn increases), with those capable of withdrawing water being nearest to sources of water within the model (that is, major rivers). Test runs with the optimization model show that if no limits are placed on ground-water withdrawals, all of the withdrawals would come from wells adjacent to sources of water within the model and at rates that are orders of magnitude higher than were pumped in 1997. Although overall optimized withdrawal would be largest for such a scenario, the distribution of wells would be unacceptable from a management standpoint because nearly all of the water production would come from wells that are adjacent to rivers, with the remaining interior wells being unable to pump at all.

For the optimization run in which an upper withdrawal limit of 100 percent of the 1997 withdrawal rate was specified (scenario 1; fig. 8), the sustainable yield from ground water for the entire study area is 360 Mft^3/d (table 3), which is only about 57 percent of the amount withdrawn in 1997 (635.6 Mft^3/d). If the upper withdrawal limit is increased to 150 percent of the 1997 withdrawal rate (scenario 2; fig. 9), the sustainable yield from ground water for the entire study area is 445 Mft^3/d (table 3), which is about 70 percent of the amount withdrawn in 1997. If the upper withdrawal limit is increased to 200 percent of the 1997 withdrawal rate (scenario 3; fig. 10), the sustainable yield from ground water for the entire study area is 526 Mft^3/d (table 3), which is about 83 percent of the amount withdrawn in 1997.

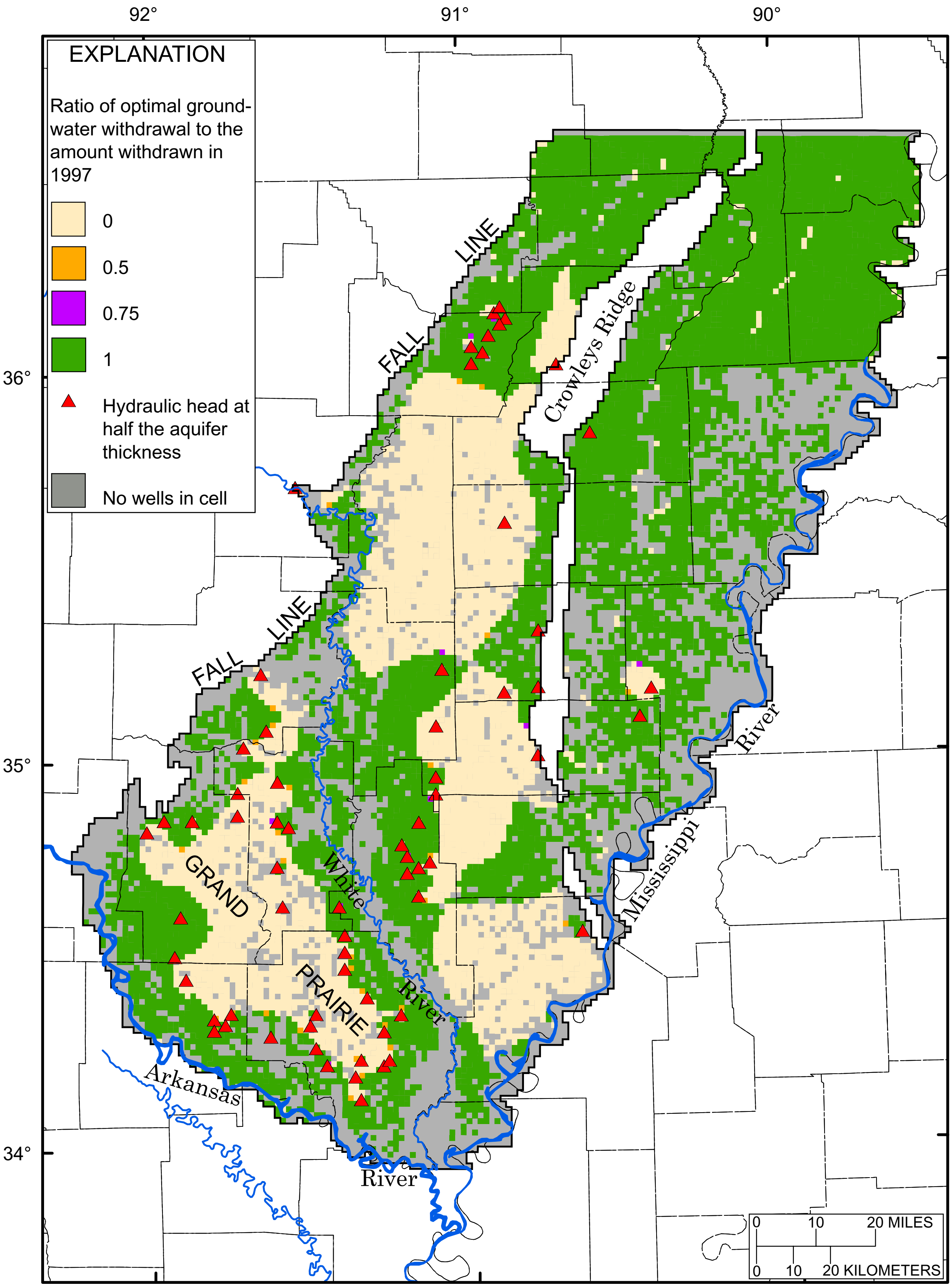
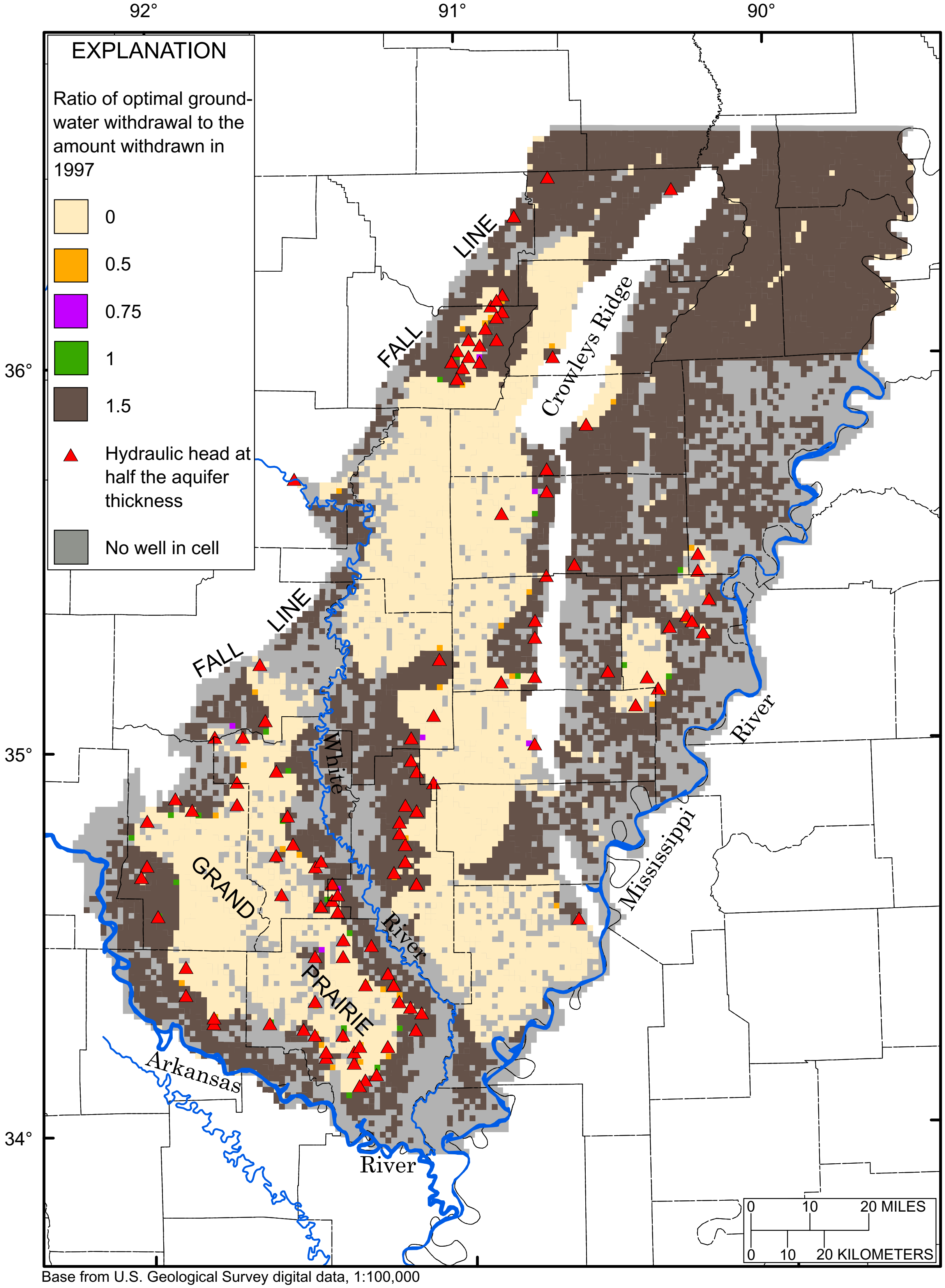


Figure 8. Location of optimal ground-water withdrawal calculated by the optimization model to the amount withdrawn in 1997 for withdrawal limits at each well set to 100 percent of the 1997 withdrawal rate.

Table 3. Sustainable yield and unmet demand for different upper limits on withdrawals and different demand rates

[Negative unmet demand values indicate surplus water availability. All values are in million cubic feet per day]

		Sustainable yield based on an upper withdrawal limit of:			Unmet ground-water demand (baseline rate minus sustainable yield) based on 1997 demand and a sustainable yield from:			Unmet ground-water demand (baseline rate minus sustainable yield) based on sustainable yields from scenarios 1, 2, and 3, respectively, and a demand of:		
County	1997 withdrawal rate (Mft^3/d)	100 percent of 1997 withdrawal rate (scenario 1)	150 percent of 1997 withdrawal rate (scenario 2)	200 percent of 1997 withdrawal rate (scenario 3)	Scenario 1	Scenario 2	Scenario 3	100 percent of 1997 withdrawal rate	150 percent of 1997 withdrawal rate	200 percent of 1997 withdrawal rate
Arkansas	51.4	24.0	27.0	29.6	27.5	24.4	21.9	27.5	50.1	73.3
Butler	4.4	4.3	6.5	8.6	0.1	-2.1	-4.2	0.1	0.1	0.2
Clay	31.4	31.0	43.4	54.9	0.3	-12.0	-23.5	0.3	3.7	7.9
Coahoma	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Craighead	49.4	26.3	30.4	34.4	23.2	19.1	15.1	23.2	43.8	64.5
Crittenden	17.3	15.1	16.9	18.4	2.2	0.4	-1.1	2.2	9.1	16.2
Cross	32.2	19.2	22.6	24.2	13.0	9.6	8.0	13.0	25.7	40.1
Desha	1.4	1.4	2.0	2.7	0.0	-0.7	-1.4	0.0	0.0	0.0
Dunklin	5.1	5.0	7.5	10.0	0.1	-2.4	-4.9	0.1	0.2	0.2
Dyer	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Greene	20.3	14.9	14.0	16.4	5.3	6.3	3.8	5.3	16.4	24.1
Independence	3.3	2.4	3.3	4.3	1.0	0.0	-1.0	1.0	1.7	2.4
Jackson	37.2	2.5	3.8	5.1	34.7	33.4	32.2	34.7	52.1	69.4
Jefferson	34.6	24.4	32.6	39.3	10.1	1.9	-4.7	10.1	19.2	29.9
Lake	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Lawrence	40.1	24.2	30.4	35.4	15.9	9.7	4.7	15.9	29.8	44.8
Lee	22.5	10.3	12.2	10.4	12.2	10.3	12.1	12.2	21.6	34.7
Lincoln	0.7	0.7	1.0	1.4	0.0	-0.3	-0.6	0.0	0.1	0.1
Lonoke	42.1	17.7	17.3	16.7	24.4	24.8	25.4	24.4	45.9	67.5
Mississippi	17.5	17.5	25.1	31.5	0.0	-7.5	-13.9	0.0	1.3	3.6
Monroe	28.3	20.8	22.0	30.0	7.5	6.3	-1.7	7.5	20.5	26.6
New Madrid	4.8	4.8	7.2	9.5	0.1	-2.3	-4.7	0.1	0.1	0.1
Pemiscot	3.9	3.9	5.8	7.7	0.1	-1.8	-3.8	0.1	0.1	0.2
Phillips	19.6	3.5	4.3	4.8	16.1	15.3	14.8	16.1	25.1	34.4
Poinsett	57.4	29.0	41.3	48.9	28.4	16.1	8.5	28.4	44.9	66.0
Prairie	27.3	13.8	16.2	18.9	13.5	11.1	8.4	13.5	24.7	35.7
Pulaski	3.1	3.1	4.7	6.3	0.0	-1.6	-3.1	0.0	0.0	0.0
Randolph	8.7	7.9	10.7	12.6	0.8	-2.0	-4.0	0.8	2.3	4.7
Ripley	0.2	0.2	0.3	0.4	0.0	-0.1	-0.2	0.0	0.0	0.0
Shelby	0.0	0.0	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0
St. Francis	27.2	9.7	9.9	11.5	17.4	17.2	15.6	17.4	30.8	42.8
Tipton	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
White	8.5	5.8	7.0	9.5	2.7	1.5	-0.9	2.7	5.8	7.6
Woodruff	35.5	16.7	19.4	22.2	18.8	16.1	13.3	18.8	33.9	48.8
Total	635.7	360.3	444.9	525.8	275.5	190.9	110.0	275.5	508.8	745.8



Base from U.S. Geological Survey digital data, 1:100,000

Figure 9. Ratio of optimal ground-water withdrawal calculated by the optimization model to the amount withdrawn in 1997 for withdrawal limits at each well set to 150 percent of the 1997 withdrawal rate.

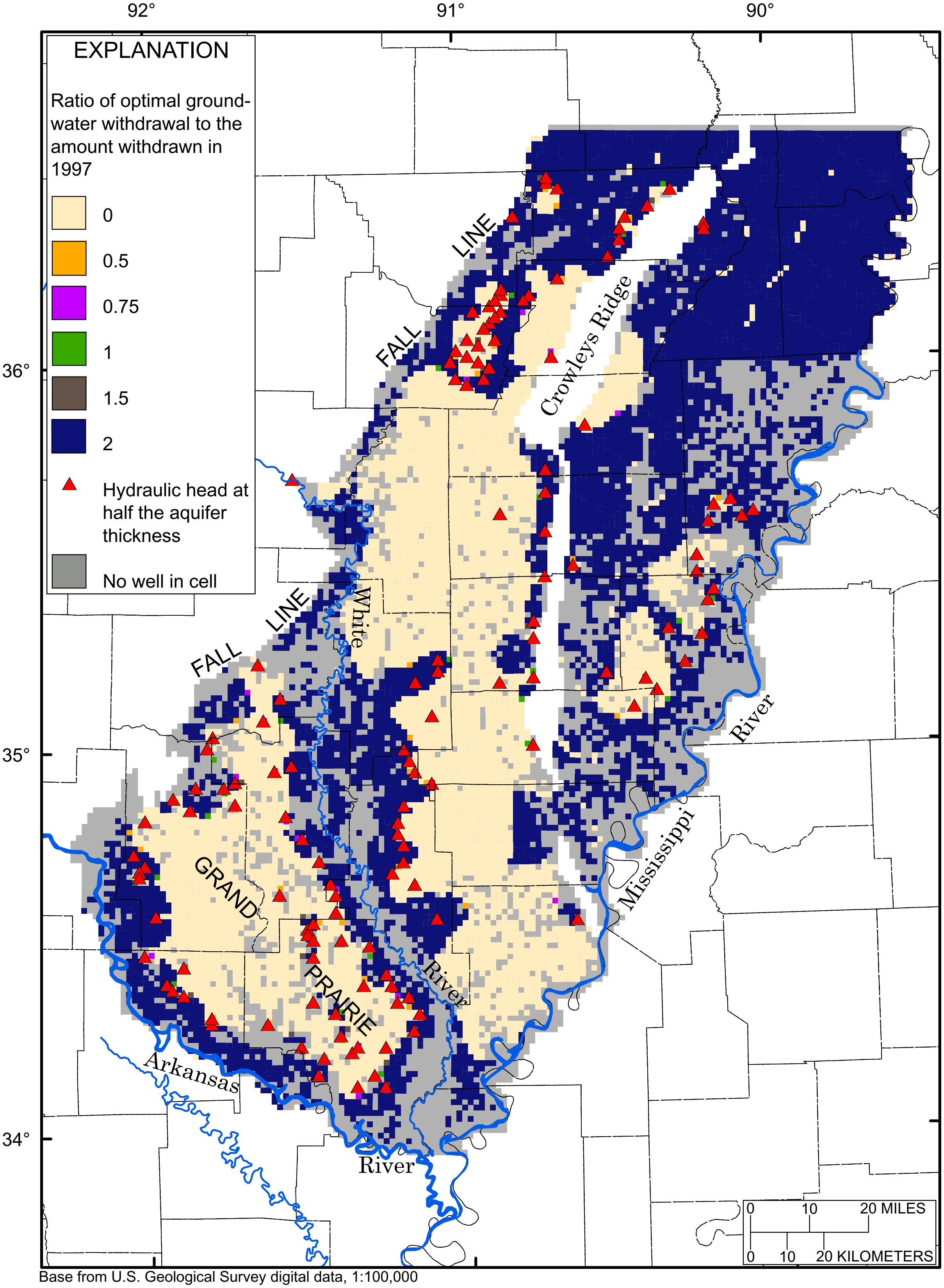


Figure 10. Ratio of optimal ground-water withdrawal calculated by the optimization model to the amount withdrawn in 1997 for withdrawal limits at each well set to 200 percent of the 1997 withdrawal rate.

Total sustainable yield from streamflow is 12.8, 12.7, and 12.6 billion ft^3/d for scenarios 1, 2, and 3, respectively (table 4), which is on the order of 20 times larger than the sustainable yield from ground water. Sustainable yield for all rivers ranged from 0 (Current and Little Red Rivers, and Bayou Meto) to almost 5 billion ft^3/d for the Arkansas River. These rates decrease as ground-water discharge to these streams decrease as the upper limit of ground-water withdrawal increases. Nonetheless, these large sustainable yields represent a potential source of water to supplement ground water and meet overall water demand, but to do so will require the construction of withdrawal and distribution facilities, which will have legal, political, economic, and social consequences.

Hydraulic-head constraints restrict where and how much ground water and, to a lesser extent, surface water can be extracted. The red triangles in figure 8 through 10 show the locations where the simulated value of hydraulic head, derived from the optimization model, reached the lower limits. In the Grand Prairie area, the red triangles delimit the boundary between cells that can produce water and those that cannot resulting from the limits imposed by the hydraulic-head constraints. Between Crowleys Ridge and the White River, a few points constrain withdrawals over large areas.

Wells in the western half of the model area, for which an optimized rate of zero was calculated, lie partly between the Arkansas and White Rivers (fig. 8). The Grand Prairie area of the model, which lies between these two rivers, contains parts of two U.S. Army Corps of Engineers irrigation project areas: Bayou Meto and Grand Prairie (fig. 2). The locations of these project areas are consistent with current and anticipated water needs, and the inability of the aquifer to supply water sustainably as demonstrated by the optimization-model results (figs. 8 through 10). Optimal sustainable yields from within the Bayou Meto irrigation project area and within the Grand Prairie irrigation project area are 18.1 and 9.1 Mft^3/d, respectively, assuming a maximum allowable withdrawal rate equal to 1997 rates. These values of sustainable yield represent 35 and 30 percent respectively of the amount pumped from these project areas in 1997.

Table 4. Optimized total streamflow withdrawals from the optimization model

River name	Sum of optimized total streamflow withdrawal (cubic foot per day)		
	100 percent of baseline rate (scenario 1)	150 percent of baseline rate (scenario 2)	200 percent of baseline rate (scenario 3)
Arkansas	4,949,237,300	4,939,029,200	4,930,806,700
Bayou Meto	0	0	0
Black	1,401,763,000	1,386,626,000	1,370,421,000
Cache	225,434,500	222,633,400	213,770,000
Current	0	0	0
L'Anguille	123,381,780	122,834,070	123,086,270
Little Red	0	0	0
Mississippi	1,110,849,200	1,102,986,600	1,096,987,400
Right Hand Chute	323,995,740	314,608,571	306,391,523
St. Francis	2,036,571,700	2,014,040,840	2,001,651,420
White	2,635,151,400	2,627,994,800	2,613,921,400
Total	12,806,384,620	12,730,753,481	12,657,035,713
Difference from scenario 1	--	-75,631,139	-149,348,907

The distribution of hydraulic-head constraint points has a substantial effect on the distribution and amount of optimal pumping. Test simulations with the model were conducted to compare the sustainable yield derived from a model simulation with hydraulic-head constraints only in place in the Bayou Meto and Grand Prairie irrigation project areas. The reason that this configuration of constraints may be of interest is that imposition of pumping restrictions within a Critical Ground-Water Area is possible only if an alternate source of water is available, which would be the case in the Bayou Meto and Grand Prairie irrigation project areas. Table 5 shows a comparison of sustainable yields obtained using that constraint configuration with one that has constraints assigned throughout the model area. Removal of constraints outside the project areas allows substantially more water to be pumped in those areas without constraints. However, doing so allows water levels outside the project areas to drop below half the thickness of the aquifer, violating Critical Ground-Water Area requirements.

Table 5. Comparison of sustainable yield obtained for hydraulic-head constraints everywhere in the model area and only in the Bayou Meto and Grand Prairie irrigation project areas

[Limit on maximum pumping for both cases was set at 1997 pumping rates]

Irrigation project area	Sustainable yield (million cubic feet per day)	
	Hydraulic-head constraints everywhere	Hydraulic-head constraints only in project areas
Bayou Meto	18.1	17.9
Grand Prairie	9.1	5.0
Outside of Bayou Meto and and Grand Prairie project areas	333	434

Additional testing with the optimization model showed that specification of non-zero lower limits on withdrawals at every pumping well in the model area led to an infeasible solution, regardless of the value of the lower limit that was specified. This was done to test if at least some water could be produced in those areas for which an optimal withdrawal rate was zero. Attempts at further subdividing areas, where some of the cells were specified with a non-zero lower limit on withdrawals (for example, applying a lower limit of 10 percent of the rate pumped in 1997 only to cells in the Bayou Meto or Grand Prairie project areas), also resulted in infeasible solutions. However, an exhaustive application of this approach to areas with optimized pumping rates of zero (figs. 8 through 10) was not done.

A test was done to evaluate the effect of optimizing ground-water withdrawals without optimizing withdrawals from streams, while still maintaining the minimum streamflow constraints. Sustainable yield from ground water (with a maximum limit of ground-water withdrawals set at 100-percent of the 1997 rate) for all wells increased by 13.6 percent to 409 Mft^3/d.

Unmet Demand

Unmet demand is defined as the difference between the sustainable yield of ground water (or optimized withdrawal rate), and the anticipated demand:

$$U = D - S \qquad (7)$$

where U is the unmet demand, in cubic feet per day;
D is the anticipated demand, in cubic feet per day; and
S is the sustainable yield, in cubic feet per day.

For example, if the anticipated demand is 635 Mft^3/d (the amount withdrawn in 1997), and the sustainable yield is calculated to be 360 Mft^3/d, the unmet demand is the difference of these two values, or 275 Mft^3/d. Note that the sustainable yield is an independent calculation based on model results that is not affected by the demand. Therefore, unmet demand is not solely a function of the sustainable yield.

Although none of the three scenarios that were considered provided a sustainable yield for the entire model area that met the 1997 demand, sustainable yield for some counties did meet or exceed the demand (table 3). These counties (Coahoma, Desha, Dyer, Lake, Mississippi, Pulaski, Shelby, and Tipton) tended to have the lowest withdrawal rates specified in the flow model and also are located near a large river. Mississippi County had the largest 1997-withdrawal rate of those counties whose demand rate was met and exceeded by sustainable yield. Arkansas, Jackson, Lonoke, and Poinsett Counties consistently have the largest unmet demand for the different scenarios considered, although the ranking for these four counties changes depending on the scenario.

Unmet demand was tallied for the six irrigation project areas (table 6) based on different upper limits

Table 6. Sustainable yield and unmet demand relative to 1997 withdrawal rates in six irrigation project areas and the remainder of the model area

[mi^2, square mile; ft^3/d, cubic foot per day]

Project area	Area (mi^2)	Number of model pumping cells	Baseline 1997 withdrawal rate (ft^3/d)	Sustainable yield based on an upper withdrawal limit of:			Unmet ground-water demand based on 1997 demand and a sustainable yield from:			Unmet demand based on sustainable yields from scenarios 1, 2, and 3, respectively, and a demand of:		
				100 percent of baseline rate (scenario 1)	150 percent of baseline rate (scenario 2)	200 percent of baseline rate (scenario 3)	Scenario 1	Scenario 2	Scenario 3	100 percent of baseline rate	150 percent of baseline rate	200 percent of baseline rate
Bayou Meto	661	601	52,014,938	18,098,444	17,244,950	15,847,078	33,916,494	34,769,988	36,167,860	33,916,494	60,777,457	88,182,798
Grand Prairie	570	432	30,196,068	9,120,136	10,219,743	10,740,002	21,075,932	19,976,325	19,456,066	21,075,932	35,074,359	49,652,134
North Prairie	176	137	9,507,588	5,379,767	5,208,230	6,849,813	4,127,821	4,299,358	2,657,775	4,127,821	9,053,152	12,165,363
Little Red River	129	103	5,433,200	2,819,435	2,866,419	3,231,044	2,613,765	2,566,781	2,202,156	2,613,765	5,283,381	7,635,356
L’Anguille River	241	215	26,778,165	10,366,204	12,377,300	13,032,644	16,411,961	14,400,865	13,745,521	16,411,961	27,789,948	40,523,686
Bayou Deview	188	169	21,062,884	1,677,939	928,016	1,237,360	19,384,945	20,134,868	19,825,524	19,384,945	30,666,310	40,888,408
Non-project areas	12,139	8,322	490,788,396	312,808,687	396,044,423	474,880,308	177,979,709	94,743,973	15,908,088	177,979,709	340,138,171	506,696,484
Total	14,104	9,979	635,781,239	360,270,612	444,889,081	525,818,247	275,510,627	190,892,158	109,962,992	275,510,627	508,782,778	745,744,231

on pumping from scenarios 1, 2, and 3, and different demand rates. A comparison of the sustainable yield and the 1997 withdrawal rates within the six irrigation project areas is listed in table 6. Sustainable yield is substantially less in all project areas than the rate withdrawn in 1997, resulting in unmet demand ranging from 2.2 to 88.1 Mft^3/d for all the project areas. Bayou Meto project area consistently has the largest unmet demand of all the project areas, followed by the Grand Prairie project area. However, unmet demand for non-project areas was considerably larger for scenarios 1 and 2.

Optimal Simulated Hydraulic-Head Altitude

Substantial differences occur between simulated hydraulic-head altitudes (or water-level altitudes) for steady-state flow-model simulations using 1997 withdrawal rates and for steady-state simulations using optimal withdrawal rates resulting in sustainable yield (fig. 11). Because the 1997 withdrawal rates are unsustainable, large areas of the model area are dry, particularly in the area between the Arkansas and White Rivers (the area that includes the Bayou Meto and Grand Prairie irrigation project areas). In contrast, the optimal hydraulic-head altitude using sustainable yield shows a gradual decline from the rivers to the troughs caused by ground-water withdrawals.

Nonlinear Effects

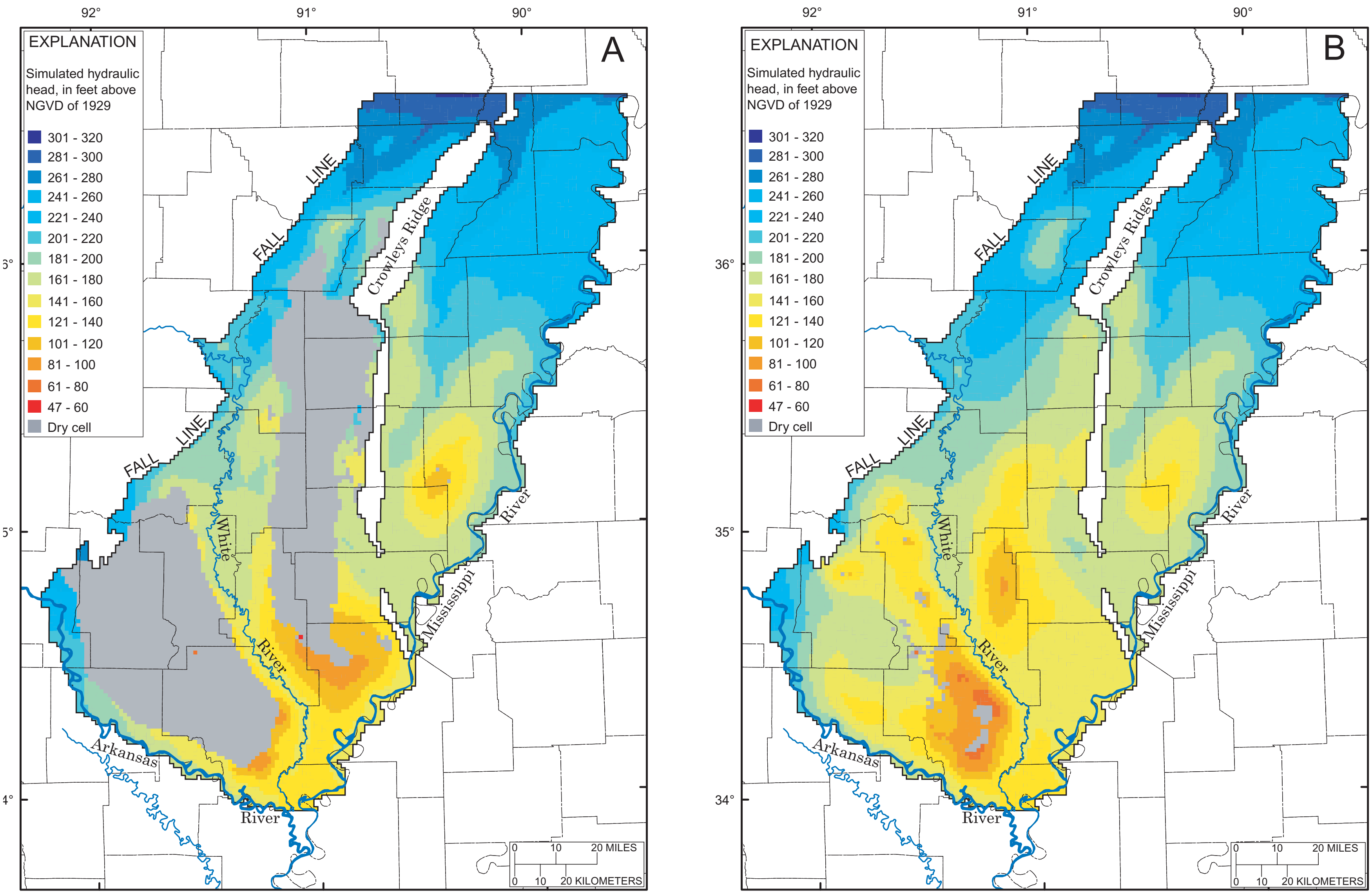
Because sustainable yield is obtained with the assumption that the model behaves linearly (that is, the change in hydraulic head is a constant multiple of the change in withdrawal rate, regardless of the withdrawal rate), it is important to compare the resulting simulated hydraulic-head values from the flow model derived using sustainable yield, to the altitudes corresponding to the hydraulic-head constraints specified in the optimization model. Such a comparison is provided in figure 12, which shows the cumulative percentage of model cells with values less than or equal to the difference between simulated hydraulic head and the altitude corresponding to half the aquifer thickness. Values to the left of zero represent cells with hydraulic head below half the aquifer thickness and represent less than 5 percent of the total. The comparison was made for data sets consisting of the 2,804 hydraulic-head constraint points and for all of the active model cells. There is no appreciable difference. The comparison indicates that (1) additional hydraulic-head constraint locations would have had little effect on the simulation results as a whole; and (2) the optimized pumping distribution is a good approximation of sustainable yield, despite non-linear behavior inherent in the model.

Limitations

The values of sustainable yield should be considered maximum rates, in that head constraints are violated in some areas because of non-linear responses in hydraulic head to incremental changes in withdrawal rates within the flow model. When the sustainable yield rates are used in the flow model, a few cells have hydraulic heads at steady state that are below the hydraulic-head constraints, which could have been corrected by reducing withdrawal rates further. This was not done, however, because of the few points where this occurred. From a management standpoint, however, the values might be considered to be conservative because they apply to steady-state conditions that will not be reached for possibly hundreds of years.

Sustainable yield results from the optimization model should be used cautiously, mindful that the model represents a simplification of a complex system. The assumption that the flow system behaves linearly is likely the largest discrepancy from actual conditions. Nonetheless, the optimization model does provide estimates of sustainable yield from both the ground-water and surface-water sources that result in hydraulic-head values remaining at or above an altitude corresponding to half the thickness of the aquifer throughout the bulk of the model area, and maintaining streamflows at or above specified minimum amounts.

The spatial distribution of the difference between simulated hydraulic head and the altitude corresponding to half the aquifer thickness is shown in figure 13. Over the vast majority of the model area, simulated hydraulic head is at or above the constraint. Those areas where this is not the case occur where large changes in saturated thickness have occurred when compared to predevelopment conditions, which has caused the largest change in aquifer transmissivity. This is particularly true in the Grand Prairie area.



Base from U.S. Geological Survey digital data, 1:100,000

Base from U.S. Geological Survey digital data, 1:100,000

Figure 11. Simulated hydraulic head at steady state using (A) 1997 withdrawal rates; and (B) sustainable yield.

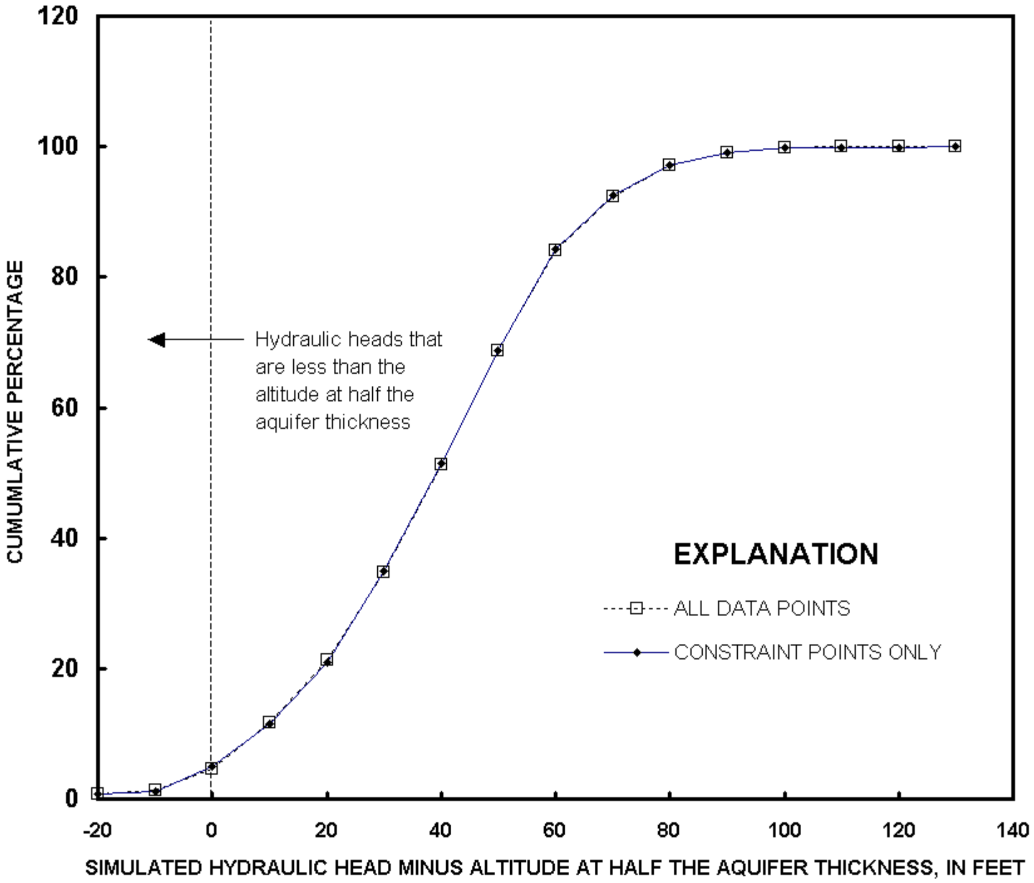
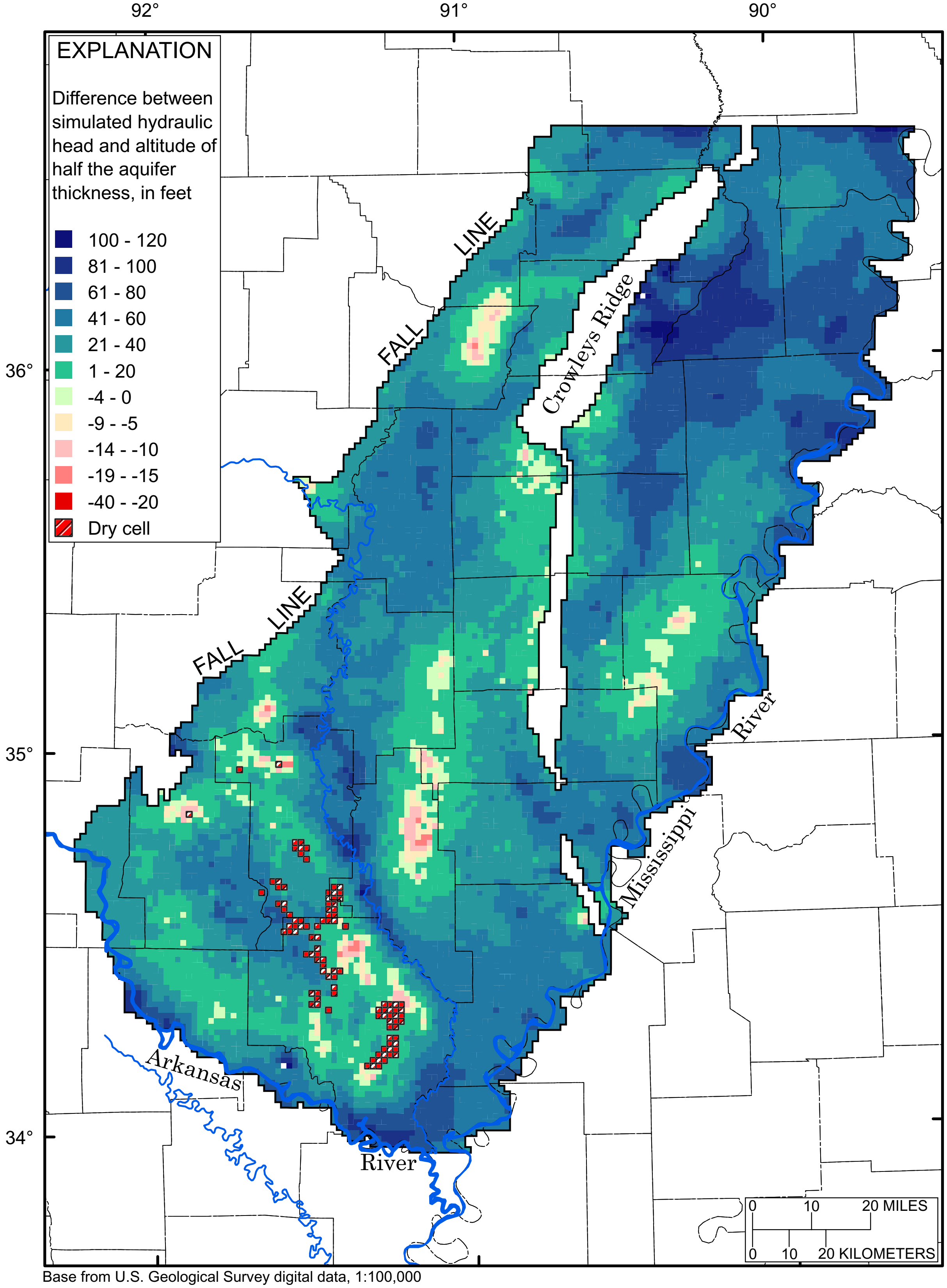


Figure 12. Cumulative percentage of model cells with values less than or equal to the difference between simulated hydraulic head and the altitude corresponding to half the aquifer thickness.



Base from U.S. Geological Survey digital data, 1:100,000

Figure 13. Difference between simulated hydraulic head and altitude of half the aquifer thickness.

SUMMARY

The Mississippi River Valley alluvial aquifer is a water-bearing assemblage of gravels and sands that underlies about 32,000 mi^2 of Missouri, Kentucky, Tennessee, Mississippi, Louisiana, and Arkansas. The Mississippi River Valley alluvial aquifer supplies large volumes of water for agriculture in Arkansas. Because of the heavy demands placed on the aquifer, several large cones of depression over 100 ft deep have formed in the potentiometric surface, resulting in lower well yields and degraded water quality in some areas. Several counties in the Grand Prairie area, which are within the extent of the alluvial aquifer, have been designated Critical Ground-Water Areas by the Arkansas Soil and Water Conservation Commission (ASWCC). These criteria state that if water levels drop below half the original saturated thickness of the formation, then a “critical ground-water area” may be designated.

A ground-water flow model of the alluvial aquifer was developed for an area covering 14,104 mi^2, extending northeast from the Arkansas River into the northeast corner of Arkansas and parts of southeast Missouri. The flow model showed that continued ground-water withdrawals at rates commensurate with those of 1997 could not be sustained indefinitely without causing water levels to decline below half the original saturated thickness of the formation. To develop estimates of withdrawal rates that could be sustained relative to the constraints of critical ground-water area designation, the U.S. Geological Survey, in cooperation with the Arkansas Soil and Water Conservation Commission, and Memphis District of the U.S. Army Corps of Engineers, applied conjunctive-use optimization modeling to the flow model of the Mississippi River Valley alluvial aquifer in northeastern Arkansas. Conjunctive-use optimization modeling is a technique that simulates maximum withdrawal rates from both surface water and ground water while honoring constraints with respect to water levels and streamflow. These withdrawal rates form the basis for estimates of sustainable yield from the alluvial aquifer and from rivers specified within the alluvial aquifer model.

The purpose of the optimization model described in this report is to: (1) obtain maximum withdrawal rates from model cells at which ground-water withdrawals would have occurred in 1997; (2) obtain maximum withdrawal rates from model cells at stream locations; (3) maintain ground-water levels at or above specified levels; and (4) maintain streamflow at or above specified rates. A management problem was formulated as one of maximizing the sustainable yield from all ground-water and surface-water withdrawal cells within limits imposed by plausible withdrawal rates, and within specified constraints involving hydraulic head and streamflow. Steady-state conditions were selected (as opposed to transient conditions) because the maximized withdrawals are intended to represent sustainable yield of the system (a rate that can be maintained indefinitely). The optimization model was used to generate response coefficients for each specified withdrawal cell in the model. After all the response coefficients were calculated, they were combined to form a data-input set along with hydraulic-head and streamflow constraints, and formulated as a linear program in mathematical programming system format. Optimal sustainable yield values were obtained by running the linear program under MINOS.

Optimal sustainable yield values are affected by the rate of recharge and limits to potential withdrawals assigned within the optimization model. For obtaining estimates of sustainable yield, specified recharge was assumed to be the same as for predevelopment conditions. Optimal sustainable yield is a function of limits assigned to ground-water withdrawals, which in this report were set to multiples of 100, 150, and 200 percent of 1997 rates. Some areas represented in the model likely could pump at rates higher than those in 1997. However, if no limit is placed on withdrawals, the majority of water production will be from wells near model water sources such as rivers or general-head boundaries, depriving wells of water that are distant from these sources of water. The maximum potential withdrawal from wells was limited to twice the amount withdrawn in 1997 because even at the 1997 rate, many areas within the model went dry. No limit on withdrawals led to unrealistic optimal withdrawal from wells adjacent to rivers and most interior withdrawal cells having no withdrawal.

Within the optimization model, 11 rivers are specified. Surface-water diversion rates that occurred in 2000 were subtracted from specified overland flow at the appropriate river cells. Included in these diversions were the planned diversions of 63,339,248 ft^3/d for the Bayou Meto project area and 55,078,367 ft^3/d for the Grand Prairie project area, which factor in an additional 30 and 40 percent transmission loss, respectively. Streamflow constraints were specified at all 1,165 river cells based on average 7-day minimum flows for 10 years. Sustainable yield for all rivers ranged from 0 (Current, Little Red, and Bayou Meto

Rivers) to almost 5 billion ft^3/d for the Arkansas River. Total sustainable yield from all rivers combined was 12.8 billion ft^3/d. Nonetheless, these large sustainable yields represent a potential source of water to supplement ground water and meet overall water demand, but to do so will require the construction of withdrawal and distribution facilities, which will have legal, political, economic, and social consequences.

Sustainable-yield estimates are affected by the allowable upper limit on withdrawals from wells specified in the optimization model. Withdrawal rates were allowed to vary up to 200 percent of the withdrawal rate in 1997. As the overall upper limit is increased, the sustainable yield generally increases because wells closer to water sources can produce more water. Tests with the optimization model show that without limits on pumping, wells adjacent to sources of water would have optimized withdrawal rates that were orders of magnitude larger than 1997 rates. Not only is it physically unlikely that individual wells could pump that much more water, but construction of sufficient additional wells in the one-square mile cells is also unlikely. The sustainable yield from ground water for the entire study area with the maximum upper limit set as the amount withdrawn in 1997 is 360 Mft^3/d, which is only about 57 percent of the amount withdrawn in 1997 (635.6 Mft^3/d). Optimal sustainable yields from within the Bayou Meto irrigation project area and within the Grand Prairie irrigation project area are 18.1 and 9.1 Mft^3/d, respectively, assuming a maximum allowable withdrawal rate equal to 1997 rates. These values of sustainable yield represent 35 and 30 percent respectively of the amount pumped from these project areas in 1997.

Unmet demand (defined as the difference between the optimized withdrawal rate, or sustainable yield, and the anticipated demand) was calculated using different demand rates based on multiples of the 1997-withdrawal rate. Sustainable-yield values were based on upper limits of ground-water withdrawals set at 100, 150, and 200 percent of the 1997 withdrawal rate.

Hydraulic-head constraints have the largest effect on sustainable yield estimates in the areas in which the hydraulic-head constraints are applied. Removal of head constraint outside the Bayou Meto and Grand Prairie irrigation project areas had a much smaller effect on those areas than in the areas distant from them.

A check of sustainable-yield rates was performed by applying these rates in the flow model run to steady state and comparing simulated hydraulic-head values to hydraulic-head constraints. In 95 percent of the model area, application of the sustainable-yield rates resulted in hydraulic heads that were above the constraint values. In those areas where this was not the case, deviation from linear model response (that is, a unit incremental change in withdrawal rate results in a unit incremental change in hydraulic head) is suspected, largely because of changes in transmissivity resulting from substantial change in saturated thickness between the starting hydraulic-head distribution used in the optimization model and hydraulic head for steady-state conditions under optimized sustainable yield.

Sustainable-yield results from the optimization model should be used cautiously, mindful that the model represents a simplification of a complex system. The assumption that the flow system behaves linearly is likely the largest discrepancy from actual conditions. Nonetheless, the optimization model does provide estimates of sustainable yield from both the ground-water and surface-water sources that result in hydraulic-head values remaining at or above an altitude corresponding to half the thickness of the aquifer throughout the bulk of the model area, and maintaining streamflows at or above specified minimum amounts.

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