Feature Detection

12 February 2003

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What Is Feature Detection?

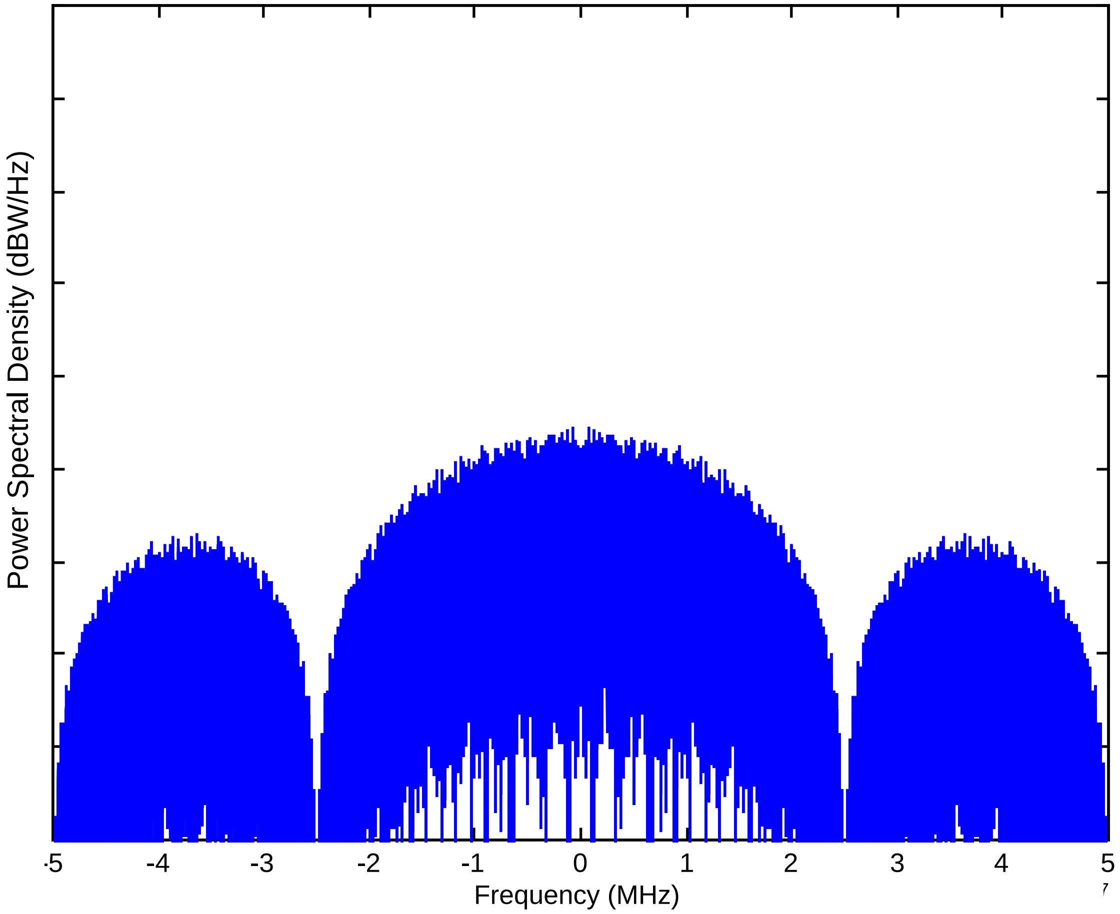
- Most transmitted radio frequency (RF) signals exhibit structure, or “features”
- One class of features is known rates in the signal
  - Carrier frequency
  - Keying
- Signal presence can be determined through detection of these features
  - More sensitive than demodulation in some cases
  - Better discrimination and more robustness than energy detection in some cases

Presentation Objectives

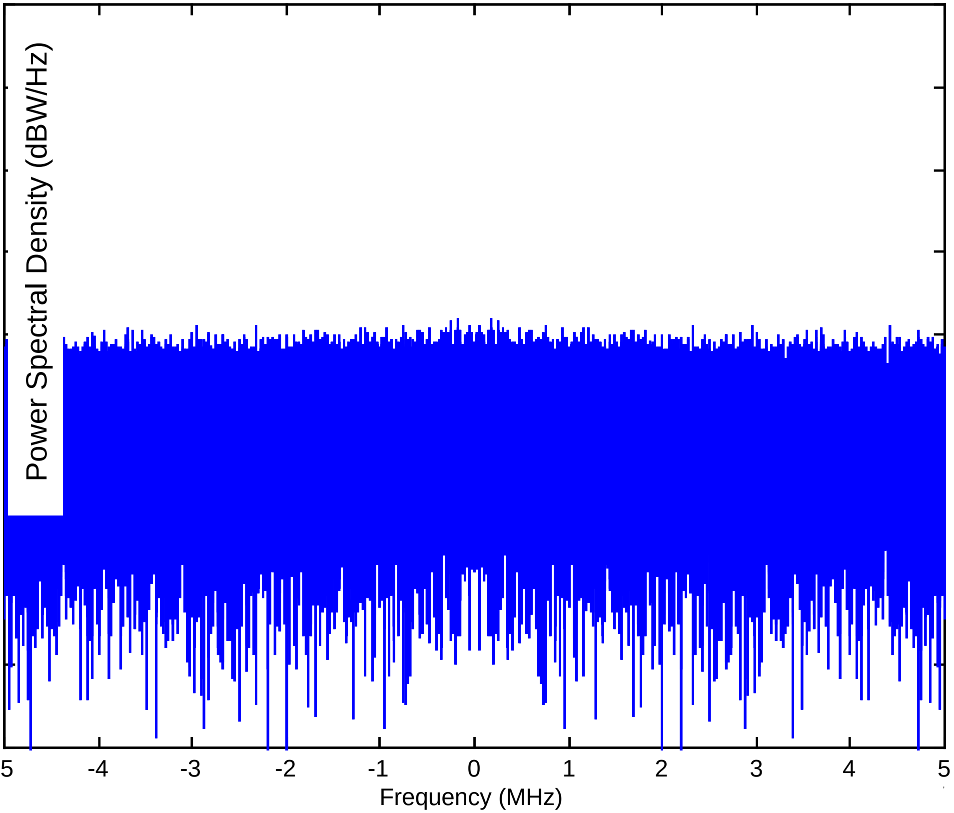
- **Presentation attempts to:**
  - **Provide background on cyclostationarity**
  - **Describe cyclostationary feature detectors**
  - **Describe performance of cyclostationary feature detectors for some classes of modulations**

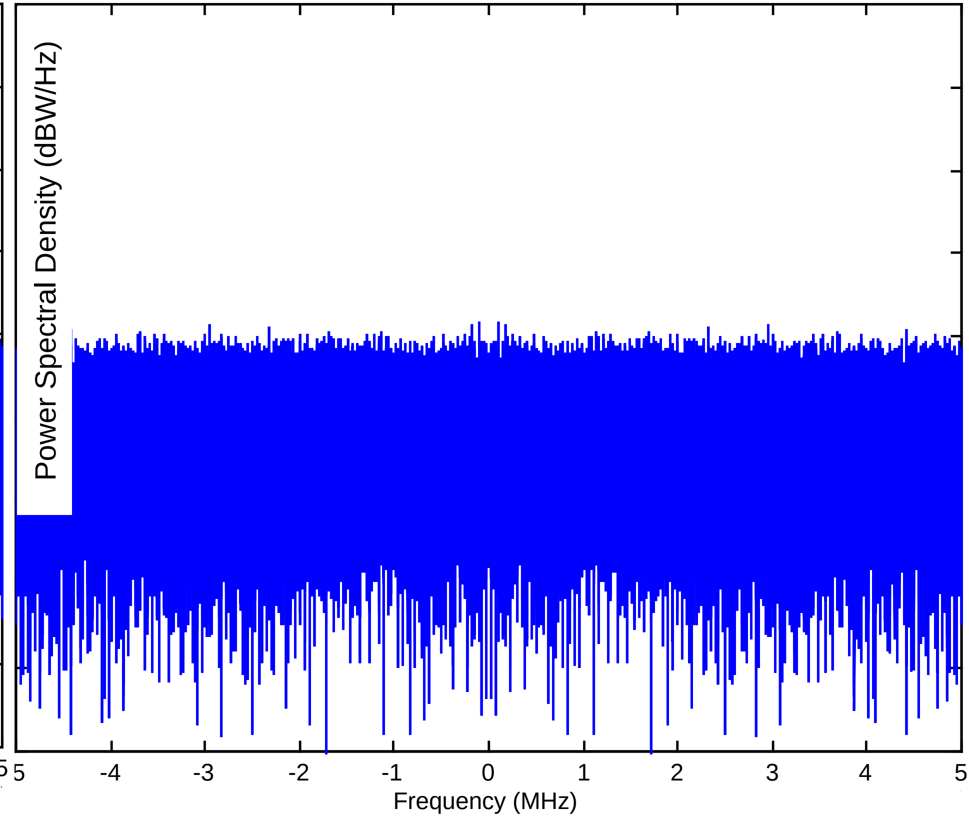
- **Presentation does *NOT* intend to:**
  - **Design or analyze feature detectors for specific modulations used in digital television**
  - **Address overall aspects of listen-before-talk protocols**
  - **Assess the utility of cyclostationary feature detection in listen-before-talk protocols**

Spectrum of M-ary PSK Signal, 2.5 M symbol/s



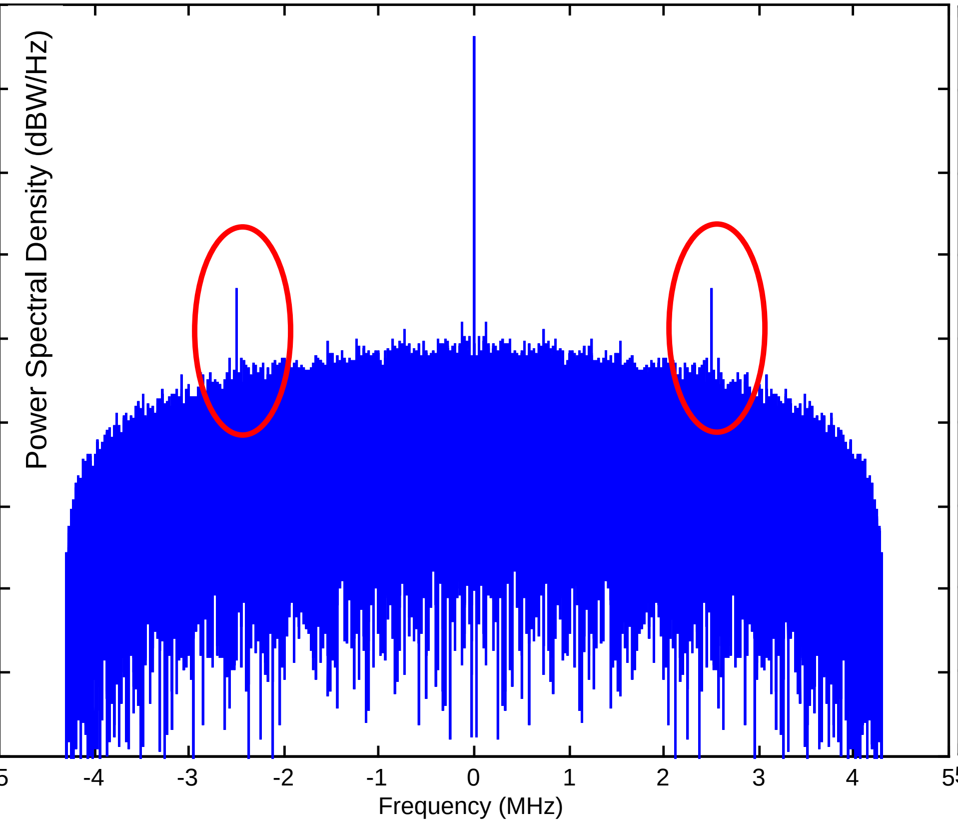
Spectrum of Noise Only and 2.5 M symbol M-ary PSK Signal in Noise

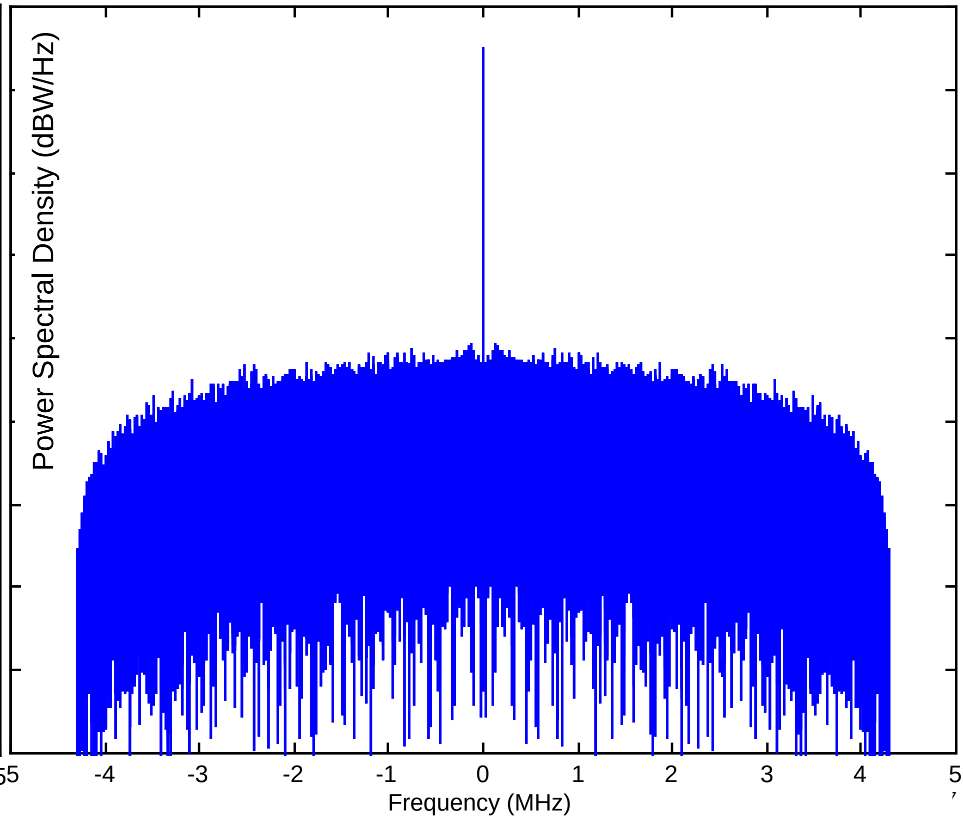




- **Input SNR (Energy Per Bit)/(Noise Density) is –10 dB**

Spectrum of Feature Detector Outputs: Noise Only and 2.5 M symbol M-ary PSK Signal in Noise





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Feature Detection

- Cyclostationary Processes
- Cyclostationary Feature Detection
  - Processing Structures
  - Performance
- Practical Considerations
- Summary

Mathematical Models of Communications Waveforms

- **Narrowband signal**

$$x(t) = x_{\mathrm{r}}(t)\cos\left(2\pi f_{\mathrm{c}} t\right) - x_{\mathrm{i}}(t)\sin\left(2\pi f_{\mathrm{c}} t\right)$$

- **Complex envelope representation**

$$x(t) = \Re\{y(t)\}$$

$$y(t) = z(t)e^{i2\pi f_{\mathrm{c}} t}$$

- $y(t)$ **and** $z(t)$ **are complex-valued**

Stationary Processes

- **Much of communications and signal processing relies on modeling noise and signals as a special class of stochastic processes known as *stationary processes***
  - **Statistics do not vary over time**
- **Many practical applications in signal processing involve first-order and second-order moments of stationary processes**
  - **Mean** $m = E\{z(t)\}$
  - **Variance** $\sigma^2 = E\left\{z(t)z^*(t)\right\}$
  - **Correlation functions**

$$R^1(\tau) = E\left\{z(t)z^*(t-\tau)\right\}$$

$$R^0(\tau) = E\{z(t)z(t-\tau)\}$$

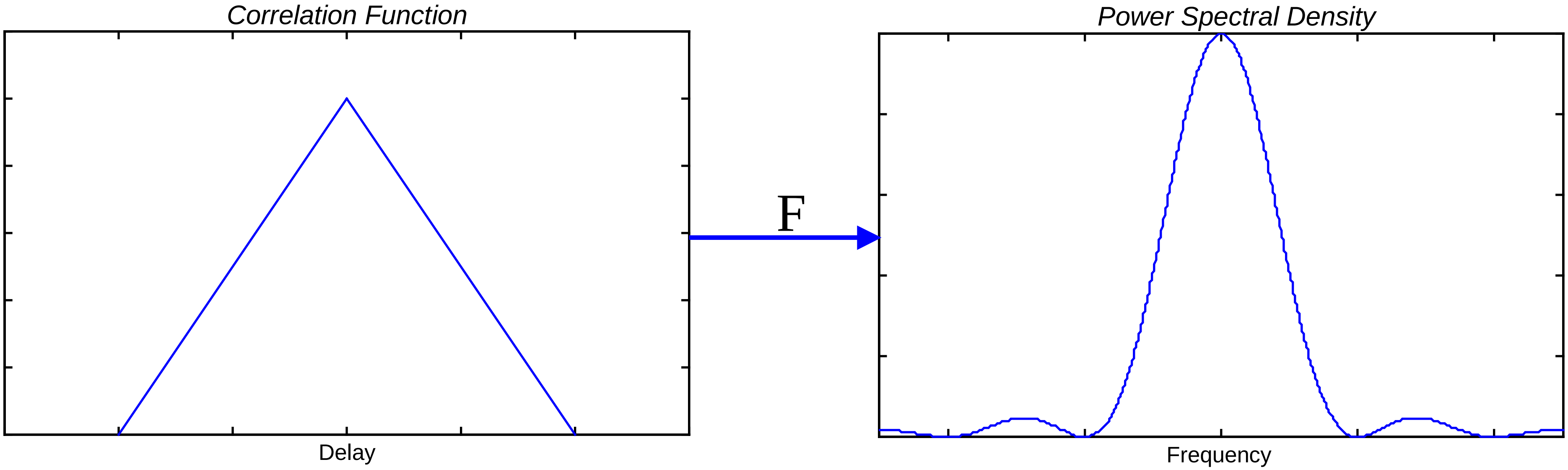
  - **Power spectral density**

$$G^1(f) = \mathrm{F}_\tau\left\{R^1(\tau)\right\}$$

- **Ergodicity (equivalence of time averages and ensemble averages) can be important is assumed**

Second-Order Moments for Zero-Mean Wide-Sense Stationary Process

- **Like all moments, the correlation function and power spectral density are deterministic functions that (incompletely) describe a stochastic process**
  - **There exists an infinite number of stochastic processes that have the same correlation function and power spectral density**



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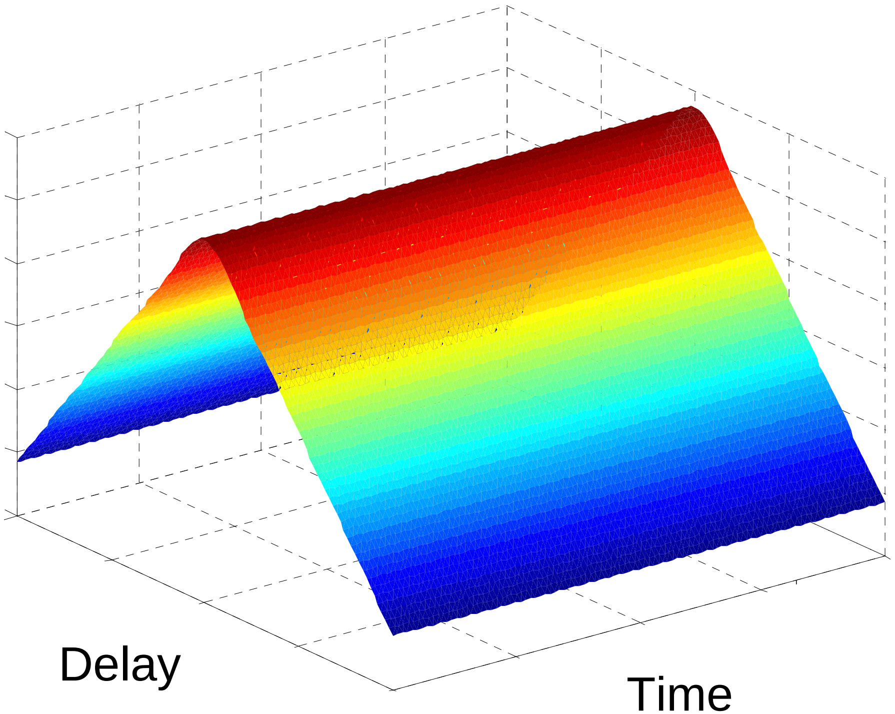
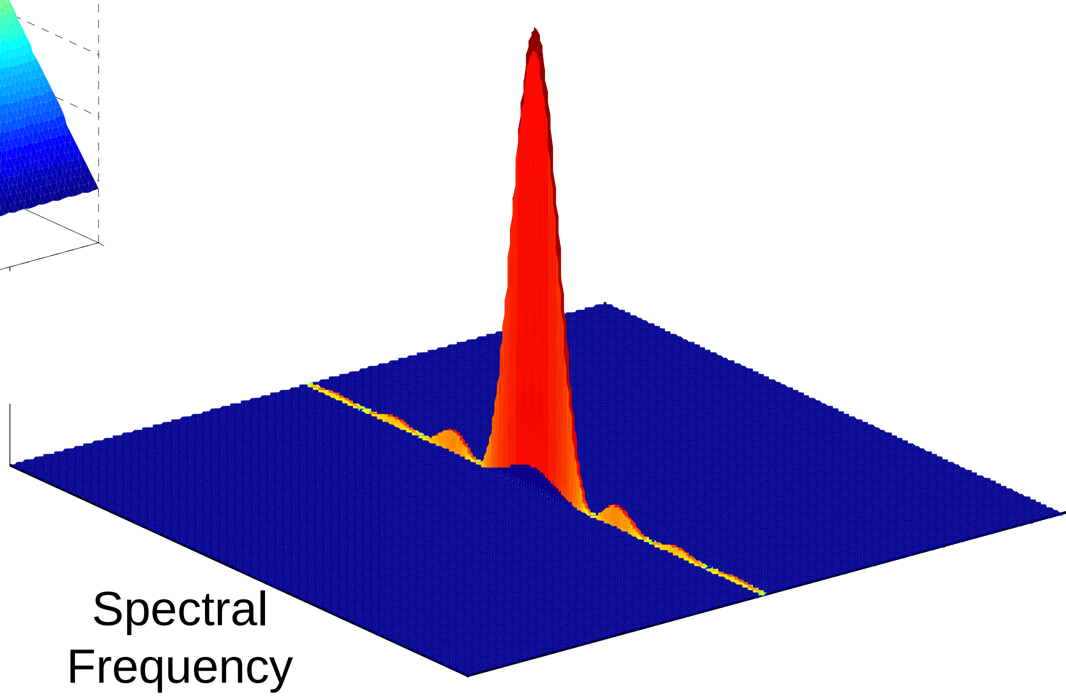
Second-Order Moments for Zero-Mean Wide-Sense Stationary Process (Concluded)

Correlation Function



Power Spectral Density



Cyclostationary Processes

- Model of actual data as stationary becomes limited as statistics vary over time
- Statistics of some time series vary periodically over time—cyclostationary processes
- First-order and second-order moments of cyclostationary processes
  - Mean $m(t) = E\{z(t)\} = \sum_{k=-\infty}^{\infty} \mu_k e^{i2\pi k\beta t}$
  - Variance $\sigma^2(t) = E\left\{z(t)z^*(t)\right\} = \sum_{k=-\infty}^{\infty} \chi_k e^{i2\pi k\alpha t}$
  - Cyclic correlation functions

$$R^1(t,\tau) = E\left\{z(t)z^*(t-\tau)\right\} = \sum_{k=-\infty}^{\infty} \chi_k^1(\tau) e^{i2\pi k\alpha^1 t}$$

$$R^0(t,\tau) = E\{z(t)z(t-\tau)\} = \sum_{k=-\infty}^{\infty} \chi_k^0(\tau) e^{i2\pi k\alpha^0 t}$$

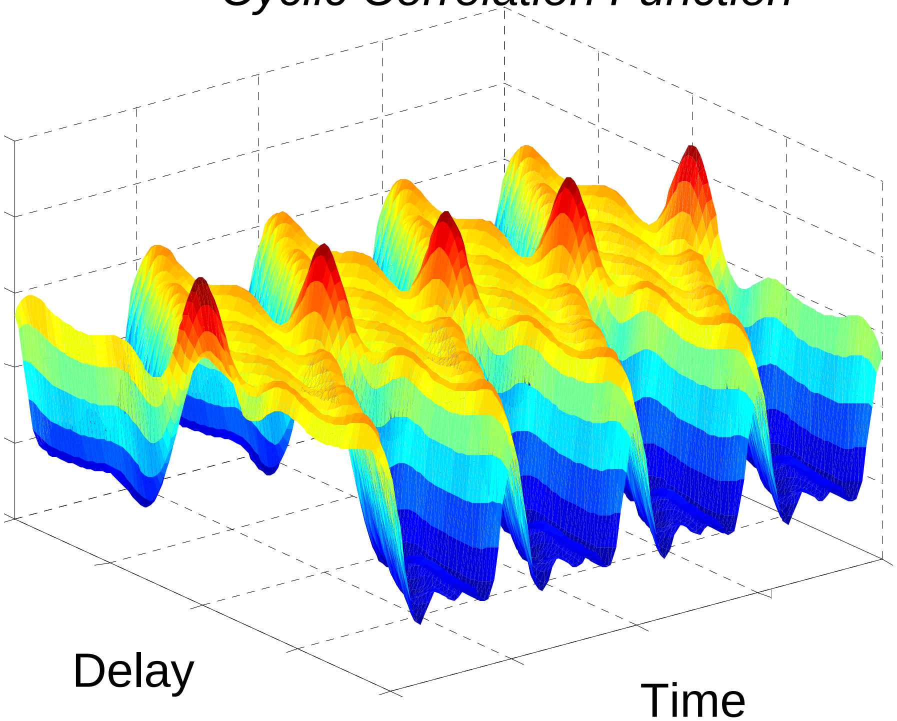
  - Cyclic power spectral densities

$$G^1(\phi, f) = \mathrm{F}_{t,\tau}\left\{R^1(t,\tau)\right\} = \sum_{k=-\infty}^{\infty} \mathrm{X}_k^1(f)\delta(\phi - k\alpha)$$

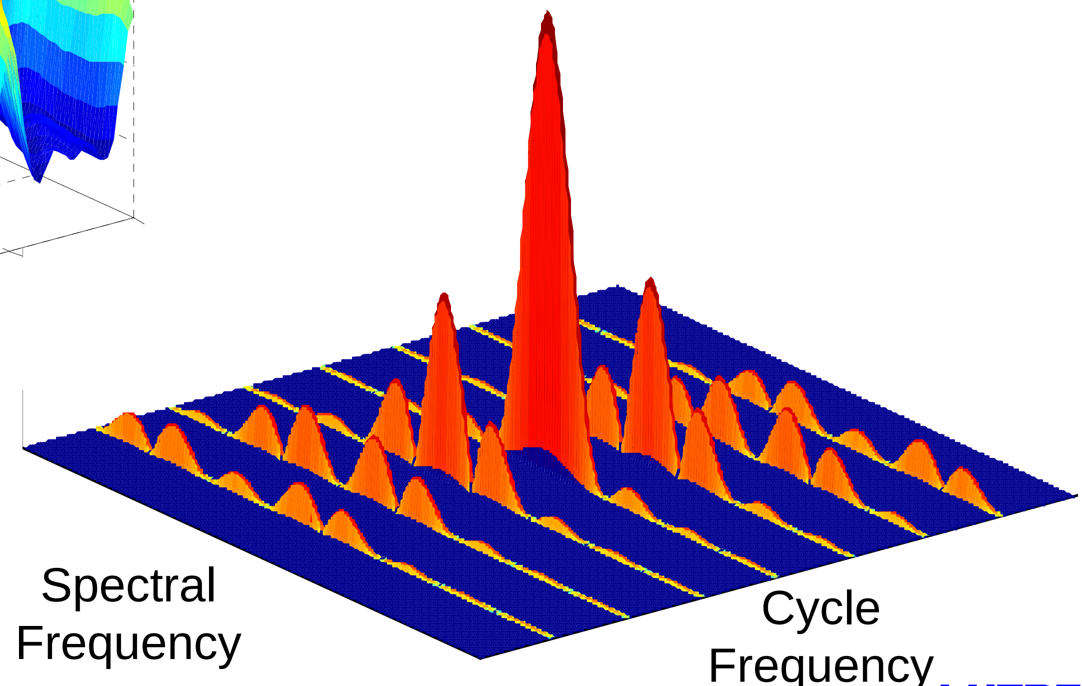
$$G^0(\phi, f) = \mathrm{F}_{t,\tau}\left\{R^0(t,\tau)\right\} = \sum_{k=-\infty}^{\infty} \mathrm{X}_k^0(f)\delta(\phi - k\alpha)$$

Second-Order Statistics of Cyclostationary Processes

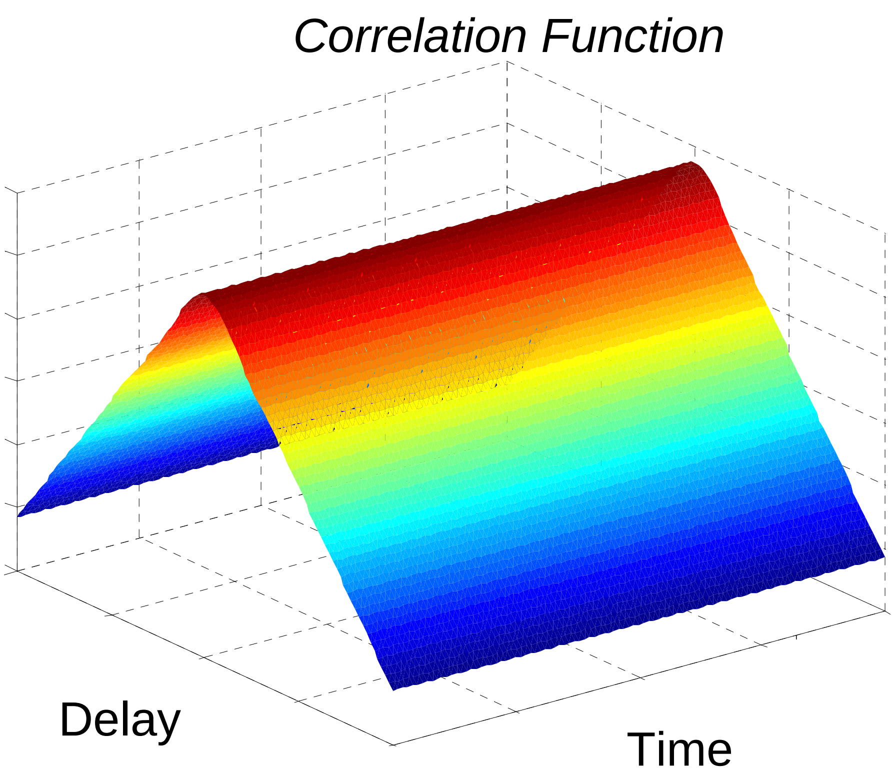
Cyclic Correlation Function

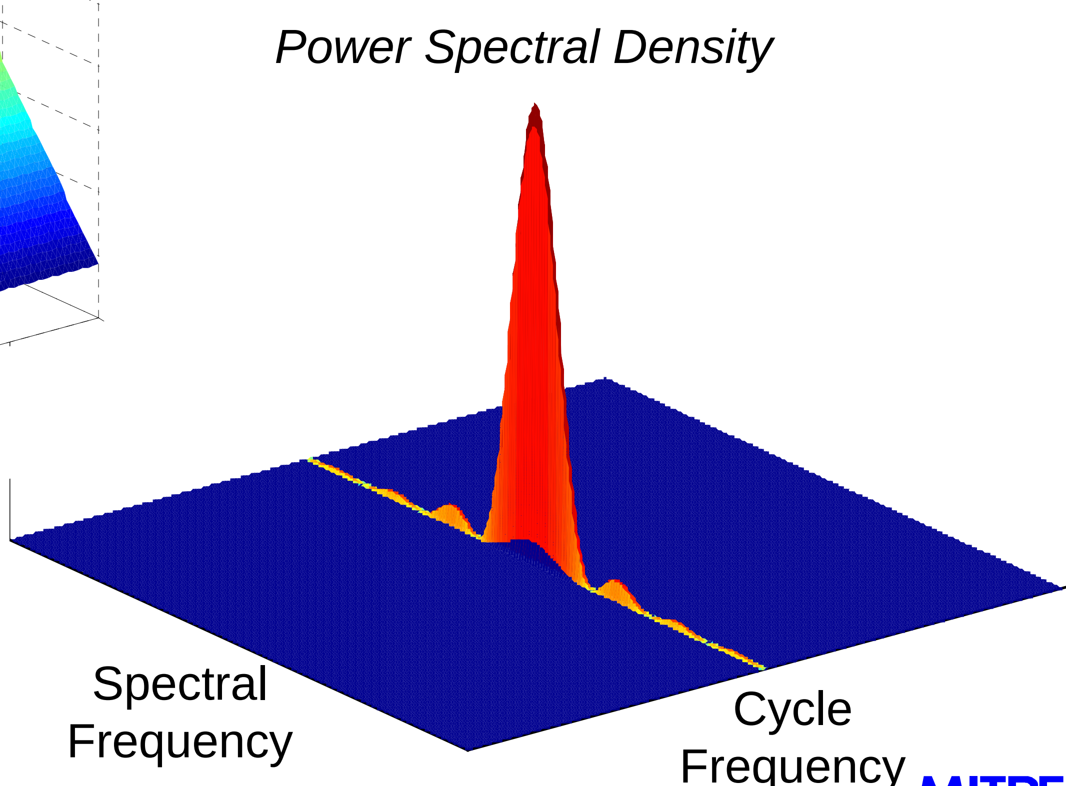


Cyclic Spectral Density



Time-Averaged Second-Order Statistics of Cyclostationary Process





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Applications of Cyclostationarity

- Filtering: estimation of signals from noise and interference
- Prediction
- Parameter estimation
- System identification
- Equalization
- Detection

Feature Detection Outline

- Cyclostationary Processes
- Cyclostationary Feature Detection
  - Processing Structures
  - Performance
- Practical Considerations
- Summary

Fundamental Detection Problem

- Decide between two hypotheses
  - Null hypothesis: only Gaussian noise
  - Alternative hypothesis: Gaussian cyclostationary signal in Gaussian noise
- A priori knowledge:
  - Power spectrum of noise, including total power
  - Second-order cyclostationary statistics of signal
    - Can accommodate unknown timing (phasing of periodicities in statistics)
- Optimal test statistic is sum of two terms
  - Energy detector based on stationary statistics
  - Cycle frequency detector
    - Detects periodicities at all delays in the sample cyclic correlation functions
    - Detects peaks in the sample cyclic spectral density

Detection with Noise Power Uncertainty

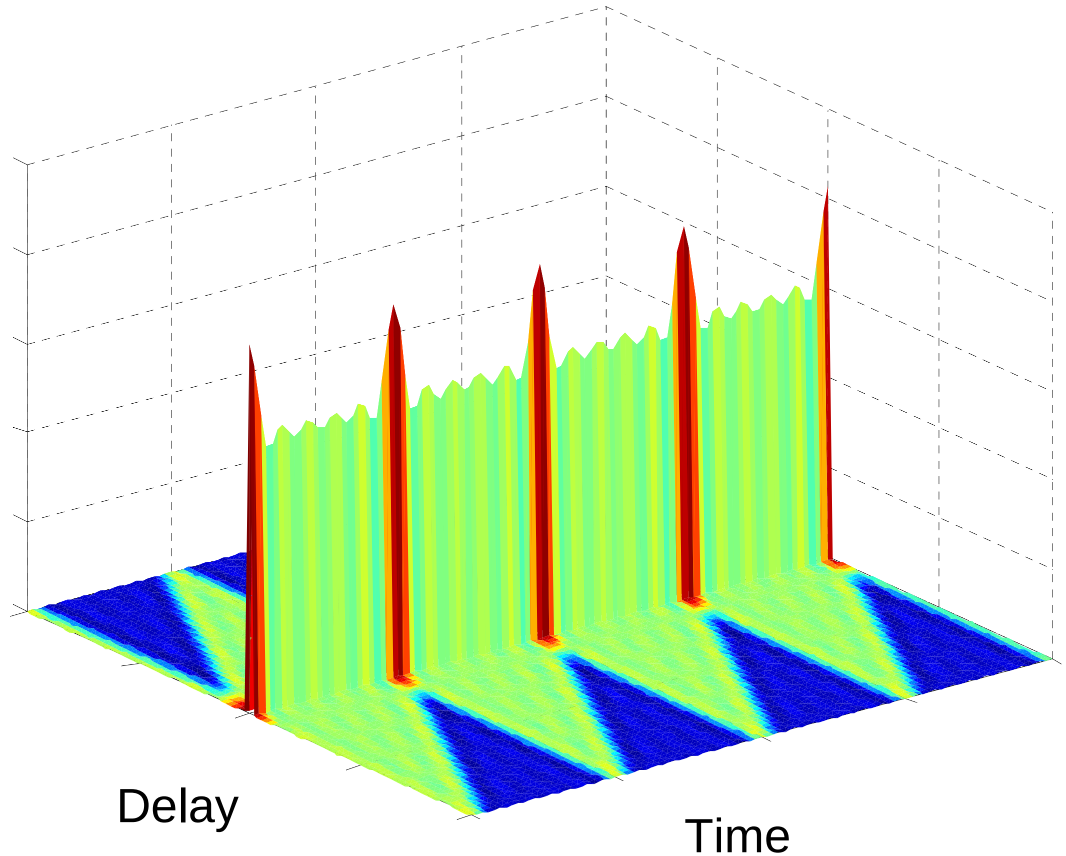
- Decide between two hypotheses
  - Null hypothesis: only Gaussian noise
  - Alternative hypothesis: Gaussian cyclostationary signal in Gaussian noise
- A priori knowledge:
  - Power spectrum of noise: shape known but not total power not known precisely
  - Second-order cyclostationary statistics of signal, except for timing (phase of periodicities in statistics)
- Optimal test statistic is merely:
  - Cycle frequency detector that detects periodicities at all delays in the sample cyclic correlation function

Simpler Cycle Frequency Detector: "Cyclostationary Feature Detector"

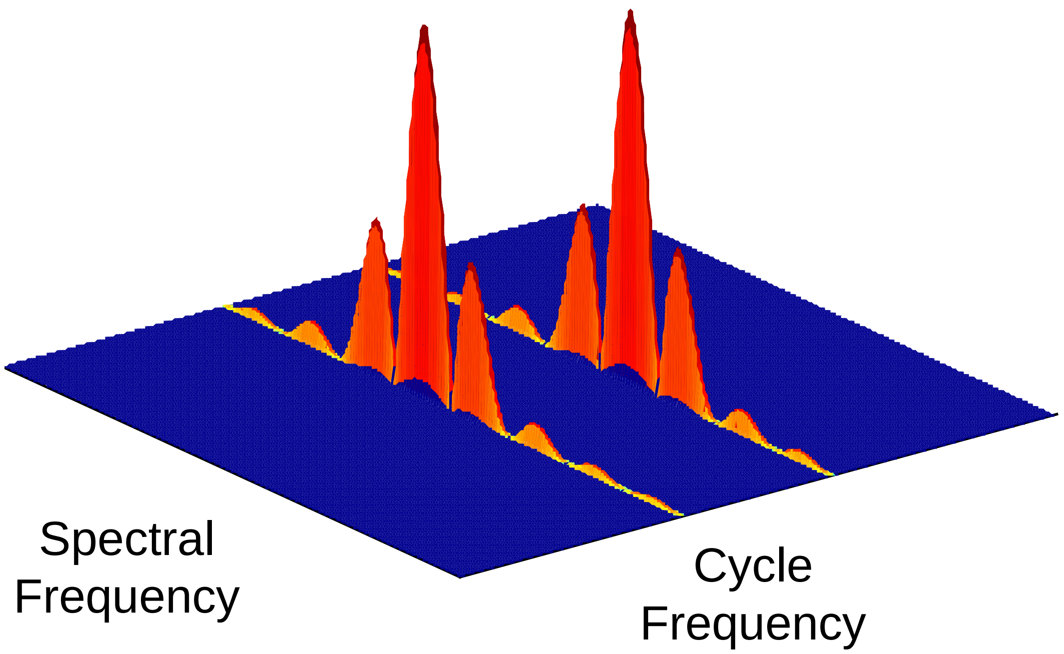
- **Detect periodicities in one cyclic correlation function evaluated at a single delay**

- **Remainder of this presentation focuses on**

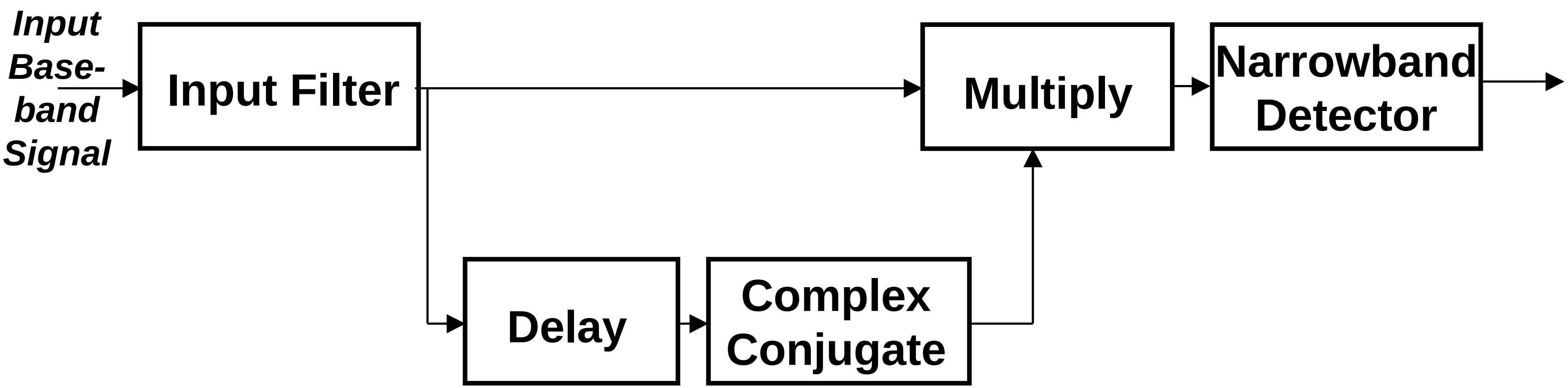
$$R^1(t,\tau) = E\left\{ z(t)z^*(t-\tau) \right\}$$



Simpler Cycle Frequency Detector: "Cyclostationary Feature Detector" (Concluded)



Canonical Structure for Feature Detector



- Joint optimization to maximize output signal-to-noise ratio (SNR)
  - Input filter shapes signal and noise
  - Delay
- Narrowband detector isolates energy at selected cycle frequency
  - Narrowband filter with energy detector
  - Filter bank (FFT) searches multiple frequencies in parallel
  - Can use a combination of coherent and noncoherent integration

Input Filter Is Critical

BPSK with Random Data, Null-to-Null Filtered

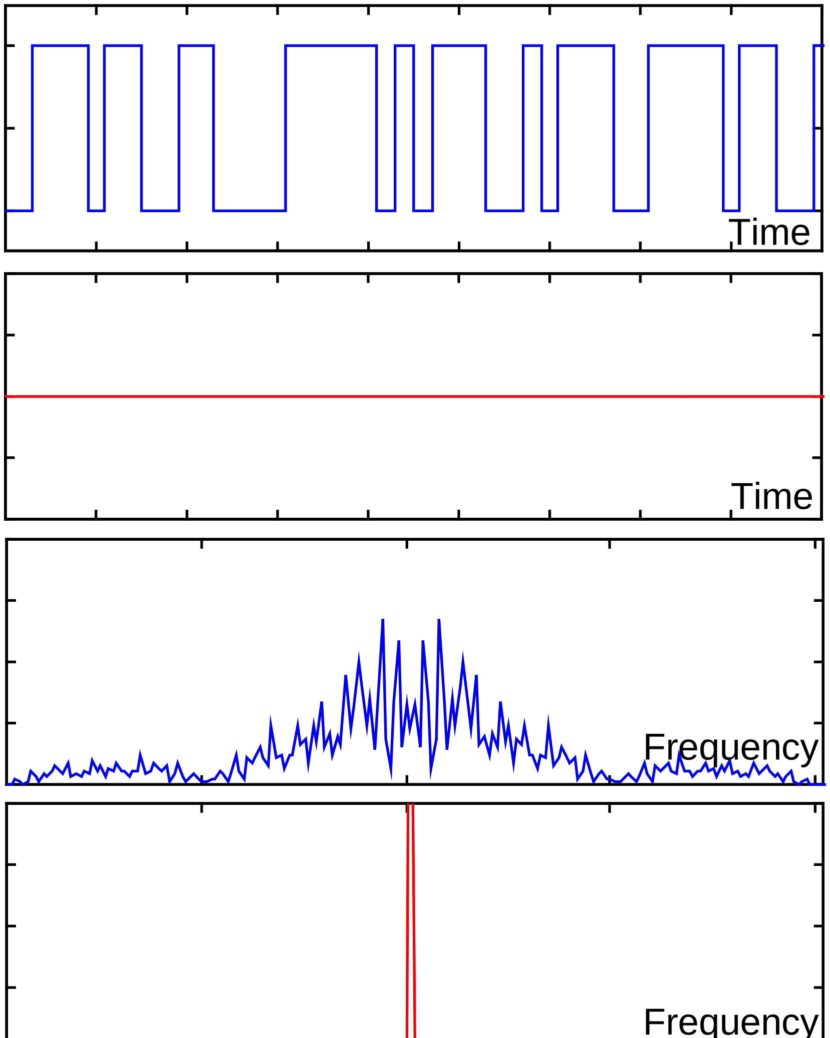
Wideband | **Null-to-Null Filtered**

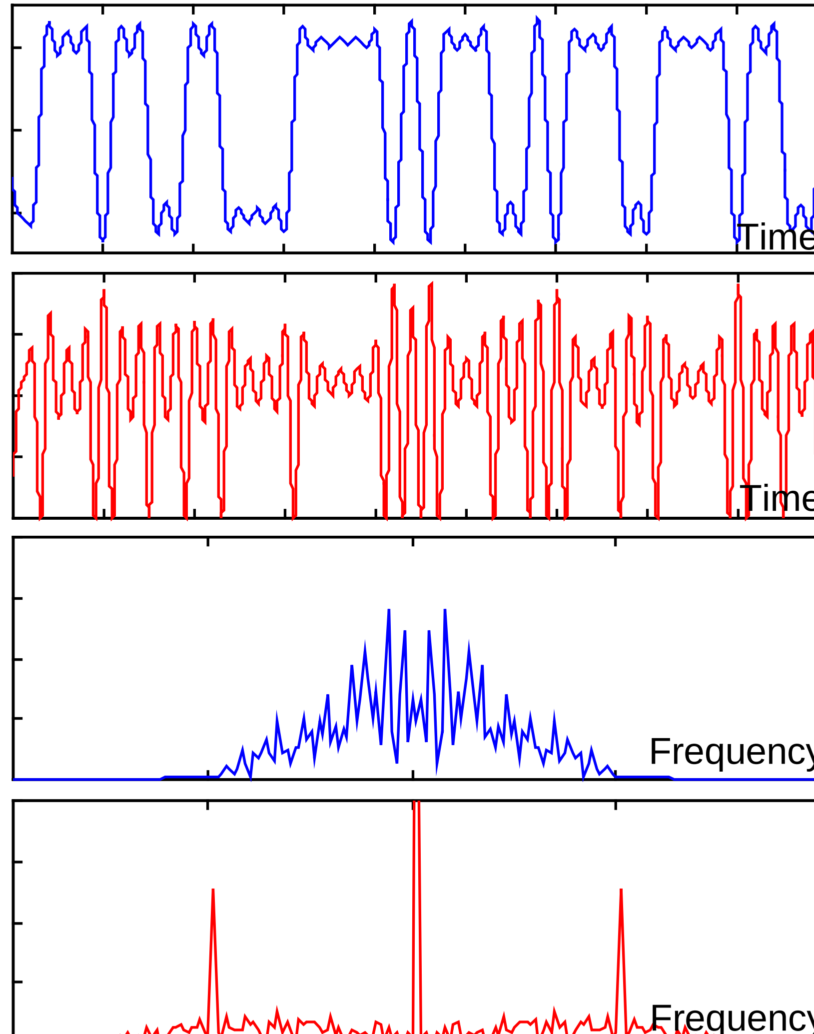
Signal

Signal Squared

Magnitude Fourier Transform of Signal

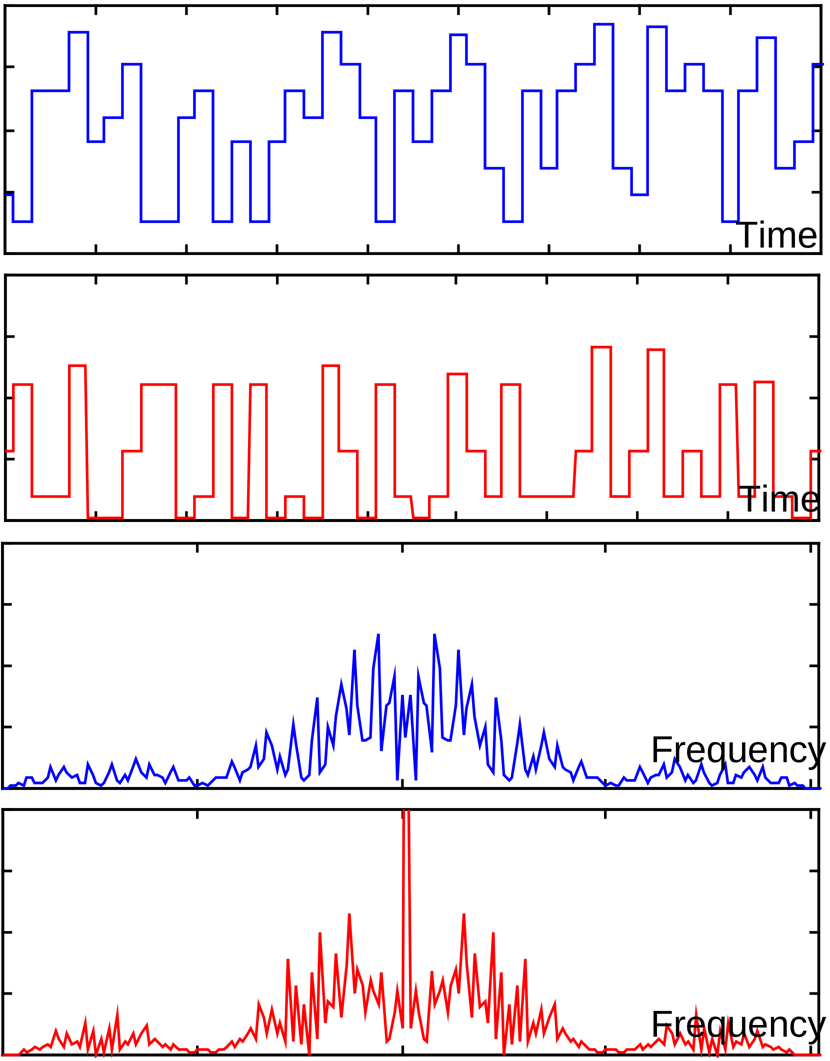
Magnitude Fourier Transform of Signal Squared





8-PAM with Random Data and No Trellis Coding, Null-to-Null Filtered

Wideband



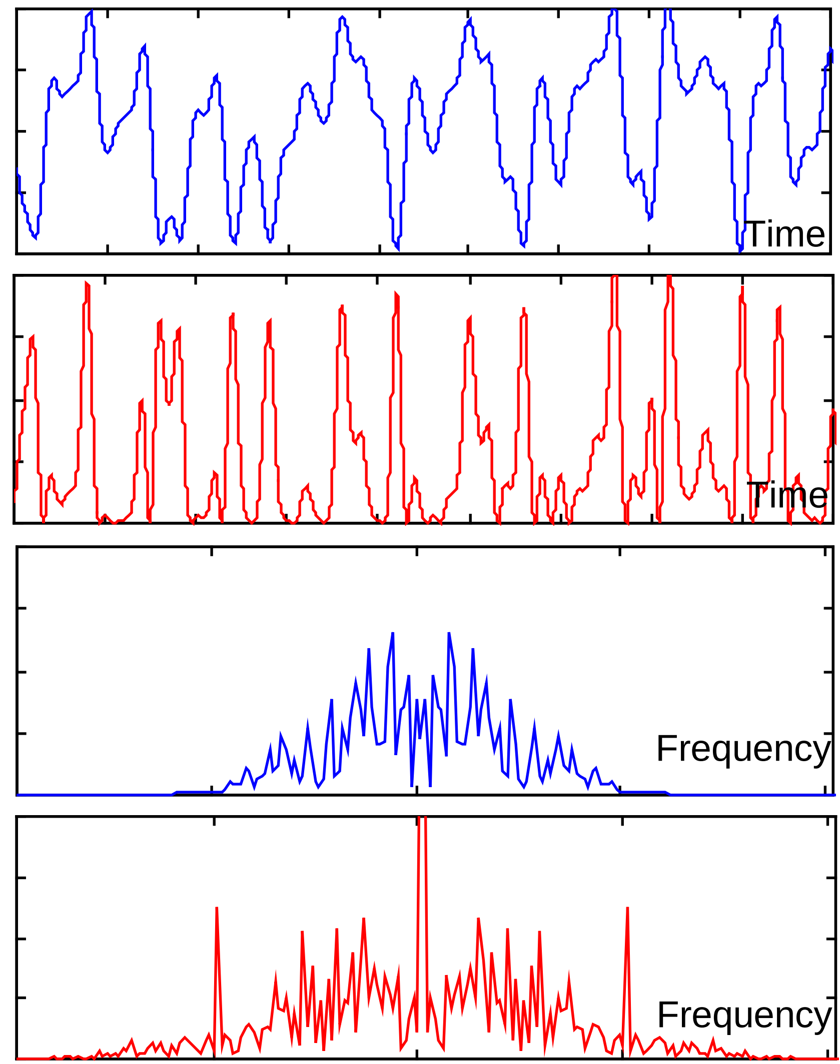
Signal

Signal Squared

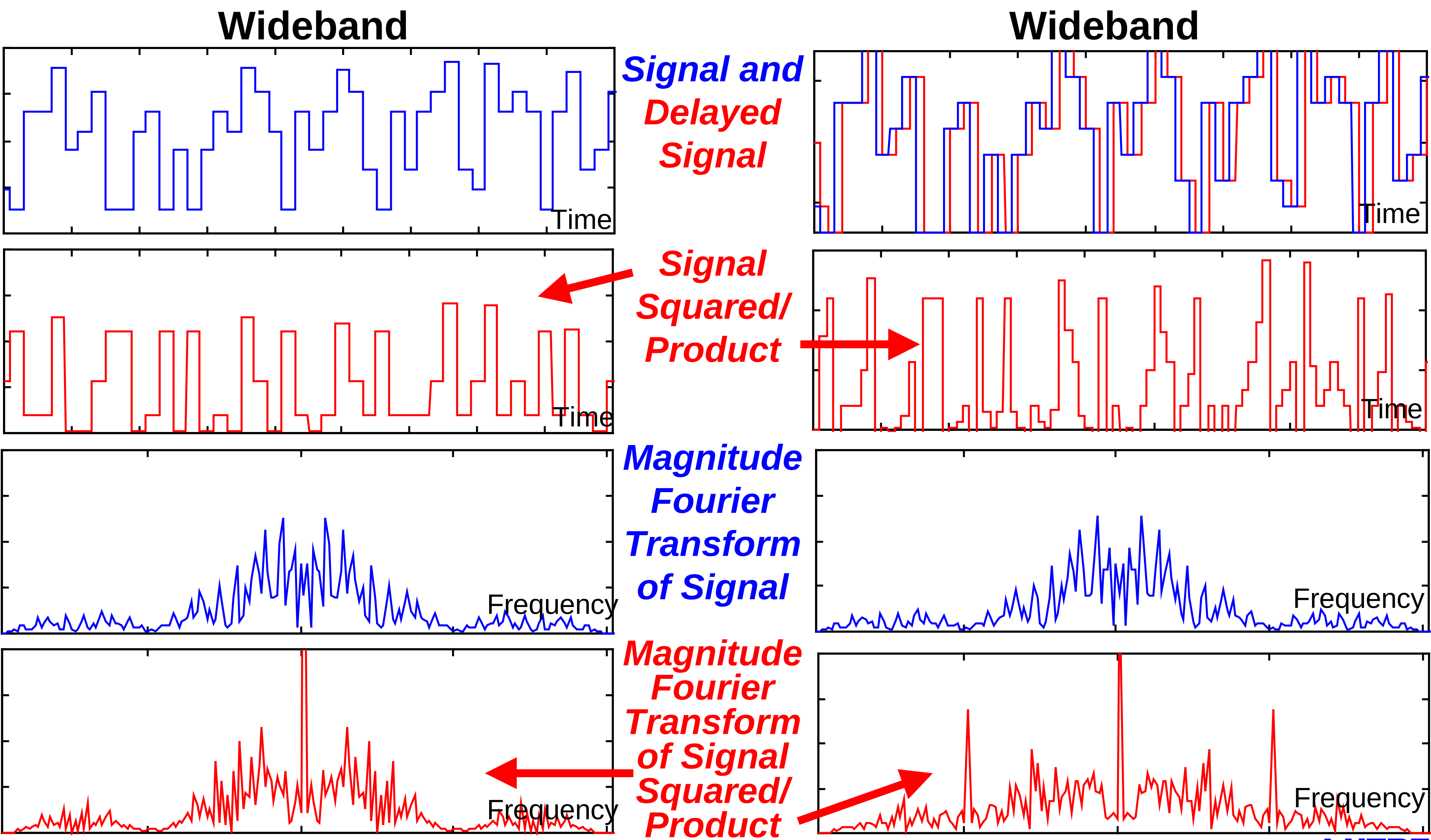
Magnitude Fourier Transform of Signal

Magnitude Fourier Transform of Signal Squared

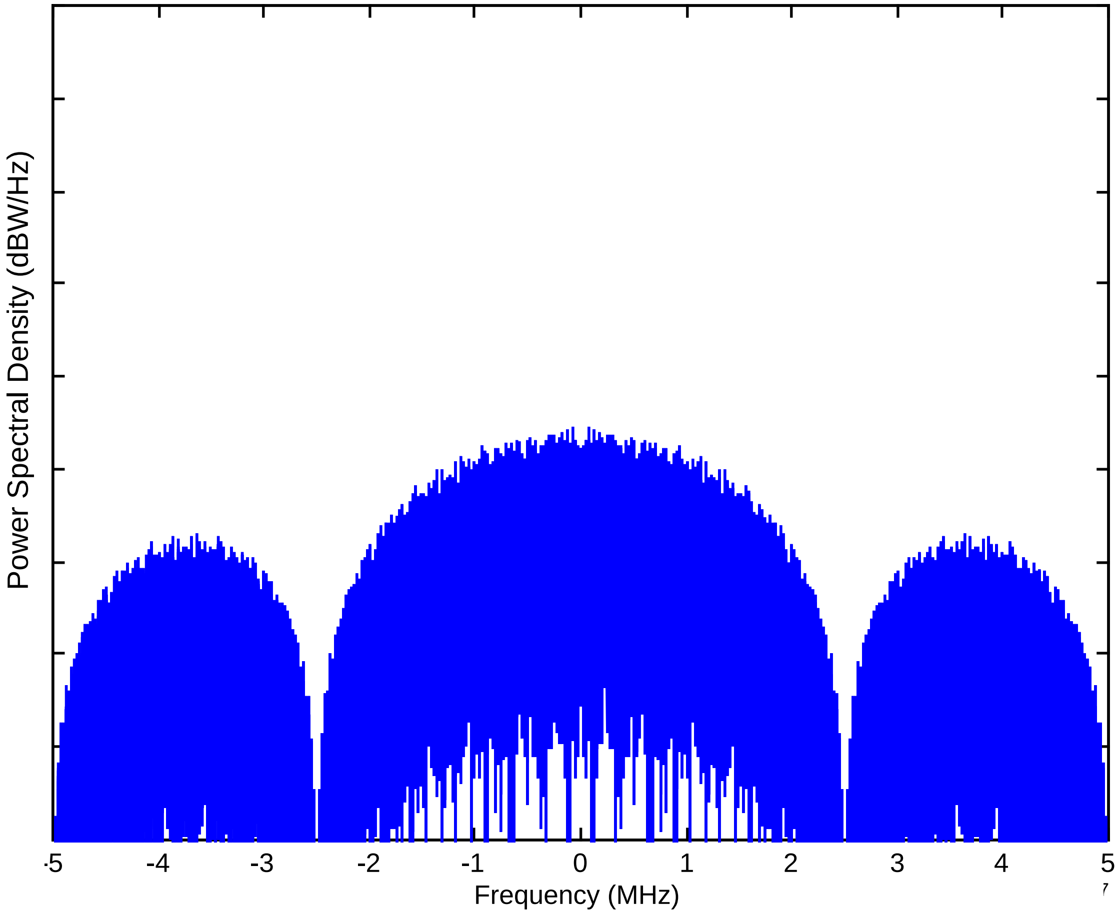
Null-to-Null Filtered



8-PAM with Random Data and No Trellis Coding, Delay Instead of Filtering



Spectrum of M-ary PSK Signal, 2.5 M symbol/s



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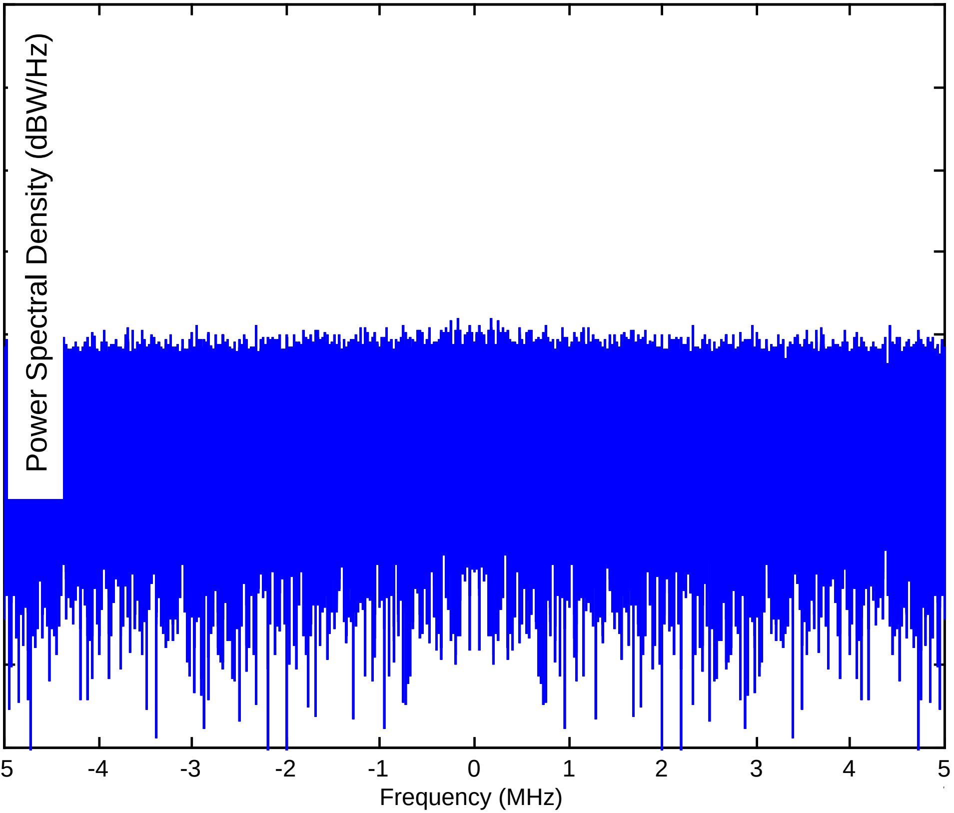
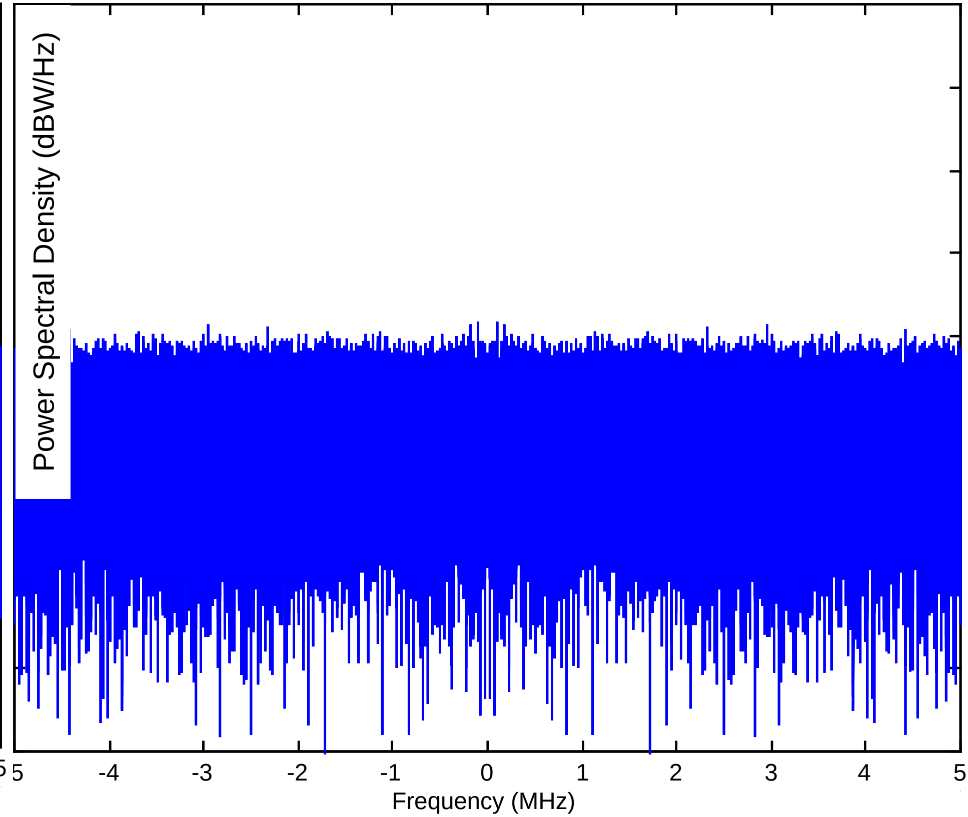
Spectrum of Noise Only and M-ary PSK Signal in Noise





- **Input SNR (Energy Per Symbol)/(Noise Density) is –10 dB**

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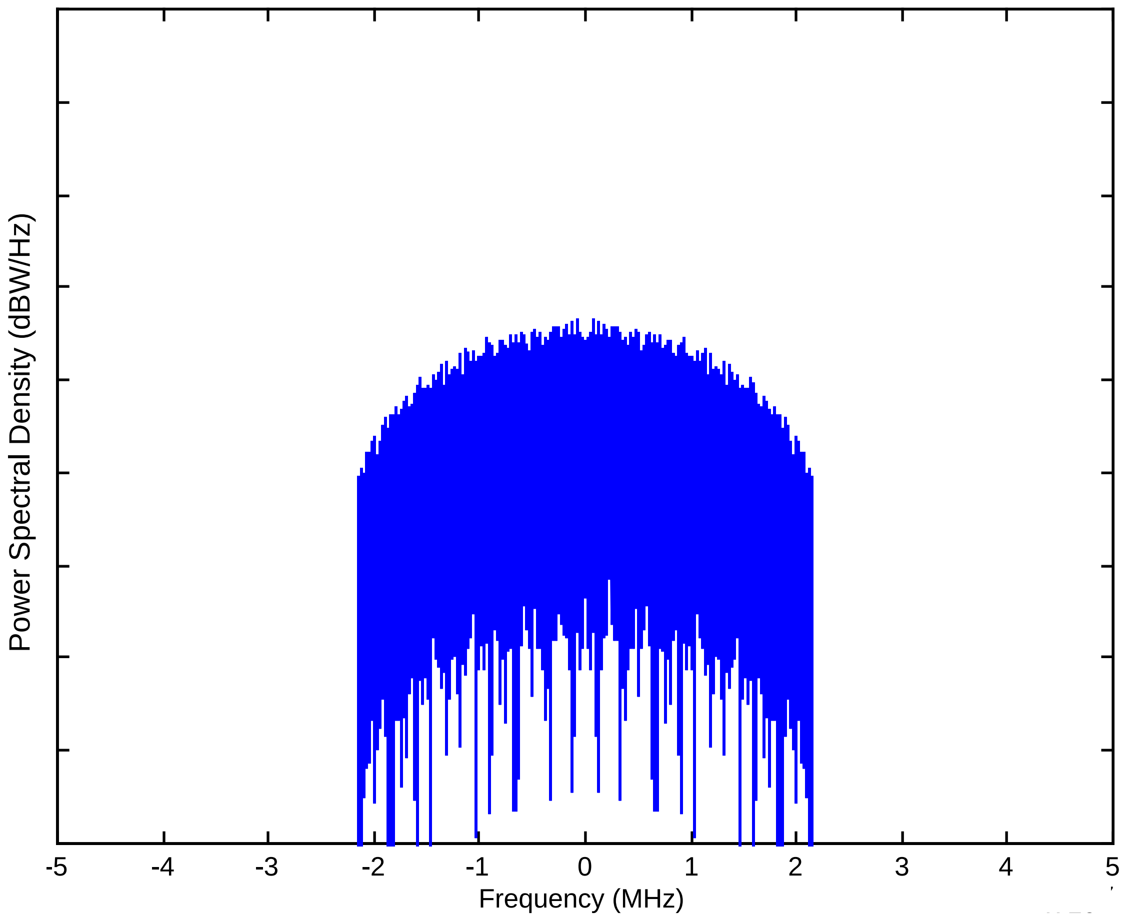
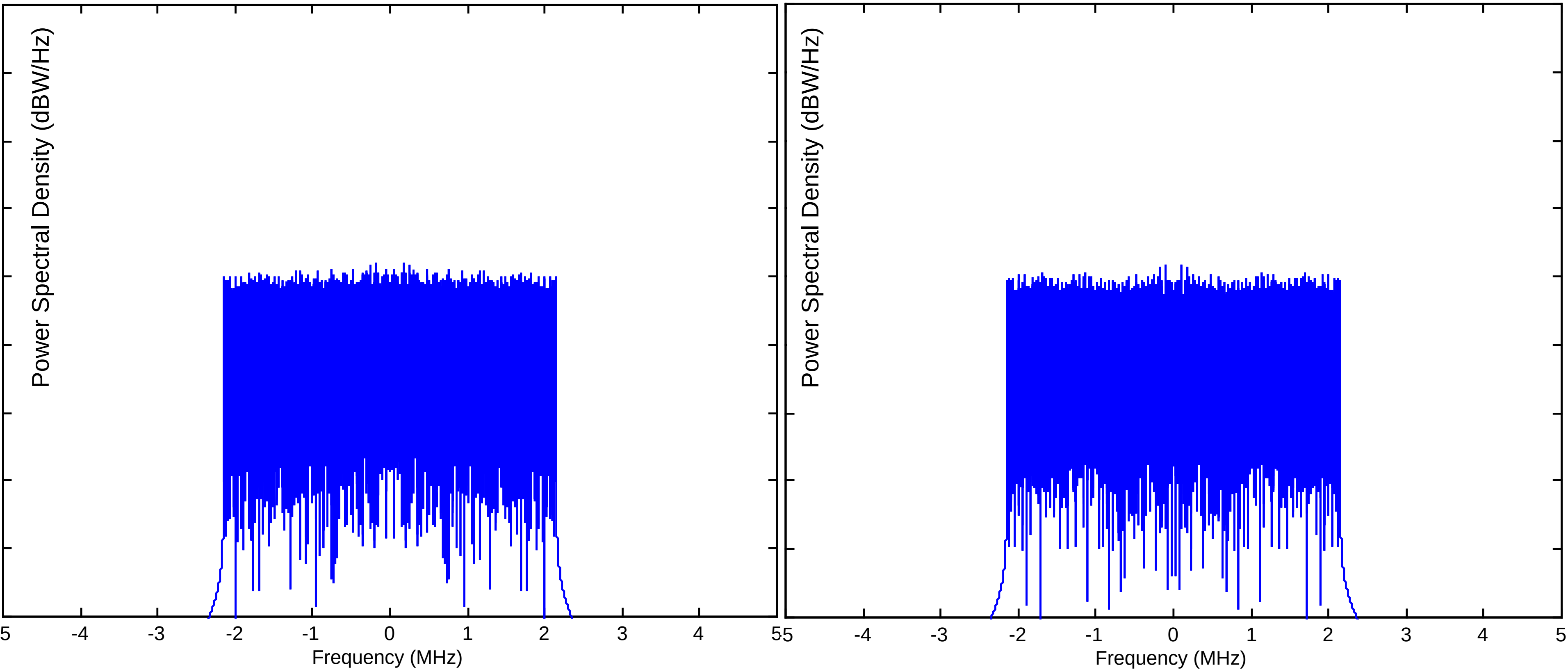
Spectrum of M-ary PSK Signal after Input Filter



Spectrum of Noise Only and M-ary PSK Signal in Noise after Input Filter



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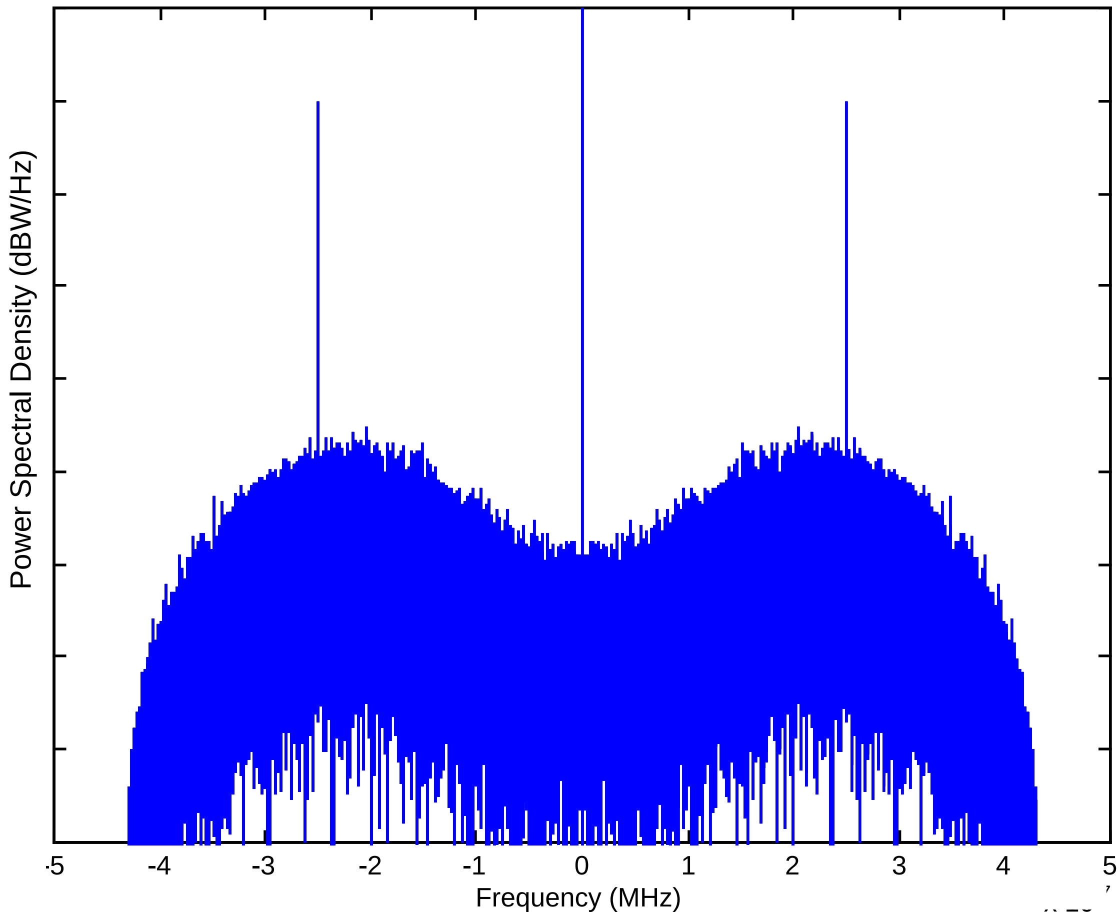
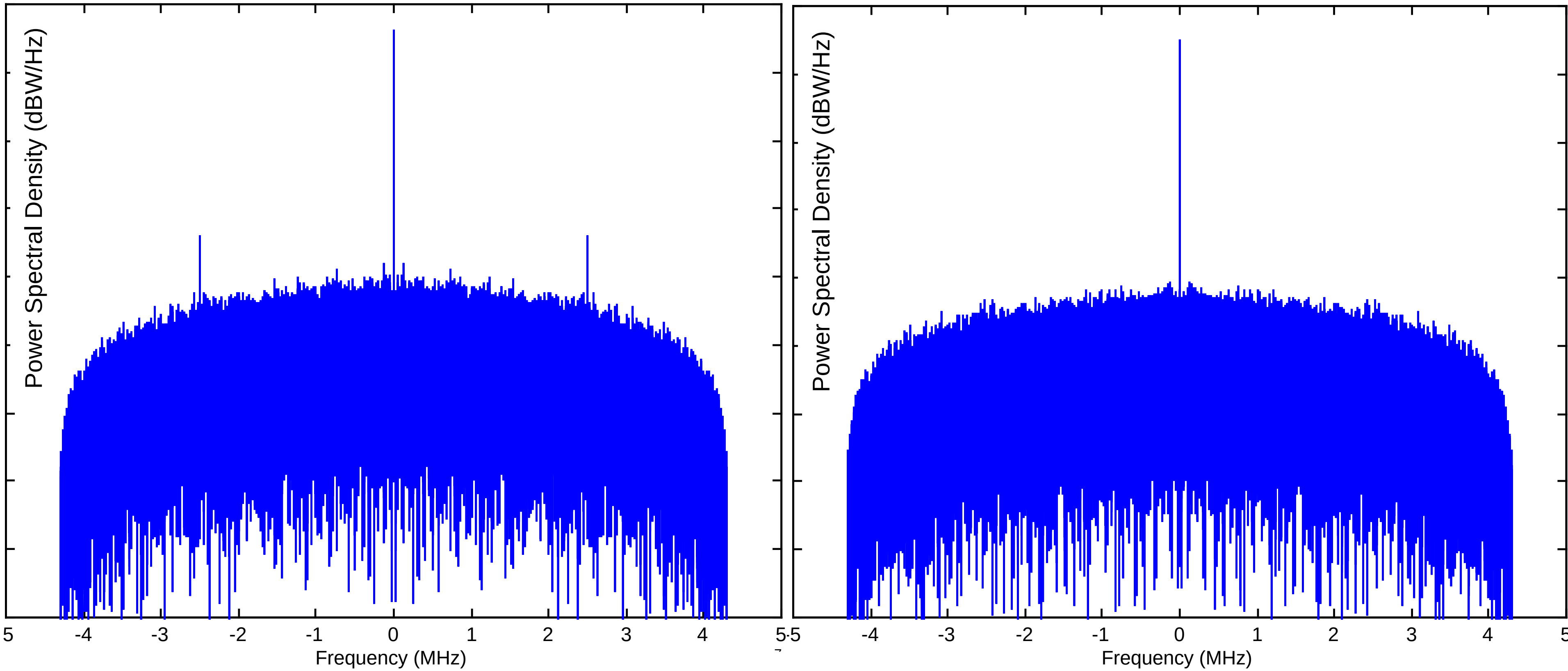
Spectrum of Filtered M-ary PSK Signal after Squaring



Spectrum of Filtered Noise Only and M-ary PSK Signal in Noise After Squaring



- **32768 symbols processed coherently, 3 noncoherent integrations**

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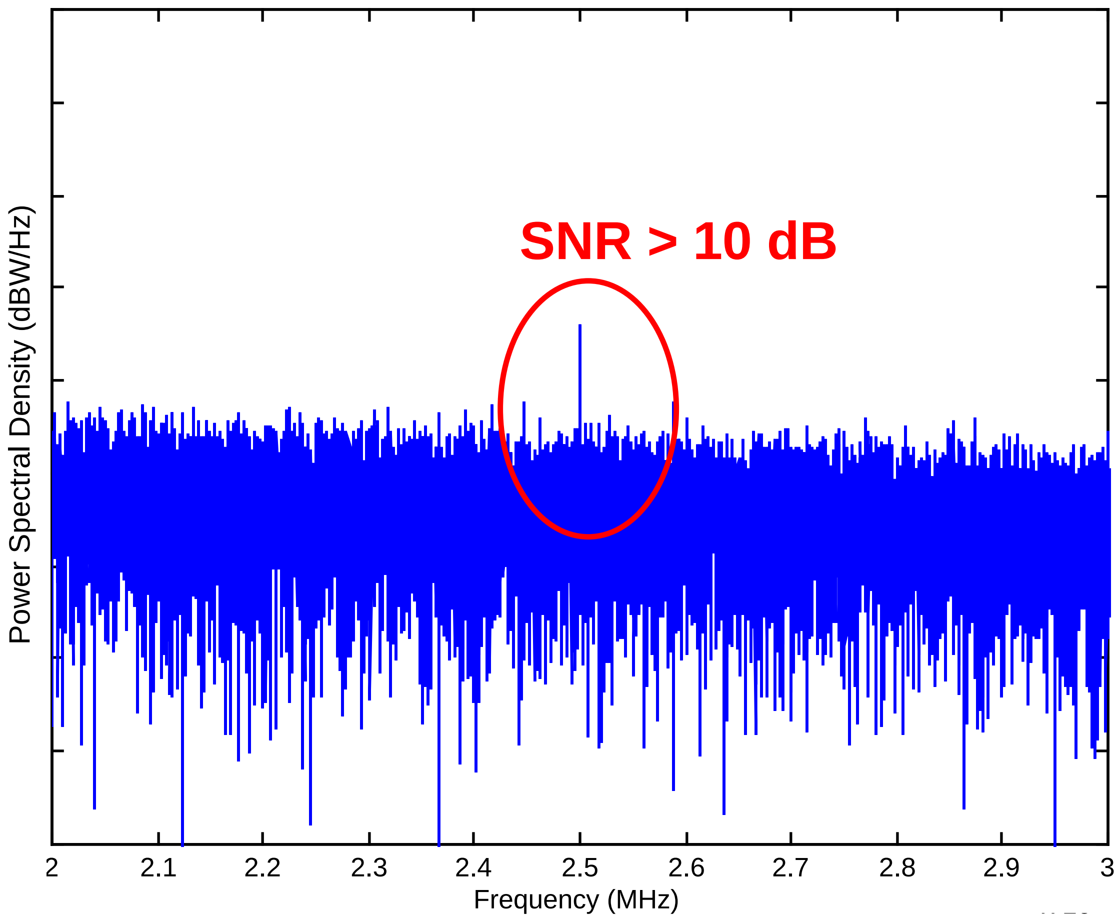
Spectrum of M-ary PSK Signal in Noise after Squaring



Feature Detector Design Process

- Using mathematical model of waveform, derive expression for cyclic correlation function
  - Account for modulation, filtering and equalization, statistics of data sequence
  - Example for bandlimited M-ary PSK with rectangular symbols and random data

$$z(t) = \sum_{k=-\infty}^{\infty} a_k p\left(t - kT_s\right)$$

$$R^1(t,\tau) = E\left\{z(t)z^*(t-\tau)\right\} = \sum_{k=-\infty}^{\infty} \chi_k^1(\tau) e^{i2\pi k\alpha^1 t}$$

$$\alpha^1 = \frac{1}{T_s}$$

$$\chi_k^1(\tau) = e^{-i\pi k} \int_{-B/2}^{B/2} \mathrm{sinc}\left[\pi f T_s\right] \mathrm{sinc}\left[\pi\left(fT_s + k\right)\right] e^{i2\pi f\tau} df$$

Feature Detector Design Process (Concluded)

- Identify and select cycle frequency to detect
  - Can repeat for different cycle frequencies
- Derive expression for output SNR after narrowband detector at selected cycle frequency, in terms of input filter and delay
- Optimal detector:
  - Input filter found from application of generalized Schwartz Inequality
  - Any delay is incorporated in transfer function of optimal input filter
- Suboptimal detector uses input filter with rectangular passband
  - Numerical search finds bandwidth and delay that jointly maximize output SNR
- Select coherent and noncoherent integration times
  - Determine relationships between input SNR, output SNR, and integration times
- Evaluate operating characteristics: detection and false alarm probabilities

General Expression for Output SNR for Detection in White Noise Using Coherent Narrowband Detector

$$\rho_o = \gamma \, N_s \, \rho_i^2$$

- ρ_o **is output SNR**
- γ **is a "processing coefficient" that depends on modulation type, choice of cycle frequency, selection of input filter and delay**
- N_s **is the number of cycle periods observed**
- ρ_i **is the input SNR: (signal energy over cycle period)/(noise density)**

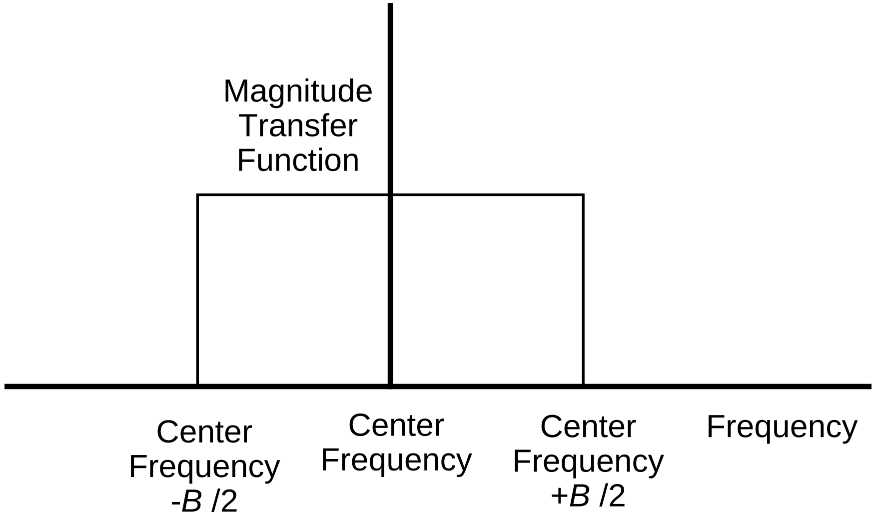
- **Expression applies for small input SNR**

Processing Coefficients for M-ary PSK Symbol Rate, Rectangular Symbols, Random Values

- **Detection of symbol rate** $1/T_S$ **in** $R^1(t,\tau)$**, where** T_S **is symbol period**
- **For optimal input filter**

$$\gamma = \int_{-\infty}^{\infty} \operatorname{sinc}^2(\pi f)\operatorname{sinc}^2(\pi(f-1))df \approx 0.10$$

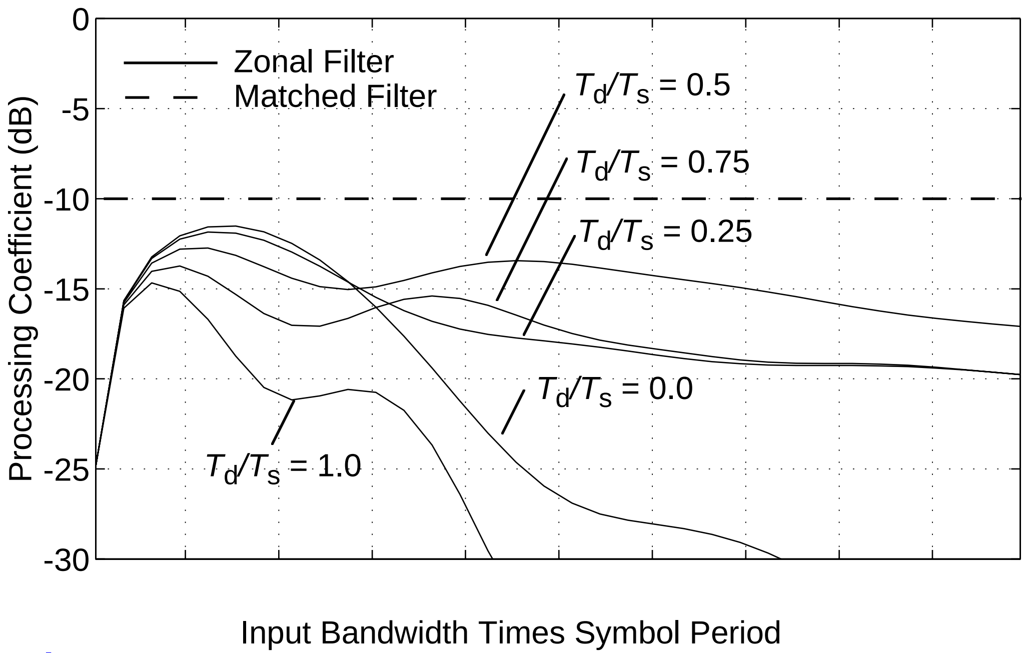
- **For rectangular input filter**



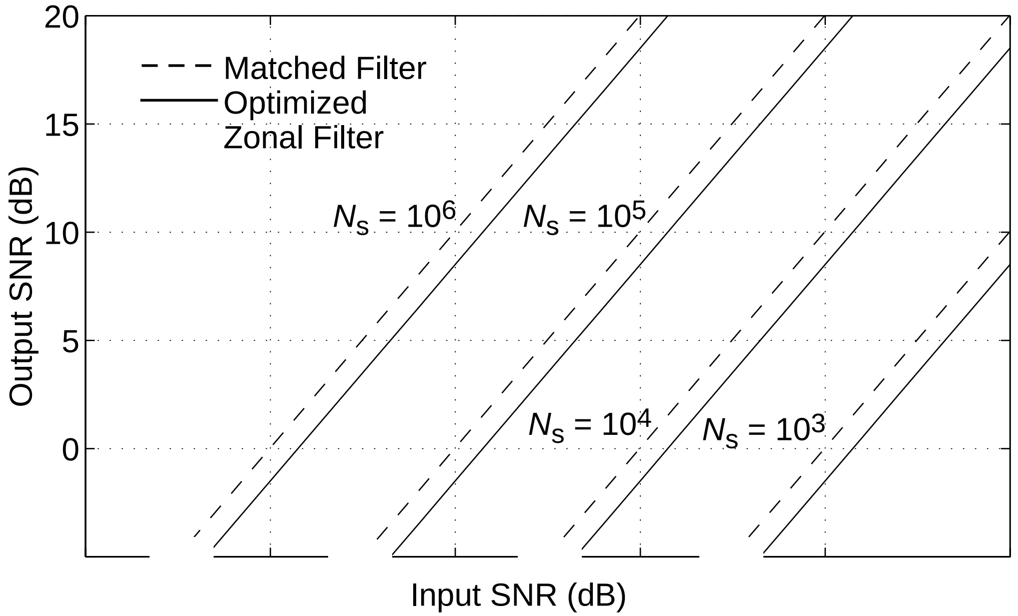
$$\gamma = \begin{cases} \dfrac{1}{BT_S - 1}\left| \displaystyle\int_{1-BT_S/2}^{BT_S/2} \operatorname{sinc}(\pi f)\operatorname{sinc}(\pi(f-1))e^{i2\pi f T_{\mathrm{d}}/T_S} df \right|^2, & BT_S > 1 \\ 0, & \text{elsewhere} \end{cases}$$

M-ary PSK Processing Coefficients for Symbol Rate

- Signal uses binary phase shift keying with rectangular symbols modulated by random (independent, equally likely) values
  - Processing coefficient with optimal input filter is –10 dB
  - Maximum processing coefficient with rectangular input filter is –11.5 dB
    - Input filter bandwidth is ~1.7 times reciprocal of symbol period
    - Delay is zero

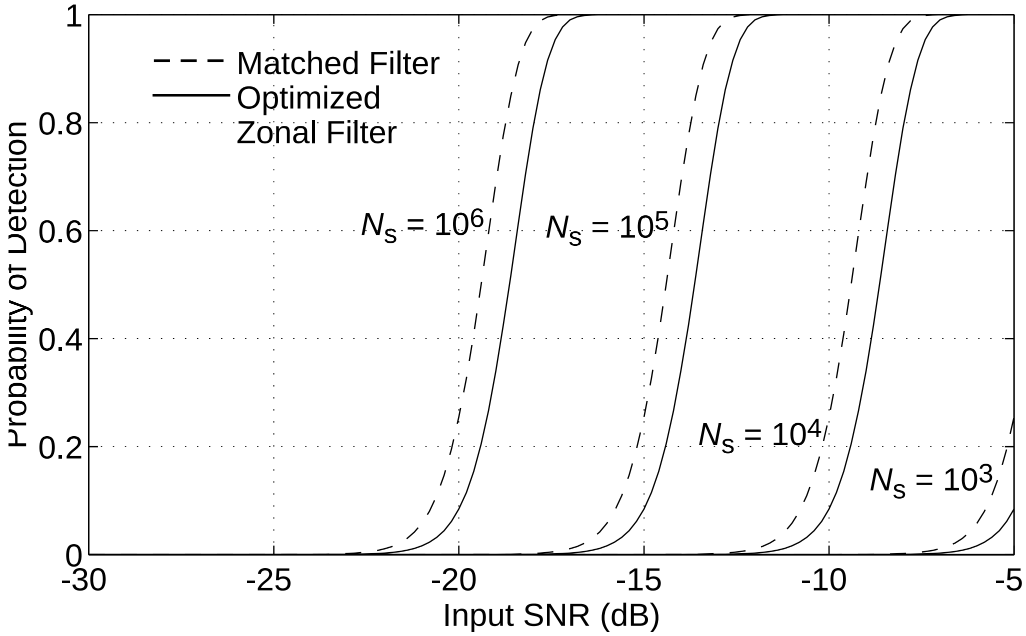


M-ary PSK Symbol Rate Detection Output SNR



M-ary PSK Detection Performance, False Alarm Probability 10^{-6}

- **When narrowband detector uses only coherent processing, resulting test statistic has Rayleigh/Rician distribution**



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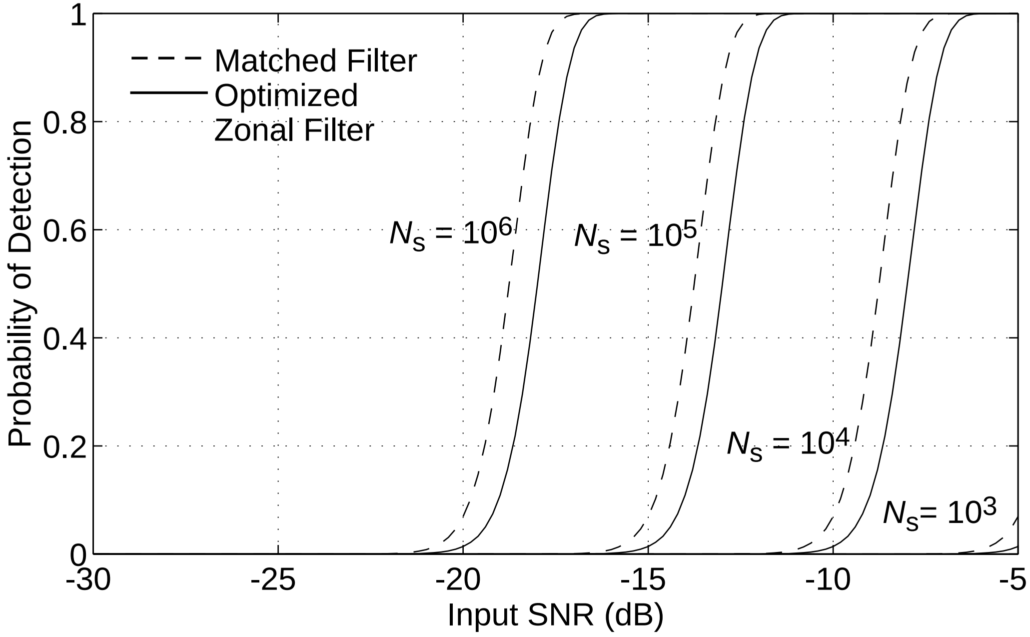
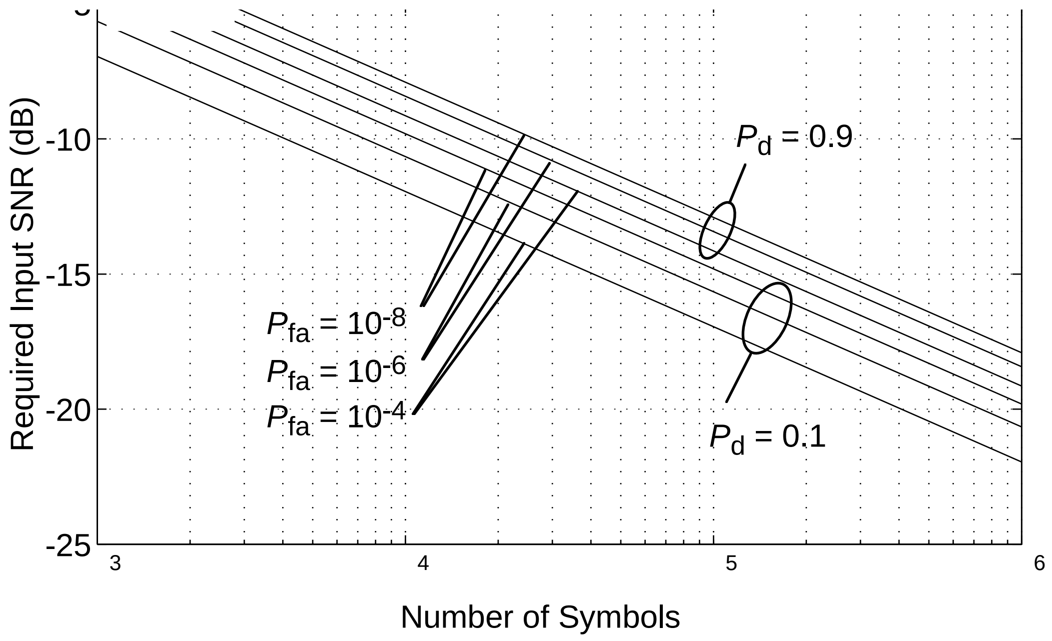
M-ary PSK Detection Performance, False Alarm Probability 10^{-8}



Relationship between Input SNR and Integration Time



Example Calculation of Detection Sensitivity

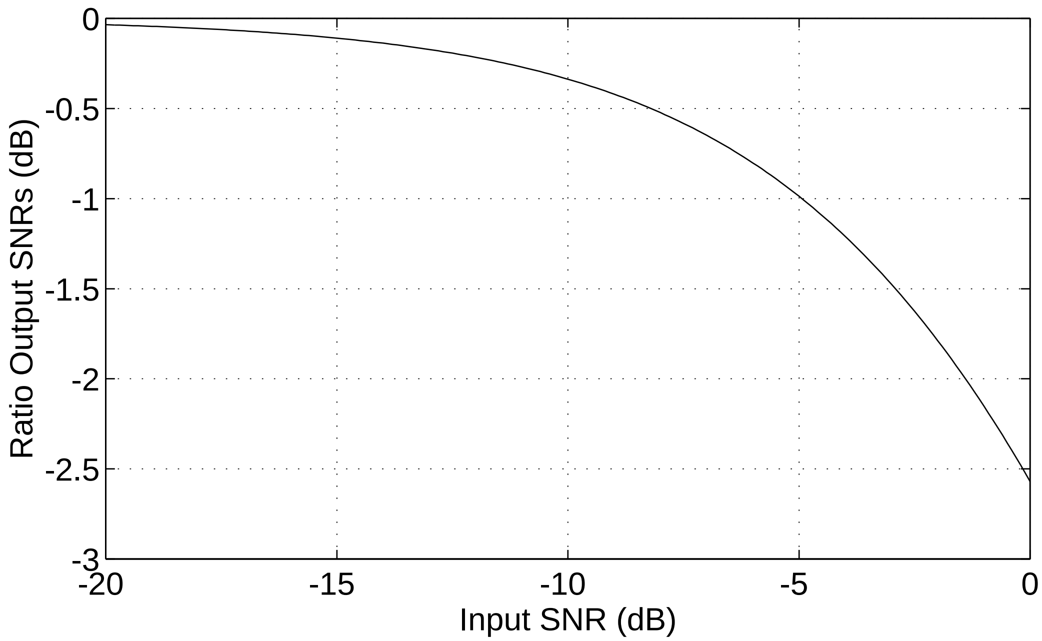
- M-ary PSK signal with symbol rate 10.79 MHz
- Produce 12 dB output SNR
  - Corresponds to detection probability near 0.9 with false alarm probability less than 0.1 over 1000 FFT bins
- Assume 5 dB implementation loss
- Coherent Integration time / Minimum Input SNR

Coherent Integration time	Minimum Input SNR
0.1 ms	–1.7 dB*
1 ms	–6.7 dB*
10 ms	–11.7 dB
100 ms	–16.7 dB

**Must confirm assumption of low input SNR*

Assumption of Small Input SNR

- As input SNR becomes larger and approaches 0 dB, actual output SNR is less than predicted by expressions that assume small input SNR
- Plot below shows ratio of actual output SNR to output SNR predicted under assumption of small input SNR



Feature Detection

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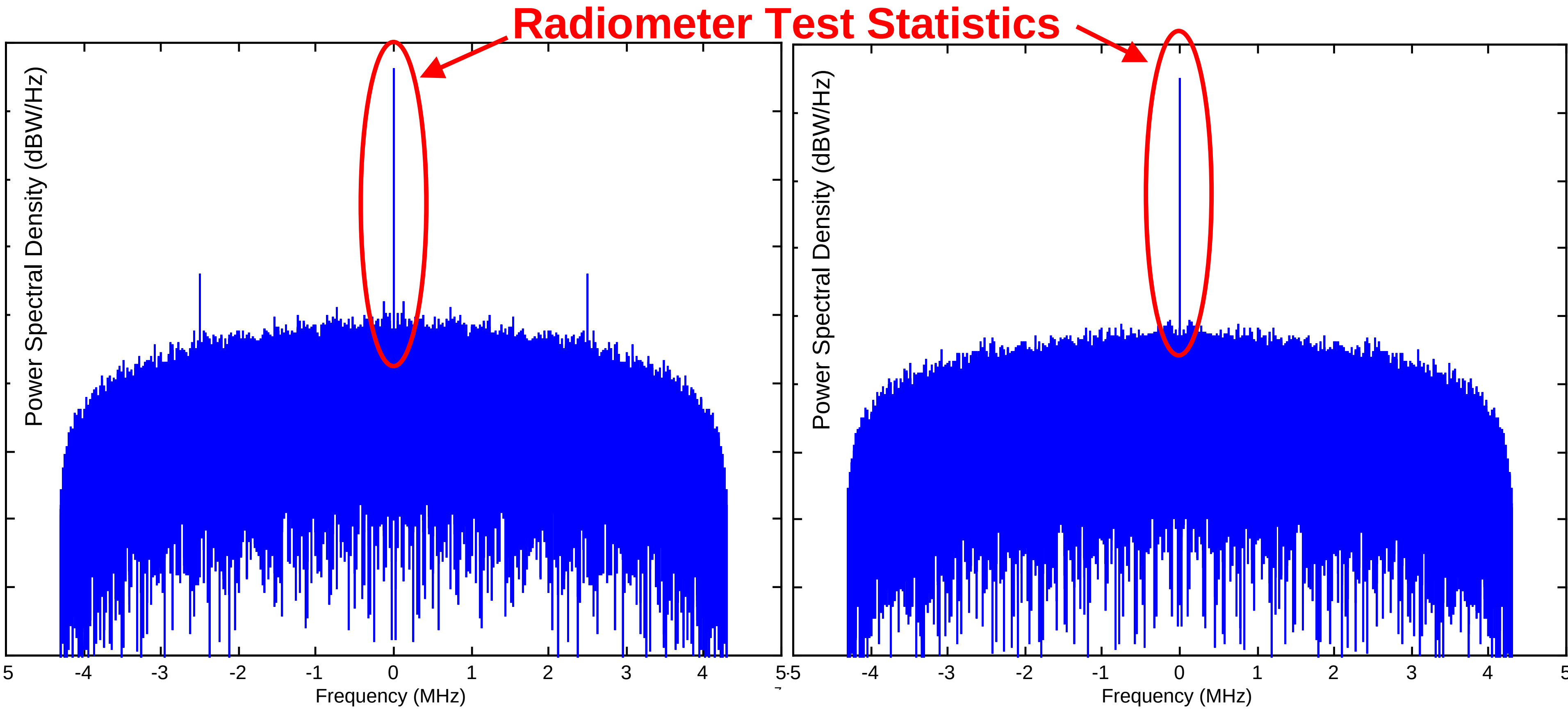
Why Use Feature Detection Over Radiometry?

- Radiometric detectors not very robust in detecting weak signals
  - Sensitive to uncertainty in the power of background noise
  - Sensitive to interference, and limited in ability to discriminate against it

Spectrum of Filtered Noise Only and M-ary PSK Signal in Noise After Squaring



- **32768 symbols processed coherently, 3 noncoherent integrations**
- **Equivalent to processing 40 mseconds of data**

Issues to Consider in Cyclostationary Feature Detection

- Implementation complexity
  - Analog hardware versus digital hardware versus DSP
  - Storage
- Signal characteristics
  - Excess bandwidth needed to produce cyclostationarity
  - Filtering
  - Equalization
- High sensitivity requires long integration times
  - Practical issues
  - Use of coherent/noncoherent integration times
- Channel effects
  - Coherence bandwidth
  - Coherence time
- Interference
- Frequency uncertainty
- Antennas

Feature Detection

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Summary

- Feature detection enables determining signal presence without demodulation
- Keyed signals can be represented as cyclostationary processes
- Cyclostationary feature detectors can detect with SNRs below 0 dB
  - Square-law relationship between integration time and input SNR at low input SNRs
  - Trade sensitivity for integration time
- Cyclostationary feature detector design methodology well-known
- Cyclostationary feature detector performance prediction well-known
- Applicability of cyclostationary feature detectors to listen-before-talk protocols involves many system-level trades
  - Practical issues in cyclostationary feature detection
  - Alternative detectors
  - Propagation

Selected References

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